DISCUSSION PAPERS IN ECONOMICS

Why Branded Firms May Benefit from Counterfeit Competition

Why Branded Firms May Benefit from Counterfeit Competition

Yucheng Ding

October, 2014

Abstract

A durable-good monopolist sells its branded product over two periods. In period 2, when there is entry of a counterfeiter, the branded firm may charge a high price to signal its quality. Counterfeit competition thus enables the branded firm to commit to a high price in period 2, alleviating the classic time-inconsistency problem under a durable-good monopoly. This can increase the branded firm's profit by encouraging consumer purchase without delay, despite the revenue loss to the counterfeiter. Total welfare can also increase, because early purchase eliminates delay cost and consumers enjoy the good for both periods.

JEL: D82, L11, L13

Keywords: Counterfeit, Durable Good, Quality Signaling

University of Colorado Boulder, yucheng.ding@colorado.edu. I am very grateful for Yongmin Chen's advice and encouragement. I would like to thank Oleg Baranov, Jin-Hyuk Kim and Keith Maskus for many helpful comments and discussions. All errors are mine.

1 Introduction

Counterfeits have become a fast growing multi-billion dollars business. In the 2007 OECD counterfeit report, the volume of counterfeits was around 200 billion dollars in international trade, 2% of world trade.[1] This figure does not include domestic consumption of counterfeits or digital products distributed via internet. The U.S. government estimated that counterfeit trade increased more than 17 fold in the past decade (U.S. Customs and Border Protection 2008).

Counterfeits are generally viewed as harmful to both the authentic producers and consumers, especially when they are deceptive, such as counterfeits of pharmaceutical products, eyeglasses, luxury goods or even normal textile products of famous brands.[2]. However, some recent empirical evidence suggests that (deceptive) counterfeits could actually benefit the branded firm. In particular, Qian (2008) finds that the average profit for branded shoes in China is higher after counterfeit entry. Qian (2011) provides further evidence that the impact of counterfeits on profit depends on the quality gap between the authentic good and the counterfeit good; the branded firm benefits from counterfeits when the quality gap is sufficiently large. In this paper, I provide a theoretical explanation of why a branded firm can indeed benefit from competition of a deceptive counterfeiter when the quality difference of their

[1] The Economic Impact of Counterfeiting and Piracy `http://www.oecd.org/industry/ind/38707619.pdf`

[2] This does not mean consumers cannot distinguish products at all. It is just hard for buyers to tell whether the good is authentic without any other information. For example, a consumer may not be able to separate a genuine Chanel bag from a fake one only by appearance. However, if one is priced at $3,000 and the other is sold for $50, she will know that the expensive one is more likely to be authentic ex post. On the other hand, non-deceptive counterfeits are those that consumers can easily recognize when purchasing, such as digital products.

products is large enough.

I consider a model with an authentic durable-good firm which sells in two periods. Without counterfeits, the branded durable-good monopolist faces the classic time-inconsistency problem (Coase, 1972): after selling to high-value consumers in the first period at a high price, it cannot resist cutting its price in the second period. But then rational consumers will delay their purchase, forcing the monopolist to reduce its price in the first period and lower the monopolist's overall profit. Now suppose that a counterfeiter will enter the market in the second period. In order to separate its product from counterfeits, the branded firm needs to set a high price to signal its quality. Thus the presence of counterfeits enables the branded firm to commit to a high price in period 2, providing a solution to the time-inconsistency problem. This then motivates more consumers to purchase in period 1 instead of waiting to buy in period 2, even if the first-period price is high. When the quality gap is sufficiently large, this "front-loading" effect will dominate the profit loss from competition in the second period. In terms of total welfare, counterfeits are likely to decrease surplus in the second period; however, first-period welfare increases due to front loaded purchases. Early purchases contribute twice the surplus compared to late purchases because consumers can use the good for two periods. Therefore, if the quality gap is not too large, it is possible for counterfeits to increase welfare.

The results in this paper shed light on the policy towards counterfeits. Both branded firms and consumers respond to counterfeits strategically. In the model, the authentic firm separates itself from the counterfeiter through high price when the quality gap is large enough. Therefore, consumers will not be fooled by counterfeits with extremely low quality. Moreover, knowing the later counterfeit entry, consumers are more inclined to purchase early, which benefits both the authentic firm and total welfare in a dynamic context.

The existing literature has investigated varies strategies by the durable-good mo-

beginning of the game. However, they are not able to tell which good is produced by the branded firm from its appearance before their purchase.[4] This contrasts with the standard assumption that consumers can trace the producer of the good.

There is a unit mass of heterogeneous consumer indexed by the taste parameter $\theta_l \sim U[0;1]$. Consumer's utility has the linear function form:

$$U_l = \theta_l Q_i - p_i; i \in \{A;C\}; \text{ where } p_i \text{ is the price of firm } i$$

The discount factors of both firms and consumers are assumed to be 1.

Let $\mu_i(p_A;p_C)$ be the probability that consumers believe the good from firm i is the authentic good, given p_A and p_C. Unlike the traditional monopoly signaling model, there are two signal senders here. Consumer belief is based on price and the number of firms charging that price. Consumers are aware that two firms sell the good and one of them is the counterfeiter. Thus, $\mu_A(p_A;p_C) + \mu_C(p_A;p_C) = 1$ in equilibrium. In a pooling equilibrium, where $p_A = p_C$, consumers cannot separate two products and $\mu_A = \mu_C = \frac{1}{2}$. In a separating equilibrium, where $p_A \neq p_C$, consumers believe that the expensive good is authentic and the cheap one is counterfeit.

Given consumer's belief, the firm's profit is represented by

$$\pi^k_{it}(p_A;p_C;\mu_i); t \in \{1;2\}; k \in \{P;S\}$$

The subscript i, t stands for firm type and time respectively. We use the superscript k to denote equilibrium values in the second period (P for Pooling Equilibrium and Sstuc28(th-2689tha for Separating Equilibrium). Also, assume that the separating equilibrium is selected when pro ts are the same for a separating and a pooling equilibrium.

The time-line of the game is as follows: the authentic rm sets the rst-period price p_1 in $t = 1$. Consumers decide whether to buy or wait. The counterfeiter enters in $t = 2$ and both rms set prices simultaneously. Then consumers observe both prices and make a purchasing decision based on their beliefs.

Before analyzing the game with counterfeit competition, let's rst review the benchmark monopoly model without entry.[5]

(i) When the monopoly lacks commitment power, it has an incentive to decrease the price to reap the residual demand in t

This gives a optimal profit $= \frac{1}{2}$ and $_1 = \frac{1}{2}$. The profit in no commitment case is lower because of the standard time-inconsistency problem: high valuation consumers will anticipate the price reduction in the future and some of them postpone purchase to the second period.

3 Equilibrium Analysis With Counterfeit Competition

In this section, I will first characterize the set of Perfect Bayesian Equilibrium (PBE) under counterfeit competition. I then show that there exists an equilibrium at which the counterfeit can increase the authentic firm's profit and social welfare.[6]

Standard backward induction is applied to analyze the counterfeit game. As in the benchmark, there is a marginal consumer $_1$, such that all consumers with taste parameter above $_1$ will purchase in the first period. The remaining consumers may purchase in the second period. $_1$ can be interpreted as the market size of the second period.

3.1 Signaling Game in Second Period

In t=2, there is a signaling game played between a pair of vertically differentiated firms and consumers. Consumers use market prices to update their beliefs. If both firms have the same price, counterfeits are indistinguishable ex post and a pooling equilibrium is sustained. If the counterfeiter sets a lower price than the branded firm and reveals itself, there will be a separating equilibrium where consumers know for

[6] In next section, I show all equilibria survive from the refinement have the desired result

products are close substitutes, the cost of signaling for the branded firm is so high that it would rather pool with the counterfeiter.

As in other signaling games, this model also has multiple equilibria. In some pooling equilibria with low price, counterfeit competition is detrimental to the branded firm's profit. In this section, I will show that there exists an equilibrium in which both the authentic firm and the society benefit from counterfeit entry under certain conditions. In the next section, it is proved that all equilibria surviving from the Competitive Intuitive Criterion refinement have similar properties.

The equilibrium I will focus on here is the one with the highest second-period profit for authentic firm, which is defined as the profit-maximizing equilibrium. It seems reasonable that consumers will believe that the authentic firm will choose the price that maximizes its second-period profit. Therefore, consumers believe the firm charging that price is the authentic firm. If both firms set that price, the good has 50% probability to be genuine. Any other price indicates a fake product. This is the pessimistic belief that supports the profit-maximizing price in t=2. Formally, consumer belief is defined as follow.

$$\mu_i(p_{A2}, p_{A2}) = \frac{1}{2}; \quad \mu_A(p_{A2}, p_2) = 1, \forall p_2 \neq p_{A2};$$
$$\mu_A(p_2, \cdot) = \mu_C(\cdot, p_2) = 0, \forall p_2 \neq p_{A2}$$

In this section, an extra asterisk is used in superscript to denote variables in the profit-maximizing equilibrium. Let p^S_{A2} and p^P_2 be the authentic price in the optimal separating and pooling equilibriumd [(coubriu)1(md [(co(in)-d2a7g)-400ts(li7oter.)]277(0)]TJv4 of signaling game in t=2. $p_{A2} = p^S_{A2} = \underline{p_2}(\ _1; C)$, $\ _{A2} = \ ^S_{A2} = \frac{4(1\ C)^2(1\ C^2)}{C^2\ 3C+4}\ ^2_1$. (ii) If the counterfeit's quality is high ($C > C_3$), the pooling equilibrium will be selected. (a) For $C_3 < C \quad C_2 \quad 0{:}702$, $p_2 = p^P_2 \ = \frac{1+C}{4}\ _1$, $\ _2 = \ ^P_{A2} = \frac{1+C}{16}\ ^2_1$; (b) For $C > C_2$, $p_2 = p^P_2 \ = \underline{p_2}(\ _1; C)$, $\ _2 = \ ^P_{A2} = \frac{C(1+C)(1\ C^2)}{2(C^2\ 3C+4)^2}\ ^2_1$.

Figure 1 illustrates the second-period price scheme in the pro t-maximizing equilibrium. For $C\ 2\ [0; C_3]$, the price $\underline{p_2}(\ _1; C)$, which is the minimum price that prevents the counterfeiter from mimicking the branded rm, has an inverted-U shape with respect to C and is higher than the monopoly price in benchmark. The counterfeiter's pro t in the pooling equilibrium increases faster with C than its pro t in the separating equilibrium when C is close to 0.[8] Therefore, the authentic rm is forced to increase the price in order to reduce competition and increase the competitor's pro t in the separating equilibrium. As C gets larger, the condition will be reversed and the authentic rm has no need to incur a large distortion to support the separating equilibrium. Combining these two segments give us an inverted-U shape price in the separating equilibrium. When $C\ 2\ (C_3; C_2]$, the price increases with C because of higher expected quality. When C is close to 1, the game converges to Bertrand Competition of homogeneous good, and the price goes down to 0.

3.2 The Dynamic Game

In this subsection, I will analyze the dynamic game and illustrate why the entry of counterfeiter may generate higher pro t for the incumbent. Given the second-period consumer surplus and the rst-period price, the marginal buyer in the rst period will be determined. The authentic rm's decision is to choose this marginal consumer to maximize total pro t.

[8]When C is close to 0; $\frac{d\ ^P_{C2}}{dC} = \frac{1}{(1+C)^2}p^2_{A2} \quad \frac{d\ ^S_{C2}}{dC} = \frac{1}{4(1\ C)^2}p^2_{A2}$.

The authentic rm's maximization problem is:

$$\max_{1} \ {}^{P}_{A}(\ _1) = (1 \quad _1)(2\ _1 \quad \frac{1+C}{2}\ _1 + p_2^P\ ) + \frac{1}{2}(\ _1 \quad \frac{2p_2^P}{1+C})p_2^P$$

The marginal buyer $_1^P$ and equilibrium pro t $_A^P$ are:

$$_1^P = \begin{cases} \dfrac{1+\frac{3\quad C}{4}}{2(1+\frac{11\quad 5C}{16}} \quad) \\ \end{cases}$$

3/F17 11.9552 Tf16.054 3.12668Td (1)]TJ/F14 11.3552 T producer, the branded rm takes a larger share of the total pro t compared to the head-to-head competition in the pooling equilibrium. The mechanism of the front-loading e ect is slightly di erent. Consumers will not be fooled ex post but face a super monopoly price in the second period as Lemma 2 indicated. Now, the marginal buyer $^{P}_{1}$ faces two options in the second period | buy the authentic good or the counterfeit.

$$2\ {}_1 \quad p_1 = \max\{\ {}_1 \quad \underline{p_2}(C;\ {}_1);\ C\ {}_1 \quad \frac{C}{2}\underline{p_2}(C;\ {}_1)\}$$

However, the buyer who is indi erent between a genuine product and a counterfeit in the second period must below ${}_1$. Therefore, the outside option is purchasing the authentic good in t=2. The incumbent's pro t maximization is as follow.

$$\max_{1} \quad {}_A(\ {}_1) = (1 \quad {}_1)(\ {}_1 + \underline{p_2}(C;\ {}_1)) + \ {}^S_{A2}(\ {}_1)$$

In equilibrium,

$$^S_1 = \frac{1 + \frac{2(1 \quad C^2)}{C^2 \quad 3C+4}}{2[1 + \frac{2(1 \quad C^2)(\quad C^2+C+2)}{(C^2 \quad 3C+4)^2}]}$$

$$^S_A = \frac{[1 + \frac{2(1 \quad C^2)}{C^2 \quad 3C+4}]^2}{4[1 + \frac{2(1 \quad C^2)(\quad C^2+C+2)}{(C^2 \quad 3C+4)^2}]}$$

The left segment of lower curve in Figure 2 informs that $^S_1$ monotonically decreases with C. As the quality gap closes, the branded rm's pro t in the second period decreases. It would be better to assign less weight in the second period by decreasing $^S_1$.

Pro t Comparison

Proposition 1. In the pro t-maximizing equilibrium, the authentic rm's pro t will be higher than the monopoly benchmark if the counterfeit quality is su ciently

Figure 2: Marginal Buyer in t=1

low ($C < C_4 \approx 0.188$). When the counterfeit quality is above that threshold, at any equilibrium in the second period, competition always decreases the incumbent's profit.

Figure 3 illustrates Proposition 1: when the pooling equilibrium emerges in the second period, the competition effect is too strong and always dominates the front-loading effect. The authentic firm suffers from the counterfeit entry. In the first segment of the pooling equilibrium, the front-loading effect gets weaker when the quality increases (θ_1^P increases with C) and the time-inconsistency problem is reinforced. However, the high-quality counterfeit also weakens the competition effect and raises the second-period profit. In the second segment, the competition effects gets too strong and the front-loading effect disappears.

However, if the separating equilibrium is sustained, the branded firm's profit has

In the monopoly benchmark, total surplus is given by the following equation.

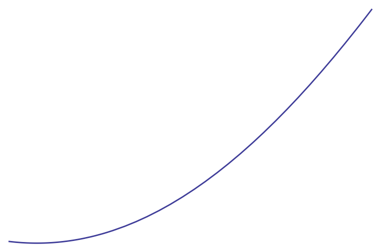
$$TS^{M} = \int_{{}_1^M}^{1} 2\, d + \int_{{}_2^M}^{{}_1^M} d$$

The rst (second) term represents the surplus created by rst (second) period transaction[9]. The total surplus decreases with $_1$, which is implied by the fact that early buyer enjoy double surplus. Given the marginal buyer in each period, $TS^{M} = 0{:}775$.

The welfare in the presence of deceptive counterfeit competition is a piecewise function.

$$TS(C) = \begin{cases} TS^{S}(C) = \int_{{}_1^S}^{1} 2\, d + \int_{-{}_2^S}^{{}_1^S} d + \int_{-{}_2^S}^{-{}_2^S} C\, d & \text{if } C \quad C_3 \\ TS^{P}(C) = \int_{{}_1^P}^{1} 2\, d + \int_{{}_2^P}^{{}_1^P} \frac{1+C}{2}\, d & \text{if } C > C_3 \end{cases}$$

Figure 4: Welfare Difference



typical criticism against counterfeits. However, if the first-period welfare is taken into account, the result will be quite different. As Figure 2 shows, there are always more sales in $t = 1$ once $C > 0$. The front-loading effect pushes consumers to buy in $t = 1$ either because the higher price or lower expected quality in $t = 2$. The

The left segment is the welfare in the separating equilibrium. In Figure 2, as the counterfeit quality improves, the positive effect increases with C roughly at the same speed ($\frac{d^2 \, {}_1^S}{dC^2}$ is close to 0). The second-period welfare decreases because of upward distorted prices. Since the second-period price has an inverted-U shape, the welfare in that period will be an U shape curve. Combining these two effects, it is clear why total welfare also has a U shape. When the counterfeit quality is 0, the model coincides with the benchmark. When C is small, unlike the pooling equilibrium, ${}_1$ is close to the benchmark value and decreases slower compared to the second-period welfare. Therefore, when the counterfeit quality is sufficiently low, the overall welfare effect is negative.

This proposition implies that deceptive counterfeits may have a positive effect on welfare in a dynamic context, which is contrary to the traditional argument. What is more surprising is that welfare is significantly higher when counterfeits are indistinguishable ex post. The result reminds us to think deeply about the counterfeit problem. Firstly, branded firms actively adopt strategies against clones. Although counterfeits are deceptive ex ante, whether they can be recognized ex post is endogenized. If the quality of clones is low, in which case consumer confusion induced by counterfeits has a strong negative effect on welfare, the authentic firm will signal by price and rational consumer will not be fooled. If consumers cannot distinguish counterfeits from authentic goods ex post, it must be that the quality gap is close enough. Even if consumers are diverted to counterfeits in that case, the welfare loss is relatively small. Secondly, consumers respond rationally to the problem. In the present paper, they are aware that surplus associated with future purchase is lowered by the counterfeit competition. Thus, more people buy earlier, which is beneficial for both the branded firm and welfare. However, as I point out, when the authentic firm decides to separate itself by distorted price, the counterfeiter can also charge a higher price in the second period. This \price collusion" created by quality signaling might decrease welfare.

4 Equilibrium Re nement and Robustness

The pro t-maximizing equilibrium discussed above is only one of equlibria in our model. In this section, the Intuitive Criterion (Cho and Kreps, 1987) is applied to re ne equilibria. Since there are two signal senders here, I will use a competitive version as Bontems et al. (2005) and Yehezkel (2008). We will show that all pooling equilibria are eliminated with a tiny adjustment. The re nement is not applicable to separating equilibria because both rms' prices are informative.[10] However, it is proved that our general conclusion that counterfeit competition may increase the branded rm's pro t and social welfare holds in all separating equilibria.

In previous discussion, both rms are assumed to have zero marginal cost. Now, let the authentic rm has a slightly higher marginal cost > 0 which is arbitrarily close to 0. This is just a tie-breaker that helps us to eliminate all pooling equilibria. By continuity of all functions in the paper, this modi cation will not alter any of our results except for the existence of pooling equilibria. For convenience, I only explicitly state this adjustment in the re nement.

[10] The Intuitive Criterion requires unilateral deviation. However, since the other rm charges the equilibrium price, consumers can use that information to construct the out of equilibrium belief. Therefore, I cannot simply assume a belief towards the deviating rm while the other one prices at the equilibrium path. Bester and Demuth (2011), Bontems et.al (2006) and Hertzendorf and Overgaard (2001) have discussed this issue.

4.1 Equilibrium Re nement

Pooling Equilibrium

The basic logic of the Intuitive Criterion is equilibrium dominance: an equilibrium should be eliminated if there exists an out-of-equilibrium price such that given consumer's most favorite belief, one type of rm would be better o by deviating from the equilibrium price to that out-of-equilibrium price, while the other type of rm cannot bene t from such deviation.

In terms of pooling equilibria, the Competitive Intuitive Criterion requires that there is no p^0, such that

$$ {}_{A2}(p^0; p_2^P; 1) \qquad {}_{A2}(p_2^P; p_2^P; \frac{1}{2}) \tag{3} $$

$$ {}_{C2}(p_2^P; p^0; 1) < \quad {}_{C2}(p_2^P; p_2^P; \frac{1}{2}) \tag{4} $$

However, for every pooling equilibrium, there must exist a p^0 such that both equations hold, which means all pooling equilibria are eliminated. The reason is similar to the re nement in the monopoly signaling game: The authentic rm with a higher marginal cost , no matter how small it is, has a lower cost to signal its quality. Since the the pro t function satis es single-crossing property, I can always nd an upward distorted price such that the authentic rm is willing to deviate to that price if consumers believe its high quality, while the counterfeiter is not willing to deviate even if people believe it produces genuine products at that price. The detail can be found in Proof of Proposition 3.

Separating Equilibrium

Since the Intuitive Criterion cannot be applied to separating equilibria, Hertzendorf and Overgaard (2001) and Yehezkel (2008) use a stronger re nement named

Resistance to Equilibrium Defections (REDE) to select the unique and most intuitive separating equilibrium in the duopoly signaling game, which is similar to the unprejudiced equilibrium in Bagwell and Ramey (1991).[11] Only the least distorted equilibrium survives that refinement, which is the profit-maximizing equilibrium investigated in the previous section. However, we don't need to impose that extra refinement since our main results hold in all separating equilibria, which is proved in next subsection.

the counterfeit quality is below C_4, the authentic rm's pro t is always higher with the presence of counterfeits, no matter which separating equilibrium emerges in the second period.

In terms of the impact on welfare, there is not such a nice monotonicity property among equilibria because the welfare in t = 2 may be too low when the price is
secon93(termi)-333(uition9(257)-292(58pact
high in that period. Howevey be toam4suc27(d.)-(ys)-39(e)-433(ialw)ed.1eatt285J/F34 7.27(d.)-(with)

Exchange.

Firstly, the incumbent has no incentive to provide an inferior good in the second period. Deneckere and MacAfee (1996) points out the linear utility function fails the condition that damaged goods help to raise profit. In my model, no matter what inferior quality the branded firm chooses, the optimal decision is to sell zero damaged version in t=2. The second-period price and profit are the same as monopoly benchmark. Since the price is not higher than the monopoly price, the front-loading effect does not exist. Therefore, the total profit can never be higher than the benchmark. If there is any fixed cost associate with product line introduction, the profit is always lower than the monopoly case.

Secondly, damaged goods introduced in the first period is not profitable as well. Hahn (2006) discusses the benefit of introducing damaged goods in durable-good model. In his paper, part of high (low) type consumers buy a high (low) quality good in each period, which changes the ratio of consumer type. Since some low types have purchased damaged goods in earlier period, the firm has less incentive to decrease price sharply later, which relaxes the competition between two versions and alleviates the time-inconsistency problem. However, with continuous consumer type, it can be proved that if anyone buys a damaged good in the first period, then all consumers with higher must purchase (a damaged or premium version) in that period as well. Therefore, introducing damaged version in $t = 1$ only makes some higher type consumers who would purchase the premium version select damaged version, which decreases profit for sure.[12] The mechanism that helps to solve Coase Conjecture in Hahn (2006) disappears in my model and the firm would rather just offer the original version.

[12] The detailed proof is available upon request

5 Conclusion

References

[1] K. Bagwell and G. Ramey. Oligopoly limit pricing. The RAND Journal of Economics, pages 155{172, 1991.

[2] H. Bester and J. Demuth. Signalling rivalry and quality uncertainty in a duopoly. Technical report, SFB/TR 15 Discussion Paper, 2013.

[3] P. Bontems, V. Meunier, and K. Universitet. Advertising and price signaling of quality in a duopoly with endogenous locationsInstitute of Economics, University of Copenhagen, 2005.

[4] J. I. Bulow. Durable-goods monopolists. The Journal of Political Economy, pages 314{332, 1982.

[5] D. A. Butz. Durable-good monopoly and best-price provisions. The American Economic Review, pages 1062{1076, 1990.

[6] Y. Chen and M. H. Riordan. Price-increasing competition. The RAND Journal of Economics, 39(4):1042{1058, 2008.

[7] R. H. Coase. Durability and monopoly. JL & Econ., 15:143, 1972.

[8] R. J. Deneckere and R. Preston McAfee. Damaged goods. Journal of Economics & Management Strategy, 5(2):149{174, 1996.

[9] C. Fluet and P. G. Garella. Advertising and prices as signals of quality in a regime of price rivalry. International Journal of Industrial Organization, 20(7):907{930, 2002.

[10] X. Gabaix, D. Laibson, and H. Li. Extreme value theory and the e ects of competition on pro ts. Massachusetts Institute of Technology, Working Paper, 2005.

[11] G. M. Grossman and C. Shapiro. Counterfeit-product trade, 1988.

[12] J.-H. Hahn. Damaged durable goods. The RAND Journal of Economics, 37(1):121{133, 2006.

[13] M. N. Hertzendorf and P. B. Overgaard. Price competition and advertising signals: Signaling by competing senders. Journal of Economics & Management Strategy, 10(4):621{662, 2001.

competition is intense. The unconstrained optimal is higher than $\underline{p_2}(\ _1; C)$. Since the pro t function is a concave parabola as well, $p_2^P \ = \underline{p_2}(\ _1; C)$.

As Lemma 1 indicates, when $C \quad C_1$, both types of equilibria exist and $\ _{A2} = Max \ f \ ^S_{A2}; \ ^P_{A2} g$. Given $C_1 < C_2$, the price of the optimal pooling equilibrium is $p_2^P \ = \frac{1+C}{4} \ _1$. Since $\frac{d \ ^S_{A2}}{dC} < 0$ and $\frac{d \ ^P_{A2}}{dC} > 0$, there is a cut-o quality $C_3 \quad 0{:}512$ such that the optimal separating equilibrium is chosen if $C \quad C_3$ and the pooling equilibrium would be selected for $C_3 < C \quad C_1$. :2When the counterfeit quality is0

If $p_2^P \ = \frac{1+C}{4}$,

$$\frac{d\ {}^P_A}{dC} = \frac{(1+\frac{3\ \ C}{4})(\ \ 2+\frac{35C\ \ 53}{64})}{(1+\frac{11\ \ 5C}{16})^2} < 0; 8C\ 2\ (C_3; 1)$$

Therefore, $8C\ \ \ C_3;\ {}^P_A(C)\ \ \ {}^P_A(C_3)$. Since ${}^P_A(C_3) < \ {}^M$, we have ${}^P_A(C) < \ {}^M$, $8C\ \ \ C_3$.

(2) When $C\ \ \ C_3$, the separating equilibrium is supported in the second period. If $C = 0$, the model is degenerated to the monopoly benchmark. ${}^M = {}^S_A$.

Let $\ (C) = {}^S_A\ \ \ {}^M$, then

total welfare. When $p_2^P = \frac{1+C}{4}\ {}_1$, ${}_2^P = \frac{1}{2}\ {}_1^P$.

$$TS^P(C) = \int_{{}_1^P}^{1} 2\ d\ + \int_{{}_2^P}^{{}_1^P} \frac{1+C}{2}\ d$$

$$= 1 \quad ({}_1^P)^2(1 \quad \frac{3(1+C)}{16})$$

$$TS^P(C) = TS^P(C) \quad TS^M(C) = \frac{5}{8}({}_1^M)^2 \quad ({}_1^P)^2(1 \quad \frac{3(1+C)}{16})$$

$$\frac{d \quad TS^P(C)}{dC} = \frac{8(1+C)}{(27 \quad 5C)^3} > 0$$

Therefore, $TS^P(C) \quad TS^P(C_3)$, $8C > C_3$. Since, $TS^P(C_3) > 0$, deceptive counterfeits always yield a higher welfare under the pooling equilibrium.

(2) In the separating equilibrium,

$$\bar{}_2^S = \frac{2 \quad C}{2(1 \quad C)}\underline{p_2}(C;\ {}_1);\ \underline{\ }^S_0$$

$_{A2}(p;p;\frac{1}{2})$ and $_{A2}(p + (1 \quad C) \ _1;p;1) = 0 < \ _{A2}(p;p;\frac{1}{2})$. Therefore, by the continuity of profit function, there must exist a $p < p^{0} < p + (1 \quad C) \ _1$ that makes $_{A2}(p^{0};p;1) = \ _{A2}(p;p;\frac{1}{2})$.

Plug p^{0} and Eq(3) into Eq(4),

$$
\begin{aligned}
& _{C2}(p;p^{0};1) \qquad _{C2}(p;p;\frac{1}{2}) \\
=& (\ _1 \quad \frac{p^{0} \quad p}{1 \quad C})p^{0} \quad \frac{1}{2}(\ _1 \quad \frac{2p}{1+C})p \\
=& (\ _1 \quad \frac{p^{0} \quad p}{1 \quad C})(\frac{p \quad p^{0}}{p}) < 0
\end{aligned}
$$

Hence, for every pooling equilibrium, there is a price p^{0} that the authentic firm wants to deviate and the counterfeiter does not given consumer's best belief.

Now let's make some preliminary definition for separating equilibria

$$
\underline{p_2}(C; \ _1) = \frac{2(1 \quad C^2)}{C^2 \quad 3C + 4} \ _1; \overline{p_2}(C; \ _1) = \frac{(4 \quad C)(1 \quad C^2)}{2(2 \quad C)(1+C) \quad C^2(1 \quad C)} \ _1
$$

For convenience, let

$$
\underline{K}(C) = \frac{2(1 \quad C^2)}{C^2 \quad 3C + 4}; \overline{K}(C) = \frac{(4 \quad C)(1 \quad C^2)}{2(2 \quad C)(1+C) \quad C^2(1 \quad C)}
$$

In any separating equilibrium, the authentic firm's price is between $\underline{K}(C) \ _1$ and $\overline{K}(C) \ _1$.

(ii) For incumbent's profit:

$$
\begin{aligned}
^{S}_{A} &= \frac{1}{4}\frac{[1 + K(C)]^2}{[1 + \frac{2 \quad C}{2(1 \quad C)}K(C)^2]} \\
\frac{@ \ ^{S}_{A}}{@K(C)} &= \frac{[1 + K(C)][1 \quad \frac{2 \quad C}{2(1 \quad C)}K(C)]}{2(1 + \frac{2 \quad C}{2(1 \quad C)}K(C)^2)^2}
\end{aligned}
$$

Since $1 - \frac{2-C}{2(1-C)}K(C) \quad 1 - \frac{2-C}{2(1-C)}\overline{K}(C) > 0$, $\frac{\partial \pi_A^S}{\partial K(C)} > 0$. The profit-maximizing equilibrium is the one that yields lowest total profit for the incumbent. In that equilibrium, when $C \quad C_4$, the profit with counterfeit entry is higher. Therefor no matter which separating equilibrium is sustained in the second period, $\pi_A^S(C) \quad 0$ if $C \quad C_4$.

(iii) For total welfare:

$$TS^S(C; K(C)) = 0.225 - \frac{1}{8}\frac{[1 + K(C)]^2[1 + \frac{4-3C}{4-4C}K(C)^2]}{[1 + \frac{2-C}{2-2C}K(C)^2]^2}$$

When $C = 0$,

$$TS^S(0; K(0)) = 0.225 - \frac{1}{8}$$