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Search, Heterogeneity, and Optimal Income Taxation

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Working Paper

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Abstract

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M (1982), H (1990) A (1999)
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), . H ,
(2001) ,
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[illegible]

2 Model

[illegible] (H) [illegible] (L) [illegible] l_k, [illegible] $k = H; L$, [illegible] q_m, [illegible] $m = H; L$. [illegible] $y_{km} > 0$, [illegible], $y_{Hm} > y_{Lm}$.

[illegible] $\delta_k \in [0; 1]$ [illegible] $c_w(\delta_k)$, [illegible] $c_w(0) = 0$, $c'_w(0) = 0$, [illegible] $\lim_{\delta_k \to 1} c'_w(\delta_k) = +\infty$. [illegible] v_m, [illegible] $c_\pi(v_m)$ [illegible].

[illegible]

[illegible]

2.1 The matching technology

[illegible]

[illegible]

, 1 – ff

. A M P (1999) ,

. H , P (1996) B D -

2.2 Output sharing

[illegible]

2.4 Private expected utility functions

$v_H q_H = \quad _m q_m$.

k

$$U_k = -c_w(\;_k) + \;_k \; M(\;) \; \frac{v_H q_H}{_m v_m q_m} \;_{kH} y_{kH} + \frac{v_L q_L}{_m v_m q_m} \;_{kL} y_{kL} \; + (1 - M(\;))0 \; + (1 - \;_k)0$$

$$U_k = -c_w(\;_k) + \;_k M(\;) E_{(m)} \;_{km} y_{km}; \qquad (5)$$

$E_{(m)}$. A , , $c(\;)$, . O , , $M(\;)$, . I , $1 - M(\;)$, , , . I , (

3 Optimal search intensity and market inefficiencies

3.1 Social Optimum

$$W = \max_{\delta,v} \; l_k U^k + q_m V^m$$

$$\ldots_k \geq 0; \quad v_m \geq 0.$$

(1), (2), (5), (6), -91.21

$$E_{(m)}y_{km} - (1 - \;)E_{(k)}E_{(m)}y_{km} = \; E_{(k)}E_{(m)}y_{km} + E_{(m)}y_{km} - E_{(k)}E_{(m)}y_{km}:$$

A [illegible] (1 [illegible]) [illegible] - [illegible] ($E_{(k)}E_{(m)}y_{km}$), [illegible] ff [illegible] ([illegible] (1) [illegible]) [illegible], $E_{(m)}y_{km}$, [illegible], $E_{(k)}E_{(m)}y_{km}$. [illegible] ([illegible]) ff [illegible].

[illegible]:

[illegible] 1 . I [illegible]

$$\begin{aligned} c'_{\pi}(\bar{v}_H) &= \frac{M(\bar{\ })}{\bar{\ }} E_{(k)}y_{kH} - \; E_{(k)}E_{(m)}y_{km} \\ c'_{\pi}(\bar{v}_L) &= \frac{M(\bar{\ })}{\bar{\ }} E_{(k)}y_{kL} - \; E_{(k)}E_{(m)}y_{km} \end{aligned} \quad \left| \begin{array}{l} \bar{\ } \lesseqgtr 1; \\ \bar{v}_H > 0;\ \bar{v}_L > 0 \end{array} \right. ; \qquad (13)$$

$$\begin{aligned} c'_{\pi}(\bar{v}_H) &= \frac{M(\ )}{} E_{(k)}y_{kH} - \; E_{(k)}E_{(m)}y_{km} \\ c'_{\pi}(0) &\geq \frac{M(\ )}{} E_{(k)}y_{kL} - \; E_{(k)}E_{(m)}y_{km} \end{aligned} \quad \left| \begin{array}{l} \bar{\ } \lesseqgtr 1; \\ \bar{v}_H > 0;\ \bar{v}_L = 0 \end{array} \right. : \qquad (14)$$

3.2 Decentralized equilibrium

A [illegible] k [illegible]

$$\max_{\delta_k} \; U_k = -c_w(\;_k) + \;_k M(\;) E_{(m)} \;_{km} y_{km} \qquad (15)$$
$$\text{. . } \;_k \geq 0;$$

[illegible] $C_k = c_w(\;_k)$ [illegible], [illegible] $B_k = \;_k M(\;)E_{(m)} \;_{km}y_{km}$ [illegible] fi [illegible].

K $_k$ ^{2}

$$-c'_w(\ _k) + M(\ )E_{(m)}\ _{km}y_{km} \leq 0$$

$$_k \geq 0 \qquad (16)$$

$$(-c'_w(\ _k) + M(\ )E_{(m)}\ _{km}y_{km})\ _k = 0;$$

$_k$, . , $M(\ )$, . F ,

ff ,

ff .

I -

. $_{LL} \neq 0$ ($_{LH} = 0$), $y_{km} > 0$,

fi (06)2 158.730 ( ) -395. 4.9812163(0),)-373( )-333(( )-350(5. 4

. D l ll K (1992), ll B B (2002),

ff ll ff ;

fi fi ,

. O fi

4. *I*

$$c'_w(\ _k) = M(\ )(1 - \ ^w_k) w_k \qquad \lesseqgtr 1; \qquad ; \qquad (22)$$
$$c'_\pi(v_m) = \frac{M(\ )}{}(1 - \ ^\pi_m)\ _m \qquad _k > 0;\ v_m > 0$$

$$c'_w(0) \geq M(\ )(1 - \ ^w_L) w_L \qquad \lesseqgtr 1; \qquad : \qquad (23)$$
$$c'_\pi(0) \geq \frac{M(\ )}{}(1 - \ ^\pi_L)\ _L \qquad _L = 0;\ v_L = 0$$

4.1 Characterizing externalities through Pigou taxes

, P ($\check{}^w_H$; $\check{}^w_L$), P ($\check{}^\pi_H$; $\check{}^\pi_L$). L ( ) $\check{R}$, P , , LS:

$$\check{R} = (\ _k\ l) M(\ ) \ \frac{_H l_H}{_k\ l}\check{}^w_H w_H + \frac{_L l_L}{_k\ l}\check{}^w_L w_L + \frac{v_H q_H}{_m v q}\check{}^\pi_H\ _H + \frac{v_L q_L}{_m v q}\check{}^\pi_L\ _L$$

$$0 = \check{R} - \ _k l_k + \ _m l_m \ LS;$$

($_k$ l)$M(\ )$ = N .

k

$$U_k = -c_w \ \frac{z^w_k}{M(\ )\, w_k} + LS + (1 - \check{}^w_k) z^w_k \qquad (24)$$

, LS, ff

. fi ,

c'_w

(P 5)

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. ,

- , P

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- , ff . I

fi , fi , L ,

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N ,

ff $(g = 0)$.

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$y_{LL} > 0$,

4.2 Optimal income taxes with positive government revenue

R,

$$W = \sum_k l_k U^k + \sum_m q_m V^m ;$$

(1), (2), (5), (6), $w_k = E_{(m)} \, {}_{km} y_{km}$, $z^w_k = {}_k M(\ )w_k$, ${}_m = E_{(k)}(1 - {}_{km}) y_{km}$, $z^\pi_m = v_m \frac{M\,\theta}{\theta} \, {}_m$ 4,

$$W = \sum_k l_k \left(-c_w \frac{z^w_k}{M(\ )\, w_k} \right) + \sum_m q_m \left(-c_\pi \frac{z^\pi_m}{\frac{M\,\theta}{\theta} \, {}_m} \right) + \left(\sum_k l\right) M(\ )\, E_{(k)} E_{(m)} y_{km};$$

$(\sum_k l) M(\ ) = N$.

R,

$$R \le \left(\sum_k l\right) M(\ ) \left(\frac{{}_H l_H}{\sum_k l} \, {}^w_H w_H + \frac{{}_L l_L}{\sum_k l} \, {}^w_L w_L + \frac{v_H q_H}{\sum_m vq} \, {}^\pi_H \, {}_H + \frac{v_L q_L}{\sum_m vq} \, {}^\pi_L \, {}_L \right); \qquad (30)$$

$(\sum_k l) M(\ ) = M$. (30),

[17]

$$W = \sum_k l_k \left(-c_w \frac{z_k^w}{M(\ ) w_k} \right) + \sum_m q_m \left(-c_\pi \frac{z_m^\pi}{\frac{M\ \theta}{\theta}\ m} \right)$$

$$+ (\ _k\ l) M(\ ) \ \frac{_H l_H}{_k\ l}(1 - \ _H^w) w_H + \frac{_L l_L}{_k\ l}(1 - \ _L^w) w_L$$

$$+ \frac{v_H q_H}{_m\ vq}(1 - \ _H^\pi)\ _H + \frac{v_L q_L}{_m\ vq}(1 - \ _L^\pi)\ _L + R:$$

[illegible] (22) (C (1978), D (1998) [illegible]). [illegible]

[illegible]

L 7 A . E , (22) [illegible]

$$\max_{\tau_k^w, \tau_m^\pi} W = \sum_k l_k \left(-c_w \frac{z_k^w}{M(\ ) w_k} \right) + \sum_m q_m \left(-c_\pi \frac{z_m^\pi}{M\ \theta} \right.$$

[illegible] P [illegible] 8 [illegible] A [illegible] [illegible] [illegible] [illegible]. I[illegible] [illegible] [illegible]. I[illegible] [illegible]

[illegible] ff [illegible], $''^{w}_{k} = 0$ [illegible] $''^{\pi}_{m} = 0$. I[illegible] [illegible]. I [illegible] (33). [illegible]. H [illegible] ff [illegible]. I [illegible] ([illegible] P [illegible] 5 [illegible] A [illegible] A). H [illegible] [illegible].

I [illegible] fi[illegible], $''^{\pi}_{m} = \infty$, [illegible]. I[illegible] [illegible] $''^{\pi}_{m} = \infty$, [illegible] ff [illegible].

4.2.2 [illegible]

I[illegible] I [illegible] 1 [illegible]

[illegible] / [illegible] ([illegible]), [illegible] ([illegible] D [illegible] (1998)). [illegible] fi [illegible] ' [illegible] (P [illegible] 11), [illegible] , [illegible] . P [illegible] 11 ([illegible]) [illegible]

12.

$$i)\ @\ \frac{w_H}{w_L} = @\ \frac{E_{(m)}\ Hm}{E_{(m)}\ Lm} < 0 \qquad (36)$$

$$ii)\ @\ \frac{w}{\pi} = @(\ ) < 0 \qquad (37)$$

5 Conclusion

[illegible]

[illegible]:

[illegible] (1) "H [illegible] Effi[illegible] F [illegible]," *I [illegible] E [illegible]*, 40, 827-849.

[illegible] (2002) "O [illegible] L [illegible]," *J [illegible] E [illegible]*, 85, 53-97.

[illegible] (1 4) "Eff[illegible] B[illegible]fi [illegible] F [illegible]," [illegible] P [illegible].

[illegible] (1) "M [illegible] Cl [illegible]," *J [illegible] E [illegible]*, 112, 141-168.

[illegible] (1) "L [illegible] - [illegible] P [illegible] F [illegible]: M [illegible] E [illegible]," *E [illegible] J [illegible]*, 109, 307-334.

[illegible] (1) "O [illegible] R [illegible]: M [illegible] R [illegible]," *A [illegible] E [illegible]*, 68, 756-768.

[illegible] (1 1) "M [illegible] C [illegible], F [illegible] Effi[illegible]," *J [illegible] E [illegible]*, .89, 798-812.

[illegible] (1 2) "A [illegible] D [illegible] M [illegible] E [illegible]," *J [illegible] E [illegible]*, .90, 881-894.

[illegible] (1 2) " [illegible] D [illegible] Effi[illegible] E [illegible]," *E [illegible]*, .49, 217-227.

[illegible] (1) "O [illegible] I [illegible]: A [illegible] E [illegible] P [illegible] O [illegible] M [illegible] R [illegible]," *A [illegible] E [illegible]*, .88, 83-95.

[illegible] (1) " [illegible] P [illegible]," *E [illegible] J [illegible]*, 7, 46-70.

[illegible] (1 3) "O [illegible] O [illegible] P [illegible] I [illegible]," *J [illegible] E [illegible]*, 2, 357-376.

[illegible] (1) "H D [illegible] P [illegible] Aff[illegible] E [illegible]? [illegible] C [illegible] O [illegible] E [illegible]," [illegible] M [illegible] P [illegible].

, . (1 0) "Effi I ," *J E* , 51, 8999.

., . (1 5) "P , , P , D E , .

, . . (1 0) "O Effi M R M ," *E* , 57, 279-298.

, . (1) " , F , ," *E* , 32, D E , .

, ., . (1) " P G E P M B ," *L E* , 3, 6580.

., . (1) "L R , G J B J ," *E* , *C* , P .

, ., . (1 3) "

., (1) "L M D Eff
," J E , 33, 457495.

, ., (2000) "A M ," E , 68, 343-369.

, ., (2001) "M , , H " A
, 1(2001), I . 1, A . 5.

. (1) "P F E P ?,"
E , 25, 223264.

. (1) "O P F L M ," L
E , 6, 435452.

, . (1) "P Effi O N N
E ," I : H E , .2.

, . (1) " - P Effi ," J
E , 6, 239262.

, . (1) " , P P D .,"
I F , 6, 239262.

, . (1) " P F E ," I : ,
.B. (E .), F C ., M M , L .

Appendices:

A Proofs of the main results

Proof of Corollary 3.

F , . -
, $_1 \neq$. I $_1 >$,
$_H(_1) > _H()$, $_L(_1) > _L()$, $v_H(_1) < v_H()$, $v_L(_1) < v_L()$. H
)$_1$)

CR

. F 1 − =

$$\check{R} = N \left[\begin{array}{l} \frac{\delta_H l_H}{k \, \delta l} (1 - \quad) E_{(k)} E_{(m)} y_{km} - E_{(m)} (1 - \quad {}_{Hm}) y_{Hm} \\ + \frac{\delta_L l_L}{k \, \delta l} (1 - \quad) E_{(k)} E_{(m)} y_{km} - E_{(m)} (1 - \quad {}_{Lm}) y_{Lm} \\ + \frac{v_H q_H}{m \, vq} (1 - (1 - \quad)) E_{(k)} E_{(m)} y_{km} - E_{(k)} \quad {}_{kH} y_{kH} \\ + \frac{v_L q_L}{m \, vq} (1 - (1 - \quad)) E_{(k)} E_{(m)} y_{km} - E_{(k)} \quad {}_{kL} y_{kL} \end{array} \right]$$

$$= N \left[\begin{array}{l} \frac{\delta_H l_H}{k \, \delta l} (1 - \quad) E_{(k)} E_{(m)} y_{km} - E_{(m)} (1 - \quad {}_{Hm}) y_{Hm} \\ + \frac{\delta_L l_L}{k \, \delta l} (1 - \quad) E_{(k)} E_{(m)} y_{km} - E_{(m)} (1 - \quad {}_{Lm}) y_{Lm} \\ + \frac{v_H q_H}{m \, vq} (1 - \quad) E_{(k)} E_{(m)} y_{km} - E_{(k)} \quad {}_{kH} y_{kH} \\ + \frac{v_L q_L}{m \, vq} (1 - \quad) E_{(k)} E_{(m)} y_{km} - E_{(k)} \quad {}_{kL} y_{kL} \end{array} \right]$$

$$\check{R} = N \quad 1 - (\quad + \quad)$$

; - , . I (L).□

Proof of Lemma 7.

I , k

$$U_k = \; - c_w (\; {}_k) + \; {}_k M(\;)(1 - \; {}^{w}_{k}) w_k$$

$$= \; - c_w \; \frac{z^w_k}{M(\;) w_k} \; + (1 - \; {}^{w}_{k}) z^w_k$$

w_k

$$\frac{@U_k}{@w_k} = -c'_w \frac{1}{M(\;) w_k} \frac{@z^w_k}{@w_k} - c'_w \frac{z^w_k}{M(\;)} \; -\frac{1}{w_k^2} \; + \frac{@z^w_k}{@w_k} (1 - \; {}^{w}_{k}) = c'_w \frac{z^w_k}{M(\;) w_k} > 0;$$

E . □

Proof of Proposition 8.

I : $w_k = E_{(m)} w_{km} = E_{(m)} \quad {}_{km} y_{km}$

$k = H; L$; $z_k^w = {}_k M(\ ) w_k$

$k = H; L$; ${}_m = E_{(k)} \quad {}_{km} = E_{(k)}(1 - \quad {}_{km}) y_{km}$ fi

e $m = H; L$; $z_m^\pi = v_m \, M$

Z π π

k

l . F fi , , l

,

$$\frac{dz_H^w}{z_H^w} \quad \frac{1}{''^w_H} + (1- \quad)\frac{_H l_H}{k \quad l} \quad = (1- \quad) \quad E_{(m)} \quad \frac{dz_m^\pi}{z_m^\pi} \quad - \frac{_L l_L}{k \quad l} \quad \frac{dz_L^w}{z_L^w} \quad - \frac{d \quad ^w_H}{1- \quad ^\pi} \quad ($$

L

$$\frac{dz^\pi_H}{z^\pi_H}\frac{1}{\varepsilon^\pi_H} = \frac{E_{(k)}\Big(\frac{dz^w_k}{z^w_k} \quad + \frac{d\tau^\pi_L}{-\tau^\pi_L} \quad \frac{v_L q_L}{{}_m vq}\varepsilon^\pi_L - \frac{d\tau^\pi_H}{-\tau^\pi_H}\Big(1 + \quad \frac{v_L q_L}{{}_m vq}\varepsilon^\pi_L}{\Delta_2} \tag{45}$$

$$\frac{dz^\pi_L}{z^\pi_L}\frac{1}{\varepsilon^\pi_L} = \frac{E_{(k)}\Big(\frac{dz^w_k}{z^w_k} \quad - \frac{d\tau^\pi_L}{-\tau^\pi_L}\Big(1 + \quad \frac{v_H q_H}{{}_m vq}\varepsilon^\pi_H \quad + \frac{d\tau^\pi_H}{-\tau^\pi_H} \quad \frac{v_H q_H}{{}_m vq}\varepsilon^\pi_H}{\Delta_2}; \tag{46}$$

$\Delta_2 = 1 + \quad E_{(m)}\varepsilon^\pi_m$. A (45) (46)

$$E_{(m)} \quad \frac{dz^\pi_m}{z^\pi_m} \quad = \frac{E_{(k)}\Big(\frac{dz^w_k}{z^w_k} \quad E_{(m)}\varepsilon^\pi_m - E_{(m)}\Big(\varepsilon^\pi_m \frac{d\tau^\pi_m}{-\tau^\pi_m}}{\Delta_2}; \tag{47}$$

(43) (44)

$$E_{(k)} \quad \frac{dz^w_k}{z^w_k} \quad = \frac{(1 - \quad)E_{(m)}\Big(\frac{dz^\pi_m}{z^\pi_m} \quad E_{(k)}\varepsilon^w_k - E_{(k)}\Big(\varepsilon^w_k \frac{d\tau^w_k}{-\tau^w_k}}{\Delta_1}: \tag{48}$$

F (47) (48) $E_{(m)}\Big(\frac{dz^\pi_m}{z^\pi_m}$ $E_{(k)}\Big(\frac{dz^w_k}{z^w_k}$

$$E_{(m)} \quad \frac{dz^\pi_m}{z^\pi_m} \quad = -\frac{(\Delta_2 - 1)E_{(k)}\Big(\varepsilon^w_k \frac{d\tau^w_k}{-\tau^w_k} \quad + \Delta_1 E_{(m)}\Big(\varepsilon^\pi_m \frac{d\tau^\pi_m}{-\tau^\pi_m}}{\Delta_1 + \Delta_2 - 1} \tag{49}$$

$$E_{(k)} \quad \frac{dz^w_k}{z^w_k} \quad = -\frac{\Delta_2 E_{(k)}\Big(\varepsilon^w_k \frac{d\tau^w_k}{-\tau^w_k} \quad + (\Delta_1 - 1)E_{(m)}\Big(\varepsilon^\pi_m \frac{d\tau^\pi_m}{-\tau^\pi_m}}{\Delta_1 + \Delta_2 - 1} \tag{50}$$

(49) (43) (44), (50) (45) (46) :

$$\frac{dz^w_H}{z^w_H}\frac{1}{\varepsilon^w_H} = -\frac{(1 - \quad) \quad (\Delta_2 - 1)E_{(k)}\Big(\varepsilon^w_k \frac{d\tau^w_k}{-\tau^w_k} \quad + \Delta_1 E_{(m)}\Big(\varepsilon^\pi_m \frac{d\tau^\pi_m}{-\tau^\pi_m}}{\Delta_1(\Delta_1 + \Delta_2 - 1)}$$
$$+ \frac{(\Delta_1 + \Delta_2 - 1) \quad \frac{d\tau^w_L}{-\tau^w_L}(1 - \quad)\frac{\delta_L l_L}{{}_k \delta l}\varepsilon^w_L - \frac{d\tau^w_H}{-\tau^w_H}\Big(1 + (1 - \quad)\frac{\delta_L l_L}{{}_k \delta l}\varepsilon^w_L}{\Delta_1(\Delta_1 + \Delta_2 - 1)}; \tag{51}$$

$$\frac{dz^w_L}{z^w_L}\frac{1}{\varepsilon^w_L} = -\frac{(1 - \quad) \quad (\Delta_2 - 1)E_{(k)}\Big(\varepsilon^w_k \frac{d\tau^w_k}{-\tau^w_k} \quad + \Delta_1 E_{(m)}\Big(\varepsilon^\pi_m \frac{d\tau^\pi_m}{-\tau^\pi_m}}{\Delta_1(\Delta_1 + \Delta_2 - 1)}$$
$$+ \frac{(\Delta_1 + \Delta_2 - 1) \quad -\frac{d\tau^w_L}{-\tau^w_L}\Big(1 + (1 - \quad)\frac{\delta_H l_H}{{}_k \delta l}\varepsilon^w_H \quad + \frac{d\tau^w_H}{-\tau^w_H}(1 - \quad)\frac{\delta_H l_H}{{}_k \delta l}\varepsilon^w_H}{\Delta_1(\Delta_1 + \Delta_2 - 1)}; \tag{52}$$

$$\frac{dz_H^\pi}{z_H^\pi}\frac{1}{''^\pi_H} = -\frac{\Delta_2 E_{(k)}\Big(''^w_k \frac{d\tau_k^w}{-\tau_k^w} \quad + (\Delta_1-1)E_{(m)}\Big(''^\pi_m\frac{d\tau_m^\pi}{-\tau_m^\pi}}{\Delta_2(\Delta_1+\Delta_2-1)} + \frac{(\Delta_1+\Delta_2-1) \quad \frac{d\tau_L^\pi}{-\tau_L^\pi} \quad \frac{v_Lq_L}{_k\, vq}''^\pi_L - \frac{d\tau_H^\pi}{-\tau_H^\pi}\Big(1+ \quad \frac{v_Lq_L}{_k\, vq}''^\pi_L}{\Delta_2(\Delta_1+\Delta_2-1)}; \tag{53}$$

$$\frac{dz_L^\pi}{z_L^\pi}\frac{1}{''^\pi_L} = -\frac{\Delta_2 E_{(k)}\Big(''^w_k \frac{d\tau_k^w}{-\tau_k^w} \quad + (\Delta_1-1)E_{(m)}\Big(''^\pi_m\frac{d\tau_m^\pi}{-\tau_m^\pi}}{\Delta_2(\Delta_1+\Delta_2-1)} + \frac{(\Delta_1+\Delta_2-1) \quad -\frac{d\tau_L^\pi}{-\tau_L^\pi}\Big(1+ \quad \frac{v_Hq_H}{_k\, vq}''^\pi_H \quad + \frac{d\tau_H^\pi}{-\tau_H^\pi} \quad \frac{v_Hq_H}{_k\, vq}''^\pi_H}{\Delta_2(\Delta_1+\Delta_2-1)}; \tag{54}$$

F (51)-(54) fi /.

$$\begin{aligned}
\frac{dz_H^w}{d\ _H^w}\frac{1}{z_H^w} &= \frac{''^w_H}{1- \ _H^w} \quad \frac{(1- \quad)\frac{\delta_H l_H}{_k\,\delta l}''^w_H - (\Delta_1+\Delta_2-1)}{\Delta_1+\Delta_2-1}\\
\frac{dz_L^w}{d\ _H^w}\frac{1}{z_L^w} &= \frac{\frac{\varepsilon_H^w}{-\tau_H^w}\frac{\delta_H l_H}{_k\,\delta l}''^w_L(1- \quad)}{\Delta_1+\Delta_2-1}\\
\frac{dz_H^\pi}{d\ _H^w}\frac{1}{z_H^\pi} &= -\frac{\frac{\varepsilon_H^w}{-\tau_H^w}\frac{\delta_H l_H}{_k\,\delta l}''^\pi_H}{\Delta_1+\Delta_2-1}\\
\frac{dz_L^\pi}{d\ _H^w}\frac{1}{z_L^\pi} &= -\frac{\frac{\varepsilon_H^w}{-\tau_H^w}\frac{\delta_H l_H}{_k\,\delta l}''^\pi_L}{\Delta_1+\Delta_2-1}
\end{aligned} \tag{55}$$

$$\begin{aligned}
\frac{dz_H^w}{d\ _L^w}\frac{1}{z_H^w} &= \frac{\frac{\varepsilon_L^w}{-\tau_L^w}\frac{\delta_L l_L}{_k\,\delta l}''^w_H(1- \quad)}{\Delta_1+\Delta_2-1}\\
\frac{dz_L^w}{d\ _L^w}\frac{1}{z_L^w} &= \frac{''^w_L}{1- \ _L^w} \quad \frac{(1- \quad)\frac{\delta_L l_L}{_k\,\delta l}''^w_L - (\Delta_1+\Delta_2-1)}{\Delta_1+\Delta_2-1}\\
\frac{dz_H^\pi}{d\ _L^w}\frac{1}{z_H^\pi} &= -\frac{\frac{\varepsilon_L^w}{-\tau_L^w}\frac{\delta_L l_L}{_k\,\delta l}''^\pi_H}{\Delta_1+\Delta_2-1}\\
\frac{dz_L^\pi}{d\ _L^w}\frac{1}{z_L^\pi} &= -\frac{\frac{\varepsilon_L^w}{-\tau_L^w}\frac{\delta_L l_L}{_k\,\delta l}''^\pi_L}{\Delta_1+\Delta_2-1}
\end{aligned} \tag{56}$$

$$
\begin{aligned}
\frac{dz_H^w}{d\ {}_H^\pi}\frac{1}{z_H^w} &= -\frac{\frac{\varepsilon_H^\pi}{-\tau_H^\pi}\frac{v_H q_H}{{}_m vq}\,{}''^w_H(1- \quad)}{\Delta_1+\Delta_2-1}\\
\frac{dz_L^w}{d\ {}_H^\pi}\frac{1}{z_L^w} &= -\frac{\frac{\varepsilon_H^\pi}{-\tau_H^\pi}\frac{v_H q_H}{{}_m vq}\,{}''^w_L(1- \quad)}{\Delta_1+\Delta_2-1}\\
\frac{dz_H^\pi}{d\ {}_H^\pi}\frac{1}{z_H^\pi} &= \frac{{}''^\pi_H}{1- \quad {}_H^\pi}\ \frac{{}''^\pi_H\frac{v_H q_H}{{}_m vq}-(\Delta_1+\Delta_2-1)}{\Delta_1+\Delta_2-1}\\
\frac{dz_L^\pi}{d\ {}_H^\pi}\frac{1}{z_L^\pi} &= \frac{\frac{\varepsilon_H^\pi}{-\tau_H^\pi}\frac{v_H q_H}{{}_m vq}\,{}''^\pi_L}{\Delta_1+\Delta_2-1}
\end{aligned}
\tag{57}
$$

$$
\begin{aligned}
\frac{dz_H^w}{d\ {}_L^\pi}\frac{1}{z_H^w} &= -\frac{\frac{\varepsilon_L^\pi}{-\tau_L^\pi}\frac{v_L q_L}{{}_m vq}\,{}''^w_H(1- \quad)}{\Delta_1+\Delta_2-1}\\
\frac{dz_L^w}{d\ {}_L^\pi}\frac{1}{z_L^w} &= -\frac{\frac{\varepsilon_L^\pi}{-\tau_L^\pi}\frac{v_L q_L}{{}_m vq}\,{}''^w_L(1- \quad)}{\Delta_1+\Delta_2-1}\\
\frac{dz_H^\pi}{d\ {}_L^\pi}\frac{1}{z_H^\pi} &= \frac{\frac{\varepsilon_L^\pi}{-\tau_L^\pi}\frac{v_L q_L}{{}_m vq}\,{}''^\pi_H}{\Delta_1+\Delta_2-1}\\
\frac{dz_L^\pi}{d\ {}_L^\pi}\frac{1}{z_L^\pi} &= \frac{{}''^\pi_L}{1- \quad {}_L^\pi}\ \frac{{}''^\pi_L\frac{v_L q_L}{{}_m vq}-(\Delta_1+\Delta_2-1)}{\Delta_1+\Delta_2-1}
\end{aligned}
\tag{58}
$$

I ... , ... L ..., ... , ...

$$
\begin{aligned}
\max_{\tau_k^w,\tau_m^\pi} W = \ & \ l_k \ -c_w\ \frac{z_k^w}{M(\ )\,w_k} \ + \ q_m \ -c_\pi\ \frac{z_m^\pi}{\frac{M\ \theta}{\theta}\ {}_m}\\
&+ {}_H l_H c_w'\ \frac{z_H^w}{M(\ )\,w_H} + {}_L l_L c_w'\ \frac{z_L^w}{M(\ )\,w_L} + v_H q_H c_\pi'\ \frac{z_H^\pi}{\frac{M\ \theta}{\theta}\ {}_H} + v_L q_L c_\pi'\ \frac{z_L^\pi}{\frac{M\ \theta}{\theta}\ {}_L} + R\\
&+ (\ {}_k\ l) M(\ )\ \frac{{}_H l_H}{{}_k\ l}\ {}_H^w w_H + \frac{{}_L l_L}{{}_k\ l}\ {}_L^w w_L + \frac{v_H q_H}{{}_m vq}\ {}_H^\pi\ {}_H + \frac{v_L q_L}{{}_m vq}\ {}_L^\pi\ {}_L\ ;
\end{aligned}
$$

... . D ...

$$
a = \frac{{}_H l_H}{{}_k\ l}\ {}_H^w w_H + \frac{{}_L l_L}{{}_k\ l}\ {}_L^w w_L \qquad b = \frac{v_H q_H}{{}_m vq}\ {}_H^\pi\ {}_H + \frac{v_L q_L}{{}_m vq}\ {}_L^\pi\ {}_L :
$$

$\quad {}^{w}_{H}$

$$\frac{@L}{@\ {}^{w}_{H}} =$$

$$= \sum_k l_k \left(-\frac{c'_w}{M(\ )\, w_k} \frac{dz^w_k}{d\ {}^{w}_{H}} \right) + \sum_m q_m \left(-\frac{c'_\pi}{\frac{M\ \theta}{\theta}\ {}_m} \frac{dz^\pi_m}{d\ {}^{w}_{H}} \right)$$

$$+ \frac{dz^w_H}{d\ {}^{w}_{H}} \frac{1}{M(\ )\, w_H} l_H c'_w \quad \frac{z^w_H}{M(\ )\, w_H} \quad + \ {}_H l_H c''_w \quad \frac{z^w_H}{M(\ )\, w_H} \quad \frac{1}{M(\ )\, w_H} \frac{dz^w_H}{d\ {}^{w}_{H}}$$

$$+ \frac{dz^w_L}{d\ {}^{w}_{H}} \frac{1}{M(\ )\, w_L} l_L c'_w \quad \frac{z^w_L}{M(\ )\, w_L} \quad + \ {}_L l_L c''_w \quad \frac{z^w_L}{M(\ )\, w_L} \quad \frac{1}{M(\ )\, w_L} \frac{dz^w_L}{d\ {}^{w}_{H}}$$

$$+ \frac{dz^\pi_H}{d\ {}^{\pi}_{H}} \frac{1}{\frac{M\ \theta}{\theta}\ {}_H} q_H c'_\pi \quad \frac{z^\pi_H}{\frac{M\ \theta}{\theta}\ {}_H} \quad + v_H q_H c''_\pi \quad \frac{z^\pi_H}{\frac{M\ \theta}{\theta}\ {}_H} \quad \frac{1}{\frac{M\ \theta}{\theta}\ {}_H} \frac{dz^\pi_H}{d\ {}^{\pi}_{H}}$$

$$+ \frac{dz^\pi_L}{d\ {}^{\pi}_{L}} \frac{1}{\frac{M\ \theta}{\theta}\ {}_L} q_L c'_\pi \quad \frac{z^\pi_L}{\frac{M\ \theta}{\theta}\ {}_L} \quad + v_L q_L c''_\pi \quad \frac{z^\pi_L}{\frac{M\ \theta}{\theta}\ {}_L} \quad \frac{1}{\frac{M\ \theta}{\theta}\ {}_L} \frac{dz^\pi_L}{d\ {}^{\pi}_{L}}$$

$$+ \left[\sum_k \frac{l_k}{M(\ )\, w_k} \frac{dz^w_k}{d\ {}^{w}_{H}} \quad M(\ ) + (\sum_k l) M(\ ) \left(\frac{\sum_m \frac{q_m}{\frac{M(\theta)}{\theta}\pi_m} \frac{dz^\pi_m}{d\tau^w_H}}{\sum_k l} - \frac{(\sum_m vq) \sum_k \left(\frac{l_k}{M\ \theta\ \ w_k} \frac{dz^w_k}{d\tau^w_H} \right)}{(\sum_k l)^2} \right) \right] (a+b)$$

$$\Bigg[$$

1980 Td[(J)]TJ/F19 0.974 Tf/6.94 0.58m

$$+ \ (\sum_k l) M(\ )$$

$$\Bigg[$$

$$ {}_H l_H\, c''_w \;\frac{z_H^w}{M(\;)\,w_H} \quad \frac{1}{M(\;)\,w_H}\frac{dz_H^w}{d\;{}_H^w} + \;{}_L l_L\, c''_w \;\frac{z_L^w}{M(\;)\,w} $$ [redacted]

$$ +v_H q_H\, c''_\pi \;\frac{z_H^\pi}{\frac{M\;\theta}{\theta}\quad{}_H} \quad \frac{1}{\frac{M\;\theta}{\theta}\quad{}_H}\frac{dz_H^\pi}{d\;{}_H^w} + v_L q_L\, c''_\pi \;\frac{z_L^\pi}{\frac{M\;\theta}{\theta}\quad{}_L} $$ [redacted]

$$ +\;(\quad{}_k\;\; l)M(\;)\left[\;\frac{{}_k\left(\frac{l_k}{M\;\theta\;\;w_k}\frac{dz_k^w}{d\tau_H^w}\right.}{{}_k\;\;l} + \frac{M'(\;)}{M(\;)}\quad\frac{{}^m}{} \right. $$ [redacted] $$ \left.\frac{\frac{l_k}{M\;\theta\;\;w_k}\frac{dz_k^w}{d\tau_H^w}}{{}_k\;\;l}\right]\;(a+b) = $$

[redacted]

$$ = \;{}_H l_H\, c''_w \;\frac{z_H^w}{M(\;)\,w_H} \quad \frac{1}{M(\;)\,w_H}\frac{dz_H^w}{d\;{}_H^w}\frac{z_H^w}{z_H^w}\frac{M(\;)\,(}{c'_w(z_H^w=} $$ [redacted]

$$ +\;{}_L l_L\, c''_w \;\frac{z_L^w}{M(\;)\,w_L} \quad \frac{1}{M(\;)\,w_L}\frac{dz_L^w}{d\;{}_H^w}\frac{z_L^w}{z_L^w}\frac{M(\;)\,(1-\;{}_L^w)w_L}{c'_w(z_L^w=M(\;)\,w_L)} $$

$$ +v_H q_H\, c''_\pi \;\frac{z_H^\pi}{\frac{M\;\theta}{\theta}\quad{}_H} \quad \frac{1}{\frac{M\;\theta}{\theta}\quad{}_H}\frac{dz_H^\pi}{d\;{}_H^w}\frac{z_H^\pi}{z_H^\pi}\,\frac{M(\;)}{}\,\frac{(1-\;{}_H^\pi)\quad{}_H}{c'_\pi(z_H^\pi=\quad{}_H)} $$

$$ +v_L q_L\, c''_\pi \;\frac{z_L^\pi}{\frac{M\;\theta}{\theta}\quad{}_L} \quad \frac{1}{\frac{M\;\theta}{\theta}\quad{}_L}\frac{dz_L^\pi}{d\;{}_H^w}\frac{z_L^\pi}{z_L^\pi}\,\frac{M(\;)}{}\,\frac{(1-\;{}_L^\pi)\quad{}_L}{c'_\pi\left(z_L^\pi=\frac{M\;\theta}{\theta}\quad{}_L\right.} $$

$$ +\;(\quad{}_k\;\; l)M(\;)\left[\quad {}_k\left(\frac{l_k}{M\;\theta\;\;u\mathrm{wH}}\right.\right. $$

$$= \;{}_H l_H \frac{1}{''^{w}_{H}} \frac{dz^{w}_{H}}{d\;{}^{w}_{H}} \frac{1}{z^{w}_{H}}$$

$^{\pi}_{H}$, $^{\pi}_{L}$

$$(\Delta_1 + \Delta_2 - 1) \quad (1 - \quad)^{1 - \quad ^{w}_{L}}$$

F

$$= 1 - \frac{\left[\begin{array}{l} \left(\frac{\delta_H l_H}{k\,\delta l}\frac{v_L q_L}{m\,vq} + \frac{\delta_L l_L}{k\,\delta l}\frac{v_H q_H}{m\,vq}\right) w_{HH} + \left(\frac{\delta_H l_H}{k\,\delta l}\frac{v_H q_H}{m\,vq} + \frac{\delta_L l_L}{k\,\delta l}\frac{v_L q_L}{m\,vq}\right) w_{HL} \quad {}^{w}_{H} \\ + \left(\frac{\delta_H l_H}{k\,\delta l}\frac{v_H q_H}{m\,vq} + \frac{\delta_L l_L}{k\,\delta l}\frac{v_L q_L}{m\,vq}\right) w_{LH} + \left(\frac{\delta_H l_H}{k\,\delta l}\frac{v_L q_L}{m\,vq} + \frac{\delta_L l_L}{k\,\delta l}\frac{v_H q_H}{m\,vq}\right) w_{LL} \quad {}^{w}_{L} \\ + \left(\frac{\delta_H l_H}{k\,\delta l}\frac{v_L q_L}{m\,vq} + \frac{\delta_L l_L}{k\,\delta l}\frac{v_H q_H}{m\,vq}\right) {}_{HH} + \left(\frac{\delta_H l_H}{k\,\delta l}\frac{v_H q_H}{m\,vq} + \frac{\delta_L l_L}{k\,\delta l}\frac{v_L q_L}{m\,vq}\right) {}_{HL} \quad {}^{\pi}_{H} \\ + \left(\frac{\delta_H l_H}{k\,\delta l}\frac{v_H q_H}{m\,vq} + \frac{\delta_L l_L}{k\,\delta l}\frac{v_L q_L}{m\,vq}\right) {}_{LH} + \left(\frac{\delta_H l_H}{k\,\delta l}\frac{v_L q_L}{m\,vq} + \frac{\delta_L l_L}{k\,\delta l}\frac{v_H q_H}{m\,vq}\right) {}_{LL} \quad {}^{\pi}_{L} \end{array}\right]}{\frac{-\tau_H^w}{\varepsilon_H^w} w_H + \frac{-\tau_L^w}{\varepsilon_L^w} w_L + \frac{-\tau_H^\pi}{\varepsilon_H^\pi}\ {}_H + \frac{-\tau_L^\pi}{\varepsilon_L^\pi}\ {}_L} \; ;$$

$$= 1 - \frac{E_{(s)} w_{Hs}\ {}^{w}_{H} + E_{(s)} w_{Ls}\ {}^{w}_{L} + E_{(s)}\ {}_{sH}\ {}^{\pi}_{H} + E_{(s)}\ {}_{sL}\ {}^{\pi}_{L}}{\frac{-\tau_H^w}{\varepsilon_H^w} w_H + \frac{-\tau_L^w}{\varepsilon_L^w} w_L + \frac{-\tau_H^\pi}{\varepsilon_H^\pi}\ {}_H + \frac{-\tau_L^\pi}{\varepsilon_L^\pi}\ {}_L} \; ; \qquad (64)$$

s $\left(\frac{\delta_H l_H}{k\,\delta l}\frac{v_L q_L}{m\,vq} + \ldots\right.$

c'

$$'' = \frac{1}{}\frac{c''}{c'} = \frac{1}{\;-1}:$$

, A, :

.

2 $c = A(\;^{\gamma} + \;^{\beta})$ ≥ 3, $<$, $c' = A(\;^{\gamma-} + \;^{\beta-})$,

$$c'' = A(\;(\;-1)\;^{\gamma-2} + \;(\;-1)\;^{\beta-2}) > 0;$$

$$'' = \frac{\;^{\gamma-2} + \;^{\beta-2}}{(\;-1)\;^{\gamma-2} + (\;-1)\;^{\beta-2}}:$$

A , , .

$$\frac{@''}{@} = -\;^{\beta}\;^{\gamma-}\;(\;-\;)^2 < 0:$$

N ≥ 3 $=$ -1. . F ,

< 1 $= 3$ $= 2$, > 1 $= 2$ $= 1$.

$\frac{-\tau_H^w}{\varepsilon_H^w} w_H < \frac{-\tau_L^w}{\varepsilon_L^w} w_L$, (66) -

. H $(1 - \;_H^w) w_H < (1 - \;_L^w) w_L$ $_H^w >\; _L^w$

(66) $_H^w w_H >\; _L^w w_L$. N $(E_{(m)}\; _{Hm}\; _m^\pi - E_{(m)}\; _{Lm}\; _m^\pi)$. -

$_{HH} >\; _{LH}$ $_{HL} >\; _{LL}$ $(E_{(m)}\; _{Hm}\; _m^\pi - E_{(m)}\; _{Lm}\; _m^\pi > 0)$.

H (66) .

$(1 - \;_H^w) w_H > (1 - \;_L^w) w_L$, $_H >\; _L$, $''^w_H \leq ''^w_L$. □

Proof of Proposition 11.

fi (65).

fi

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(60)-(63)

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, . D

(60) (61)

$$(\Delta_1 + \Delta_2 - 1) \quad (1 - \quad) \frac{1 - \quad ^{w}}{''_{w}} w + \quad (w \quad ^{w} + \quad ^{\pi} - (1 - \quad) \bar{R}) \quad =$$

$$= (1 - \quad) \left[(1 - \quad ^{w}) w + \quad (w \quad ^{w\, ''w} + \quad ^{\pi\, ''w} - (1 - \quad) \bar{R}^{\, ''w}) \right]$$

$-$

(68), (69), P 11.

P 12 (1 −), (1 −)= ↑,

, $^{\pi}$= ^{w} ↓.□