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Search, Heterogeneity, and Optimal Income Taxation

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Search, Heterogeneity, and Optimal Income Taxation*

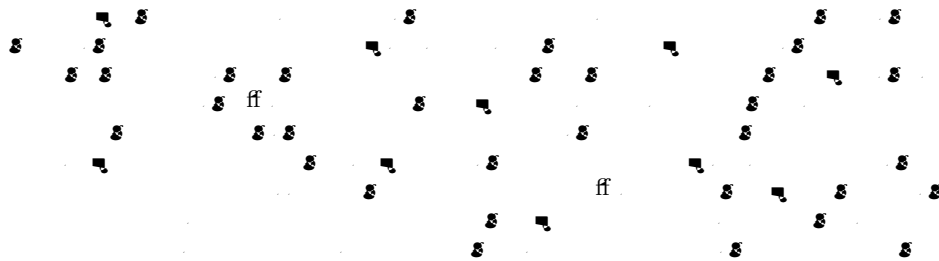
WORKING PAPER

Nikolay Dobrinov



November 9, 2009

Abstract



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The first part of the paper discusses the theoretical background of the research. It starts with a general overview of the field and then moves on to a more detailed discussion of the specific issues at hand. The second part of the paper is devoted to the empirical analysis. It begins with a description of the data used in the study and then proceeds to a series of statistical tests. The final part of the paper is a conclusion, which summarizes the main findings and discusses their implications.

The empirical analysis is based on a sample of 1000 observations. The results show that there is a significant positive relationship between the variables of interest. This finding is consistent with the theoretical predictions. The statistical tests provide strong evidence in support of the hypotheses. The results are robust to various specifications and control variables.

In conclusion, the paper provides a comprehensive analysis of the research topic. The findings have important implications for the field and warrant further research. The authors hope that this study will contribute to the understanding of the underlying mechanisms.

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2 Model

Let \mathbb{R}^n be a Euclidean space with the standard inner product $\langle \cdot, \cdot \rangle$ and the standard norm $\|\cdot\|$. Let F be a closed convex set in \mathbb{R}^n . Let H and L be two closed convex sets in \mathbb{R}^n such that $H \cap L = \emptyset$. Let I_k , $k = H; L$, be two closed convex sets in \mathbb{R}^n such that $I_k \cap I_m = \emptyset$, $m = H; L$. Let q_m , $m = H; L$, be two closed convex sets in \mathbb{R}^n such that $q_m \cap q_n = \emptyset$, $m, n = H; L$. Let $y_{km} > 0$, $y_{Hm} > y_{Lm}$. Let $I_k \in 0; 1$, $c_w(k)$, $c_w(0) = 0$, $c_w'(0) = 0$, $\lim_{\delta_k \rightarrow 0} c_w'(k) = +\infty$. Let V_m , $c_\pi(V_m)$. Let A .

On the other hand, if $\alpha \in \mathbb{R}^n$ is a vector, then $\alpha \cdot \alpha = \|\alpha\|^2$. If $\alpha \cdot \beta = 0$, then α and β are orthogonal. If $\alpha \cdot \beta = \|\alpha\| \|\beta\|$, then α and β are parallel. If $\alpha \cdot \beta = \|\alpha\| \|\beta\| \cos \theta$, then θ is the angle between α and β .

2.1 The matching technology

Let \mathcal{M} be a matching in a bipartite graph $G = (U, V, E)$. Let $u \in U$ and $v \in V$ be vertices. Let M_u be the set of vertices in U that are matched to u in \mathcal{M} . Let M_v be the set of vertices in V that are matched to v in \mathcal{M} . Let M_{uv} be the set of vertices in $U \cup V$ that are matched to either u or v in \mathcal{M} .

Let \mathcal{M}' be another matching in G . Let $u \in U$ and $v \in V$ be vertices. Let M'_u be the set of vertices in U that are matched to u in \mathcal{M}' . Let M'_v be the set of vertices in V that are matched to v in \mathcal{M}' . Let M'_{uv} be the set of vertices in $U \cup V$ that are matched to either u or v in \mathcal{M}' .

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2.2 Output sharing

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2.4 Private expected utility functions

$$U_k = -c_w(\cdot) + \dots$$

$$U_k = -c_w(\cdot) + \dots$$

$$U_k = -c_w(\cdot) + \dots$$

$E_{(m)}$

$c(\cdot)$

$M(\cdot)$

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3 Optimal search intensity and market inefficiencies

I is the search intensity, $I \in [0, 1]$. The search intensity I is chosen by the planner. The search intensity I is chosen by the planner. The search intensity I is chosen by the planner.

3.1 Social Optimum

A social planner chooses the search intensity I to maximize the social welfare. The social welfare is given by the sum of the utilities of the workers and the firm.

$$W = \int_{\delta, v} l_k U^k + q_m V^m$$

$\dots k \geq 0; v_m \geq 0:$

The first-order conditions are (1), (2), (5), (6), and (9).

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$$E_{(m)}y_{km} - (1 - \alpha)E_{(k)}E_{(m)}y_{km} = E_{(k)}E_{(m)}y_{km} + E_{(m)}y_{km} - E_{(k)}E_{(m)}y_{km}.$$

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$$\begin{aligned} c'_\pi(\bar{v}_H) &= \frac{M(\bar{v}_H)}{\bar{v}_H} E_{(k)}y_{kH} - E_{(k)}E_{(m)}y_{km} & \left| \begin{array}{l} - \\ \leq \\ \geq 1; \end{array} \right. & \\ c'_\pi(\bar{v}_L) &= \frac{M(\bar{v}_L)}{\bar{v}_L} E_{(k)}y_{kL} - E_{(k)}E_{(m)}y_{km} & \left| \begin{array}{l} \bar{v}_H > 0; \bar{v}_L > 0 \end{array} \right. & ; \end{aligned} \quad (13)$$

$$\begin{aligned} c'_\pi(\bar{v}_H) &= \frac{M(\bar{v}_H)}{\bar{v}_H} E_{(k)}y_{kH} - E_{(k)}E_{(m)}y_{km} & \left| \begin{array}{l} - \\ \leq \\ \geq 1; \end{array} \right. & \\ c'_\pi(0) &\geq \frac{M(0)}{0} E_{(k)}y_{kL} - E_{(k)}E_{(m)}y_{km} & \left| \begin{array}{l} \bar{v}_H > 0; \bar{v}_L = 0 \end{array} \right. & ; \end{aligned} \quad (14)$$

3.2 Decentralized equilibrium

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$$\delta_k U_k = -c_w(\delta_k) + \delta_k M(\delta_k) E_{(m)} y_{km} \geq 0; \quad (15)$$

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$\frac{\partial}{\partial \tau} \left(\frac{1}{M} \right) = -\frac{1}{M^2} \frac{\partial M}{\partial \tau}$.

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$$\begin{aligned}
 c'_w(w_k) &= M(1 - \frac{w}{k})w_k \\
 c'_\pi(v_m) &= \frac{M}{m}(1 - \frac{\pi}{m})v_m
 \end{aligned}
 \left| \begin{array}{l} \leq 1; \\ k > 0; v_m > 0 \end{array} \right. ; \quad (22)$$

$$\begin{aligned}
 c'_w(0) &\geq M(1 - \frac{w}{L})w_L \\
 c'_\pi(0) &\geq \frac{M}{L}(1 - \frac{\pi}{L})v_L
 \end{aligned}
 \left| \begin{array}{l} \leq 1; \\ L = 0; v_L = 0 \end{array} \right. ; \quad (23)$$

4.1 Characterizing externalities through Pigou taxes

$\tilde{R} = (1 - \frac{I}{k})M(1 - \frac{w}{k})w_k + \frac{I}{k}M(1 - \frac{w}{L})w_L + \frac{V_H q_H}{m v q} \frac{1}{H} + \frac{V_L q_L}{m v q} \frac{1}{L}$

$$\tilde{R} = (1 - \frac{I}{k})M(1 - \frac{w}{k})w_k + \frac{I}{k}M(1 - \frac{w}{L})w_L + \frac{V_H q_H}{m v q} \frac{1}{H} + \frac{V_L q_L}{m v q} \frac{1}{L}$$

$$0 = \tilde{R} - \frac{I}{k} + \frac{I}{m} LS;$$

$$U_k = -c_w \frac{Z_k^w}{M(1 - \frac{w}{k})w_k} + LS + (1 - \frac{w}{k})Z_k^w \quad (24)$$

$LS = \frac{V_H q_H}{m v q} \frac{1}{H} + \frac{V_L q_L}{m v q} \frac{1}{L}$

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$y_{LL} > 0$, $\frac{\partial y_{LL}}{\partial \tau} > 0$, $\frac{\partial y_{LL}}{\partial \tau} > 0$, $\frac{\partial y_{LL}}{\partial \tau} > 0$, $\frac{\partial y_{LL}}{\partial \tau} > 0$.

From (1), (2), (5), (6), we have $\frac{\partial y_{LL}}{\partial \tau} > 0$, $\frac{\partial y_{LL}}{\partial \tau} > 0$.

4.2 Optimal income taxes with positive government revenue

From (1), (2), (5), (6), we have $\frac{\partial y_{LL}}{\partial \tau} > 0$, $\frac{\partial y_{LL}}{\partial \tau} > 0$. From (3), (4), we have $\frac{\partial y_{LL}}{\partial \tau} > 0$, $\frac{\partial y_{LL}}{\partial \tau} > 0$.

$$W = \int_k I_k U^k + \int_m q_m V^m ;$$

From (1), (2), (5), (6), we have $\frac{\partial y_{LL}}{\partial \tau} > 0$, $\frac{\partial y_{LL}}{\partial \tau} > 0$.

From (1), (2), (5), (6), we have $\frac{\partial y_{LL}}{\partial \tau} > 0$, $\frac{\partial y_{LL}}{\partial \tau} > 0$. From (3), (4), we have $\frac{\partial y_{LL}}{\partial \tau} > 0$, $\frac{\partial y_{LL}}{\partial \tau} > 0$.

$$W = \int_k I_k - c_w \frac{Z_k^w}{M(\cdot) W_k} + \int_m q_m - c_\pi \frac{Z_m^\pi}{\frac{M\theta}{\theta} m} + (\int_k I) M(\cdot) E_{(k)} E_{(m)} y_{km};$$

From (1), (2), (5), (6), we have $\frac{\partial y_{LL}}{\partial \tau} > 0$, $\frac{\partial y_{LL}}{\partial \tau} > 0$.

From (3), (4), we have $\frac{\partial y_{LL}}{\partial \tau} > 0$, $\frac{\partial y_{LL}}{\partial \tau} > 0$.

$$R \leq (\int_k I) M(\cdot) \frac{I_H}{k} \frac{I_H}{I} W_H + \frac{I_L}{k} \frac{I_L}{I} W_L + \frac{V_H q_H}{m v q} \frac{\pi}{H} H + \frac{V_L q_L}{m v q} \frac{\pi}{L} L ; \quad (30)$$

From (1), (2), (5), (6), we have $\frac{\partial y_{LL}}{\partial \tau} > 0$, $\frac{\partial y_{LL}}{\partial \tau} > 0$.

From (3), (4), we have $\frac{\partial y_{LL}}{\partial \tau} > 0$, $\frac{\partial y_{LL}}{\partial \tau} > 0$.

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The first part of the paper is devoted to the study of the
 asymptotic behavior of the solutions of the system

$$\begin{cases}
 \dot{x} = Ax + B u \\
 \dot{y} = Cx + D u
 \end{cases}$$
 where A, B, C, D are matrices of appropriate dimensions.
 In particular, we consider the case where A is a
 Hurwitz matrix, which implies that the system is
 exponentially stable. The main result of this section is
 the following theorem:

Theorem 1. Let A be a Hurwitz matrix. Then, the
 solutions of the system (1) converge to zero as $t \rightarrow \infty$.

The proof of this theorem is based on the Lyapunov
 method. We consider the Lyapunov function

$$V(x) = x^T P x$$
 where P is a positive definite matrix. It can be
 shown that $\dot{V}(x) < 0$ for all $x \neq 0$, which
 implies that the origin is a globally asymptotically
 stable equilibrium point.

In the second part of the paper, we study the
 asymptotic behavior of the solutions of the system

$$\dot{x} = Ax + B u + f(x)$$
 where $f(x)$ is a nonlinear function satisfying
 $f(0) = 0$ and $f(x) = o(\|x\|)$ as $\|x\| \rightarrow 0$.
 The main result of this section is the following theorem:

Theorem 2. Let A be a Hurwitz matrix and $f(x)$
 be a nonlinear function satisfying the above conditions.
 Then, the solutions of the system (2) converge to zero
 as $t \rightarrow \infty$.

The proof of this theorem is based on the Lyapunov
 method. We consider the Lyapunov function

$$V(x) = x^T P x + \frac{1}{2} x^T Q x$$
 where P and Q are positive definite matrices. It
 can be shown that $\dot{V}(x) < 0$ for all $x \neq 0$,
 which implies that the origin is a globally asymptotically
 stable equilibrium point.

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$$i) @ \frac{w}{L} = @ \frac{E_{(m)} H_m}{E_{(m)} L_m} < 0 \quad (36)$$

$$ii) @ \frac{w}{\pi} = @ () < 0 \quad (37)$$

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5 Conclusion

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... (1) "P... G... E... l... B... , L... E... , 3, 6580.

... (1) "L... R... G... J... B... J... E... , C... , P... .

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Appendices:

A Proofs of the main results

Proof of Corollary 3.

For $\theta > 0$, $\theta < 1$, $\theta \neq 1$. If $\theta > 1$, $v_H(\theta) > v_H(1)$, $v_L(\theta) > v_L(1)$, $v_H(\theta) < v_H(1)$, $v_L(\theta) < v_L(1)$. If $\theta < 1$, $v_H(\theta) > v_H(1)$, $v_L(\theta) > v_L(1)$, $v_H(\theta) < v_H(1)$, $v_L(\theta) < v_L(1)$.

$$\begin{aligned}
 \check{R} &= N \left[\begin{aligned} & \frac{\delta_H l_H}{k \delta l} (1 -) E_{(k)} E_{(m)} y_{km} - E_{(m)} (1 -_{Hm}) y_{Hm} \\ & + \frac{\delta_L l_L}{k \delta l} (1 -) E_{(k)} E_{(m)} y_{km} - E_{(m)} (1 -_{Lm}) y_{Lm} \\ & + \frac{v_H q_H}{m v q} (1 - (1 -)) E_{(k)} E_{(m)} y_{km} - E_{(k)} y_{kH} \\ & + \frac{v_L q_L}{m v q} (1 - (1 -)) E_{(k)} E_{(m)} y_{km} - E_{(k)} y_{kL} \end{aligned} \right] \\
 &= N \left[\begin{aligned} & \frac{\delta_H l_H}{k \delta l} (1 -) E_{(k)} E_{(m)} y_{km} - E_{(m)} (1 -_{Hm}) y_{Hm} \\ & + \frac{\delta_L l_L}{k \delta l} (1 -) E_{(k)} E_{(m)} y_{km} - E_{(m)} (1 -_{Lm}) y_{Lm} \\ & + \frac{v_H q_H}{m v q} (1 -) E_{(k)} E_{(m)} y_{km} - E_{(k)} y_{kH} \\ & + \frac{v_L q_L}{m v q} (1 -) E_{(k)} E_{(m)} y_{km} - E_{(k)} y_{kL} \end{aligned} \right]
 \end{aligned}$$

$$\check{R} = N (1 - (+))$$

$\frac{\partial U_k}{\partial w_k} = -c_w (\frac{1}{k}) + M() (1 - \frac{w}{k}) w_k$
 $= -c_w \frac{Z_k^w}{M() w_k} + (1 - \frac{w}{k}) Z_k^w$
 $\frac{\partial U_k}{\partial w_k} = -c_w' \frac{1}{M() w_k} \frac{\partial Z_k^w}{\partial w_k} - c_w' \frac{Z_k^w}{M()} - \frac{1}{w_k^2} + \frac{\partial Z_k^w}{\partial w_k} (1 - \frac{w}{k}) = c_w' \frac{Z_k^w}{M() w_k} > 0;$
 $\frac{\partial U_k}{\partial w_k} = -c_w' \frac{1}{M() w_k} \frac{\partial Z_k^w}{\partial w_k} - c_w' \frac{Z_k^w}{M()} - \frac{1}{w_k^2} + \frac{\partial Z_k^w}{\partial w_k} (1 - \frac{w}{k}) = c_w' \frac{Z_k^w}{M() w_k} > 0;$

Proof of Lemma 7.

$\frac{\partial U_k}{\partial w_k} = -c_w (\frac{1}{k}) + M() (1 - \frac{w}{k}) w_k$

$$\begin{aligned}
 U_k &= -c_w (\frac{1}{k}) + M() (1 - \frac{w}{k}) w_k \\
 &= -c_w \frac{Z_k^w}{M() w_k} + (1 - \frac{w}{k}) Z_k^w
 \end{aligned}$$

$\frac{\partial U_k}{\partial w_k} = -c_w' \frac{1}{M() w_k} \frac{\partial Z_k^w}{\partial w_k} - c_w' \frac{Z_k^w}{M()} - \frac{1}{w_k^2} + \frac{\partial Z_k^w}{\partial w_k} (1 - \frac{w}{k}) = c_w' \frac{Z_k^w}{M() w_k} > 0;$

$$\frac{\partial U_k}{\partial w_k} = -c_w' \frac{1}{M() w_k} \frac{\partial Z_k^w}{\partial w_k} - c_w' \frac{Z_k^w}{M()} - \frac{1}{w_k^2} + \frac{\partial Z_k^w}{\partial w_k} (1 - \frac{w}{k}) = c_w' \frac{Z_k^w}{M() w_k} > 0;$$

$\frac{\partial U_k}{\partial w_k} = -c_w' \frac{1}{M() w_k} \frac{\partial Z_k^w}{\partial w_k} - c_w' \frac{Z_k^w}{M()} - \frac{1}{w_k^2} + \frac{\partial Z_k^w}{\partial w_k} (1 - \frac{w}{k}) = c_w' \frac{Z_k^w}{M() w_k} > 0;$

$$\frac{dz_H^\pi}{Z_H^\pi} \frac{1}{n_H^\pi} = \frac{E_{(k)} \left(\frac{dz_k^w}{z_k^w} + \frac{d\tau_L^\pi}{-\tau_L^\pi} \frac{v_L q_L}{m v q} n_L^\pi - \frac{d\tau_H^\pi}{-\tau_H^\pi} \left(1 + \frac{v_L q_L}{m v q} n_L^\pi \right) \right)}{\Delta_2} \quad (45)$$

$$\frac{dz_L^\pi}{Z_L^\pi} \frac{1}{n_L^\pi} = \frac{E_{(k)} \left(\frac{dz_k^w}{z_k^w} - \frac{d\tau_L^\pi}{-\tau_L^\pi} \left(1 + \frac{v_H q_H}{m v q} n_H^\pi \right) + \frac{d\tau_H^\pi}{-\tau_H^\pi} \frac{v_H q_H}{m v q} n_H^\pi \right)}{\Delta_2}; \quad (46)$$

$$\Delta_2 = 1 + E_{(m)} \frac{n_m^\pi}{m}. \quad (45) \quad (46)$$

$$E_{(m)} \frac{dz_m^\pi}{Z_m^\pi} = \frac{E_{(k)} \left(\frac{dz_k^w}{z_k^w} E_{(m)} \frac{n_m^\pi}{m} - E_{(m)} \left(\frac{n_m^\pi}{m} \frac{d\tau_m^\pi}{-\tau_m^\pi} \right) \right)}{\Delta_2}; \quad (47)$$

$$(43) \quad (44)$$

$$E_{(k)} \frac{dz_k^w}{Z_k^w} = \frac{(1 -) E_{(m)} \left(\frac{dz_m^\pi}{z_m^\pi} E_{(k)} \frac{n_k^w}{k} - E_{(k)} \left(\frac{n_k^w}{k} \frac{d\tau_k^w}{-\tau_k^w} \right) \right)}{\Delta_1}; \quad (48)$$

$$E_{(m)} \left(\frac{dz_m^\pi}{z_m^\pi} E_{(k)} \frac{n_k^w}{k} - E_{(k)} \left(\frac{dz_k^w}{z_k^w} \right) \right) \quad (47) \quad (48)$$

$$E_{(m)} \frac{dz_m^\pi}{Z_m^\pi} = - \frac{(\Delta_2 - 1) E_{(k)} \left(\frac{n_k^w}{k} \frac{d\tau_k^w}{-\tau_k^w} + \Delta_1 E_{(m)} \left(\frac{n_m^\pi}{m} \frac{d\tau_m^\pi}{-\tau_m^\pi} \right) \right)}{\Delta_1 + \Delta_2 - 1} \quad (49)$$

$$E_{(k)} \frac{dz_k^w}{Z_k^w} = - \frac{\Delta_2 E_{(k)} \left(\frac{n_k^w}{k} \frac{d\tau_k^w}{-\tau_k^w} + (\Delta_1 - 1) E_{(m)} \left(\frac{n_m^\pi}{m} \frac{d\tau_m^\pi}{-\tau_m^\pi} \right) \right)}{\Delta_1 + \Delta_2 - 1} \quad (50)$$

$$(49) \quad (43) \quad (44), \quad (50) \quad (45)$$

$$(46) \quad (45)$$

$$\frac{dz_H^w}{Z_H^w} \frac{1}{n_H^w} = - \frac{(1 -) (\Delta_2 - 1) E_{(k)} \left(\frac{n_k^w}{k} \frac{d\tau_k^w}{-\tau_k^w} + \Delta_1 E_{(m)} \left(\frac{n_m^\pi}{m} \frac{d\tau_m^\pi}{-\tau_m^\pi} \right) \right)}{\Delta_1 (\Delta_1 + \Delta_2 - 1)} + \frac{(\Delta_1 + \Delta_2 - 1) \frac{d\tau_L^w}{-\tau_L^w} (1 -) \frac{\delta_L l_L}{\delta l} n_L^w - \frac{d\tau_H^w}{-\tau_H^w} \left(1 + (1 -) \frac{\delta_L l_L}{\delta l} n_L^w \right)}{\Delta_1 (\Delta_1 + \Delta_2 - 1)}; \quad (51)$$

$$\frac{dz_L^w}{Z_L^w} \frac{1}{n_L^w} = - \frac{(1 -) (\Delta_2 - 1) E_{(k)} \left(\frac{n_k^w}{k} \frac{d\tau_k^w}{-\tau_k^w} + \Delta_1 E_{(m)} \left(\frac{n_m^\pi}{m} \frac{d\tau_m^\pi}{-\tau_m^\pi} \right) \right)}{\Delta_1 (\Delta_1 + \Delta_2 - 1)} + \frac{(\Delta_1 + \Delta_2 - 1) - \frac{d\tau_L^w}{-\tau_L^w} \left(1 + (1 -) \frac{\delta_H l_H}{\delta l} n_H^w \right) + \frac{d\tau_H^w}{-\tau_H^w} (1 -) \frac{\delta_H l_H}{\delta l} n_H^w}{\Delta_1 (\Delta_1 + \Delta_2 - 1)}; \quad (52)$$

$$\begin{aligned}
\frac{dz_H^w}{d\tau_H^\pi} \frac{1}{Z_H^w} &= - \frac{\frac{\varepsilon_H^\pi}{-\tau_H^\pi} \frac{v_H q_H}{m v q} \frac{n_w}{H} (1 -)}{\Delta_1 + \Delta_2 - 1} \\
\frac{dz_L^w}{d\tau_H^\pi} \frac{1}{Z_L^w} &= - \frac{\frac{\varepsilon_H^\pi}{-\tau_H^\pi} \frac{v_H q_H}{m v q} \frac{n_w}{L} (1 -)}{\Delta_1 + \Delta_2 - 1} \\
\frac{dz_H^\pi}{d\tau_H^\pi} \frac{1}{Z_H^\pi} &= \frac{\frac{n_H^\pi}{1 - \frac{\pi}{H}} \frac{n_H^\pi \frac{v_H q_H}{m v q} - (\Delta_1 + \Delta_2 - 1)}{\Delta_1 + \Delta_2 - 1}} \\
\frac{dz_L^\pi}{d\tau_H^\pi} \frac{1}{Z_L^\pi} &= \frac{\frac{\varepsilon_H^\pi}{-\tau_H^\pi} \frac{v_H q_H}{m v q} \frac{n_\pi}{L}}{\Delta_1 + \Delta_2 - 1}
\end{aligned} \tag{57}$$

$$\begin{aligned}
\frac{dz_H^w}{d\tau_L^\pi} \frac{1}{Z_H^w} &= - \frac{\frac{\varepsilon_L^\pi}{-\tau_L^\pi} \frac{v_L q_L}{m v q} \frac{n_w}{H} (1 -)}{\Delta_1 + \Delta_2 - 1} \\
\frac{dz_L^w}{d\tau_L^\pi} \frac{1}{Z_L^w} &= - \frac{\frac{\varepsilon_L^\pi}{-\tau_L^\pi} \frac{v_L q_L}{m v q} \frac{n_w}{L} (1 -)}{\Delta_1 + \Delta_2 - 1} \\
\frac{dz_H^\pi}{d\tau_L^\pi} \frac{1}{Z_H^\pi} &= \frac{\frac{\varepsilon_L^\pi}{-\tau_L^\pi} \frac{v_L q_L}{m v q} \frac{n_\pi}{H}}{\Delta_1 + \Delta_2 - 1} \\
\frac{dz_L^\pi}{d\tau_L^\pi} \frac{1}{Z_L^\pi} &= \frac{\frac{n_L^\pi}{1 - \frac{\pi}{L}} \frac{n_L^\pi \frac{v_L q_L}{m v q} - (\Delta_1 + \Delta_2 - 1)}{\Delta_1 + \Delta_2 - 1}}
\end{aligned} \tag{58}$$

$$\begin{aligned}
W &= I_k - c_w \frac{Z_k^w}{M(\cdot) W_k} + q_m - c_\pi \frac{Z_m^\pi}{\frac{M \theta}{\theta} m} \\
&+ {}_H I_H c'_w \frac{Z_H^w}{M(\cdot) W_H} + {}_L I_L c'_w \frac{Z_L^w}{M(\cdot) W_L} + v_H q_H c'_\pi \frac{Z_H^\pi}{\frac{M \theta}{\theta} H} + v_L q_L c'_\pi \frac{Z_L^\pi}{\frac{M \theta}{\theta} L} + R \\
&+ ({}_k I) M(\cdot) - \frac{{}_H I_H}{{}_k I} {}_H^w W_H + \frac{{}_L I_L}{{}_k I} {}_L^w W_L + \frac{v_H q_H}{m v q} \frac{\pi}{H} {}_H + \frac{v_L q_L}{m v q} \frac{\pi}{L} {}_L ;
\end{aligned}$$

$$a = \frac{{}_H I_H}{{}_k I} {}_H^w W_H + \frac{{}_L I_L}{{}_k I} {}_L^w W_L \quad b = \frac{v_H q_H}{m v q} \frac{\pi}{H} {}_H + \frac{v_L q_L}{m v q} \frac{\pi}{L} {}_L :$$

$$\begin{aligned}
& \frac{\partial L}{\partial w_H} = \\
& = l_k - \frac{c'_w}{M(\cdot) w_k} \frac{dz_k^w}{d w_H} + q_m - \frac{c'_\pi}{M \theta} \frac{dz_m^\pi}{d w_H} \\
& + \frac{dz_H^w}{d w_H} \frac{1}{M(\cdot) w_H} l_H c'_w \frac{z_H^w}{M(\cdot) w_H} + l_H c''_w \frac{z_H^w}{M(\cdot) w_H} \frac{1}{M(\cdot) w_H} \frac{dz_H^w}{d w_H} \\
& + \frac{dz_L^w}{d w_H} \frac{1}{M(\cdot) w_L} l_L c'_w \frac{z_L^w}{M(\cdot) w_L} + l_L c''_w \frac{z_L^w}{M(\cdot) w_L} \frac{1}{M(\cdot) w_L} \frac{dz_L^w}{d w_H} \\
& + \frac{dz_H^\pi}{d w_H} \frac{1}{M \theta} q_H c'_\pi \frac{z_H^\pi}{M \theta} + v_H q_H c''_\pi \frac{z_H^\pi}{M \theta} \frac{1}{M \theta} \frac{dz_H^\pi}{d w_H} \\
& + \frac{dz_L^\pi}{d w_H} \frac{1}{M \theta} q_L c'_\pi \frac{z_L^\pi}{M \theta} + v_L q_L c''_\pi \frac{z_L^\pi}{M \theta} \frac{1}{M \theta} \frac{dz_L^\pi}{d w_H} \\
& + \left[\frac{l_k}{M(\cdot) w_k} \frac{dz_k^w}{d w_H} M(\cdot) + (l_k) M(\cdot) \frac{m \frac{q_m}{M(\theta)} \frac{dz_m^\pi}{d w_H}}{k l} - \frac{(m v q)}{(k l)^2} \left(\frac{l_k}{M \theta} \frac{dz_k^w}{d w_H} \right) \right] (a + b) \\
& + (l_k) M(\cdot)
\end{aligned}$$

1980 T. IT. 1513 2674 T. 6 24 9 3876 432963 5381 35 251 1 12733 417 8 136 131 10 2 5 7 24 d (d) IT 2 4 16 1 1991 2 162 0 24 18 13 6 11 9 5 5 26 T. K. I. U. T. J. L. 47 T. K. T. J. F. H. @ 3 6 3 d 1 5 d H @ 3 q 1 9 9 1 3 0 d 1 8 9 1 0 0 1 / 0 0 1 4 0 d 1 1 5 1 1 2 6 d 1 1 1 0 5 d 0 1

$$\begin{aligned}
& + l_H c''_w \frac{z_H^w}{M(\cdot) w_H} \frac{1}{M(\cdot) w_H} \frac{dz_H^w}{d \tau_H^w} + l_L c''_w \frac{z_L^w}{M(\cdot) w_L} \\
& + v_H q_H c''_\pi \frac{z_H^\pi}{M \theta_H} \frac{1}{M \theta_H} \frac{dz_H^\pi}{d \tau_H^\pi} + v_L q_L c''_\pi \frac{z_L^\pi}{M \theta_L} \\
& + \left(\frac{l_k}{M \theta} \frac{dz_k^w}{d \tau_H^w} + \frac{M'(\cdot)}{M(\cdot)} \right) \frac{m}{I} \left(\frac{l_k}{M \theta} \frac{dz_k^w}{d \tau_H^w} + \frac{M'(\cdot)}{M(\cdot)} \right) (a + b) = \\
& = l_H c''_w \frac{z_H^w}{M(\cdot) w_H} \frac{1}{M(\cdot) w_H} \frac{dz_H^w}{d \tau_H^w} \frac{z_H^w}{z_H^w} \frac{M(\cdot)}{c'_w(z_H^w = M(\cdot) w_H)} \\
& + l_L c''_w \frac{z_L^w}{M(\cdot) w_L} \frac{1}{M(\cdot) w_L} \frac{dz_L^w}{d \tau_H^w} \frac{z_L^w}{z_L^w} \frac{M(\cdot)}{c'_w(z_L^w = M(\cdot) w_L)} (1 - \frac{w}{L}) w_L \\
& + v_H q_H c''_\pi \frac{z_H^\pi}{M \theta_H} \frac{1}{M \theta_H} \frac{dz_H^\pi}{d \tau_H^\pi} \frac{z_H^\pi}{z_H^\pi} \frac{M(\cdot)}{c'_\pi(z_H^\pi = \frac{M \theta}{H})} (1 - \frac{\pi}{H}) \frac{H}{H} \\
& + v_L q_L c''_\pi \frac{z_L^\pi}{M \theta_L} \frac{1}{M \theta_L} \frac{dz_L^\pi}{d \tau_H^\pi} \frac{z_L^\pi}{z_L^\pi} \frac{M(\cdot)}{c'_\pi(z_L^\pi = \frac{M \theta}{L})} (1 - \frac{\pi}{L}) \frac{L}{L} \\
& + \left(\frac{l_k}{M \theta} \frac{dz_k^w}{d \tau_H^w} + \frac{M'(\cdot)}{M(\cdot)} \right) \frac{m}{I} \left(\frac{l_k}{M \theta} \frac{dz_k^w}{d \tau_H^w} + \frac{M'(\cdot)}{M(\cdot)} \right)
\end{aligned}$$

$$= {}_H l_H \frac{1}{d_H^w} \frac{dz_H^w}{z_H^w}$$

$\frac{\pi}{H}$ $\frac{\pi}{L}$

$$(\Delta_1 + \Delta_2 - 1) (1 -)^{1 - \frac{w}{L}}$$

$$\begin{aligned}
& \left[\begin{aligned}
& \left(\frac{\delta_{H^l H}}{\delta l} \frac{v_L q_L}{m v q} + \frac{\delta_{L^l L}}{\delta l} \frac{v_H q_H}{m v q} \right) W_{HH} + \left(\frac{\delta_{H^l H}}{\delta l} \frac{v_H q_H}{m v q} + \frac{\delta_{L^l L}}{\delta l} \frac{v_L q_L}{m v q} \right) W_{HL} \\
& + \left(\frac{\delta_{H^l H}}{\delta l} \frac{v_H q_H}{m v q} + \frac{\delta_{L^l L}}{\delta l} \frac{v_L q_L}{m v q} \right) W_{LH} + \left(\frac{\delta_{H^l H}}{\delta l} \frac{v_L q_L}{m v q} + \frac{\delta_{L^l L}}{\delta l} \frac{v_H q_H}{m v q} \right) W_{LL} \\
& + \left(\frac{\delta_{H^l H}}{\delta l} \frac{v_L q_L}{m v q} + \frac{\delta_{L^l L}}{\delta l} \frac{v_H q_H}{m v q} \right) W_{HH} + \left(\frac{\delta_{H^l H}}{\delta l} \frac{v_H q_H}{m v q} + \frac{\delta_{L^l L}}{\delta l} \frac{v_L q_L}{m v q} \right) W_{HL} \\
& + \left(\frac{\delta_{H^l H}}{\delta l} \frac{v_H q_H}{m v q} + \frac{\delta_{L^l L}}{\delta l} \frac{v_L q_L}{m v q} \right) W_{LH} + \left(\frac{\delta_{H^l H}}{\delta l} \frac{v_L q_L}{m v q} + \frac{\delta_{L^l L}}{\delta l} \frac{v_H q_H}{m v q} \right) W_{LL}
\end{aligned} \right] \\
= & 1 - \frac{\frac{-\tau_H^w}{\varepsilon_H^w} W_H + \frac{-\tau_L^w}{\varepsilon_L^w} W_L + \frac{-\tau_H^\pi}{\varepsilon_H^\pi} W_H + \frac{-\tau_L^\pi}{\varepsilon_L^\pi} W_L}{\frac{-\tau_H^w}{\varepsilon_H^w} W_H + \frac{-\tau_L^w}{\varepsilon_L^w} W_L + \frac{-\tau_H^\pi}{\varepsilon_H^\pi} W_H + \frac{-\tau_L^\pi}{\varepsilon_L^\pi} W_L} ;
\end{aligned}$$

$$= 1 - \frac{E_{(s)} W_{Hs}^w + E_{(s)} W_{Ls}^w + E_{(s)} W_{Hs}^\pi + E_{(s)} W_{Ls}^\pi}{\frac{-\tau_H^w}{\varepsilon_H^w} W_H + \frac{-\tau_L^w}{\varepsilon_L^w} W_L + \frac{-\tau_H^\pi}{\varepsilon_H^\pi} W_H + \frac{-\tau_L^\pi}{\varepsilon_L^\pi} W_L} ; \quad (64)$$

$$\left(\frac{\delta_{H^l H}}{\delta l} \frac{v_L q_L}{m v q} + \frac{\delta_{L^l L}}{\delta l} \frac{v_H q_H}{m v q} \right) W_{HH} + \left(\frac{\delta_{H^l H}}{\delta l} \frac{v_H q_H}{m v q} + \frac{\delta_{L^l L}}{\delta l} \frac{v_L q_L}{m v q} \right) W_{HL}$$

$\Delta_1 + \Delta_2 - 1$ $(1 - \tau_w)$ $\frac{1 - \tau_w}{\tau_w} w$ $(w^w + \tau_w - (1 - \tau_w) \bar{R})$ $=$

$$\begin{aligned}
 & (\Delta_1 + \Delta_2 - 1) (1 - \tau_w) \frac{1 - \tau_w}{\tau_w} w + (w^w + \tau_w - (1 - \tau_w) \bar{R}) = \\
 & = (1 - \tau_w) [(1 - \tau_w) w + (w^w + \tau_w - (1 - \tau_w) \bar{R})]
 \end{aligned}$$

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