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Advanced-Purchase Premiums versus Discounts in the Presence of Capacity Constraints

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Those consumers with a low cost of consuming the wrong good will take the discount and buy in advance, leaving only the high-cost consumers remaining in the market. These consumers can then be charged a higher price for their preferred ight once their preferences become known.

I extend their model to include three consumer types: those who are uncertain as to their preferences, those who prefer good one, and those who prefer good two. Each consumer will have a cost of consuming their less preferred good. This cost is continuous from having no preference between the goods to having no value for the less preferred good. With a capacity constrained monopolist, preference-uncertain consumers can be forced to separate into relatively high-cost and relatively low-cost consumers being o ered an advanced-purchase discount. The low-cost, preference-uncertain consumers will be willing to take the discount since the risk of purchasing the wrong good is not a large concern. The high-cost, preference-uncertain consumers are forced to wait as the discount does not cover the risk of getting the wrong good. In the case of a discount, all preference-certain consumers will purchase their preferred good in advance. The e ectiveness of the discount will depend on both the number of preference-uncertain consumers and the distribution of the costs of mismatching.

On the other hand, if a premium is charged, preference-uncertain consumers will be unwilling to pay it[1] but the high-cost preference-certain consumers will (if the premium is not too large). These customers will purchase in advance to ensure themselves from the good being sold out if they wait. The low-cost preference-certain consumers will not pay the premium. They are willing t354(of)-35F17 11.955oaepref69298(oF17 11.2-sm46d1ds)ef preference-uncertain consumers, it is the high-cost preference-certain consumers that are now willing to purchase in advance paying the premium. The low-cost preference-certain consumers will wait. Having more high-cost, preference certain consumers makes charging a premium for advanced-purchases more effective. In the end, because of the dichotomy in how different consumer types can be discriminated against, the relative size of these types and the distribution of mismatch costs will determine whether a premium or discount is profit maximizing. If there are mostly preference-certain consumers with a high cost to consuming the less preferred good, a premium will be charged. If there are mostly preference-uncertain consumers with low costs of consuming the less preferred good, an advanced-purchase discount will prevail.

Others have examined various causes for discount and premium pricing for advanced sales. Dana (1992) showed that advanced-purchase discounts can persist in competitive markets for goods in which capacity is not storable. Gilbert and Klemperer (2000) examines the case where consumers have to make a sunk cost investment to participate in the market. Under this case committing to a price that leads to excess demand is optimal. Rosen and Rosenfield (1997) looks at intertemporal pricing issues when faced with the problem of managing capacity. They show that under constrained capacity for multiple substitute goods, like theater performances on different days, it can be optimal to queue the consumers for the early good and let prices decline over time. Finally, Spulber (1993) proves that there are multiple profit maximizing pricing strategies, from reference point pricing to priority service, for a capacity constrained monopolist selling two quality differentiated goods when faced with an unknown distribution of consumers who have multi-unit demand.

The paper proceeds in Section 2 by developing an example to give intuition for when an advanced-purchase premiums are possible. Section 3 expands the Gale and Holmes model by adding preference-certain consumers and sets up the key assumptions. Section 4 examines the behavior of consumers within the model showing how various consumer types can be discriminated against. Section 5 will develop the optimal pricing policy for the monopolist. Section 6 relates the model results back to the motivating examples of concert tickets and airlines. Section 7 summarizes and concludes.

2 An Example of Advanced-Purchase Premiums

To show how advanced-purchase premiums are possible, a motivating example is presented here. Consider a monopolist selling concert tickets to a unit mass of two consumer types: high valuation and low valuation consumers. The monopolist has .75 tickets and each set of consumer types account for half of the total consumer population. The high valuation consumers value the tickets at $50, the low valuation consumers value them at $40 each, and all consumers have unit demand. For simplicity the seller has a marginal cost of zero[2]. The monopolist can charge one price for day of concert sales and a second price for advanced-purchases but must commit to both prices in advance.

The monopolist considers three different potentially optimal strategies. First, she could set a price of $50 selling only to the .5 high valuation consumers, netting a profit of = $25.

expected surplus of waiting is $E[CS_H^{Wait}] = :5 \cdot (\$50 - \$40) = \$5$. If he purchases now he gets a surplus of $CS_H^{Now} = \$50 - \$45 = \$5$, so he is just as well o purchasing now. The low-types get zero surplus by waiting and negative surplus by purchasing now so they are happy to wait. Finally, the monopolist cannot charge a higher price in either period without losing customers. Lowering the price in either period will reduce revenue so her strategy is a best response to the consumer behavior. Charging an advanced-purchase premium of \$5 on top of the \$40 base price is the pro t maximizing strategy.

To examine what determines the size of the premium that can be charged, the example is expanded here. Let there be ${}_H$ high valuation customers who value a ticket at v_H and ${}_L$ low valuation customers who value a ticket at v_L. Using a continuum of consumers, normalize ${}_H$ and ${}_L$ to be proportions of the total population so that ${}_L + {}_H = 1$. Also, without loss of generality, $v_L < v_H$. The monopolist has a capacity of $K \in ({}_H; 1)$. This ensures that all high valuation consumers can purchase a ticket in advance and that some tickets are rationed among the low valuation consumers. The monopolist will set the day of price to v_L. If all high-type consumers purchase in advance then there will be $K - {}_H$ tickets remaining for day of show sales to be purchased by ${}_L$ low valuation customers. As such any high-type consumer who chooses to wait will get a surplus of $E[CS_H^{Wait}] = Pr(\text{Get Ticket}|\text{High Type \& Wait}) \cdot (v_H - p_{Wait}) = \frac{K \quad {}_H}{1 \quad {}_H}(v_H - v_L)$. The monopolist must give the high-type consumers this much in surplus from the advance purchase. So $p_{Advance} = v_H - E[CS_H^{Wait}] = \frac{1 \quad K}{1 \quad {}_H} v_H + \frac{K \quad {}_H}{1 \quad {}_H} v_L$. The absolute premium charged is then $p_{Advance} - p_{Wait} = \frac{1 \quad K}{1 \quad {}_H}(v_H - v_L)$, while the markup is given by $\frac{p_{Advance} \quad p_{Wait}}{p_{Wait}} = \frac{1 \quad K}{1 \quad {}_H}(\frac{v_H}{v_L} - 1)$[4].

From the solution it can be seen that a lower capacity results in more rationing, increasing the premium that high-types are willing to pay because they are less likely to receive a ticket

higher probability, a larger premium could be sustained during advanced sales.

[4] This solution is for the separating equilibrium that prevails under the given capacity restriction. The two pooling equilibria where a single price equal to v_H or v_L prevails can happen if the capacity is smaller than the number of high valuation consumers. Which one occurs is dependent on both the sizes of the consumer populations and their relative valuations.

if they wait. Likewise, having more high-type consumers reduces the day of show supply of tickets increasing the premium that can be charged. Finally, the larger the absolute spread between the high and low valuations the more that high-type customers are giving up by not receiving a ticket, and thus the higher premium they are willing to pay. The higher the relative spread, the higher the markup percentage on advanced ticket prices for the same reason. It is worth noting that unlike the DeGraba-Mohammed result bundling is not needed to get an advanced-purchase premium. While not captured in this example, risk aversion by the high valuation types would increase the premium they would be willing to pay because it increases the cost of not getting a ticket. Also, a rationing rule that lowered the probability of a high valuation consumer receiving a ticket if they waited would increase the premium that could be charged.

3 Model Description

This model extends the Gale and Holmes (1992) model of advanced purchase discounts by adding additional consumer types. Their model showed how advanced-purchase discounts arise with preference-uncertain consumers and a monopolist selling two goods. This model will extend that framework to include preference-certain consumers. There are two goods: good-A and good-B. Sales of the goods occur both in advance at $t = 0$ and immediately prior to consumption at $t = 1$. There is a continuum of measure one, risk-neutral consumers with unit demand and distribution of reservation valuations[5] of $r \sim f_r(\cdot)$ on $[0; r_{max}$ for their preferred good. Consumers vary in two dimensions: their preference over the goods and their cost of being mismatched with their preferred good. Expanding the Gale-Holmes model, there are continua of three consumer types rather than just the one. Type-A consumers have a strict preference for good-A; type-B consumers have a strict preference for good-B;

[5] The reservation value is the highest price that a consumer is willing to pay and still receive a non-negative surplus. This terminology is used to match with that of Gale and Holmes.

type-U consumers are initially uncertain as to their preferences at $t = 0$. For simplicity, equal numbers of type-A and type-B consumers is assumed. Denoting the number of consumers of each type by $_i$, $_A = _B$ and $_A + _B + _U = 1$. Henceforth, imposing the symmetry assumption, I will use $_A$ for both the number of type-A and type-B consumers.

Assumption 1. *Ex-ante Probability that Type-U Prefers Peak Good*

(a) $Pr[\text{Type-U prefers peak good}|t = 0] =$

(b) $Pr[\text{Type-U prefers o -peak good}|t = 0] = 1 -$

(c) $> \frac{1}{2}$

At $t = 1$, $> \frac{1}{2}$ of these uncertain consumers will prefer the peak good[6] while the remaining $1 -$ uncertain consumers will prefer the o -peak good. Ex-ante each good is equally likely to be the high demand (peak) good.

Assumption 2. *Ex-ante Peak Demand*

$\Pr[\text{Good-}A \text{ is Peak}|t = 0] = \Pr[\text{Good-}B \text{ is Peak }|t = 0] = \frac{1}{2}$

Consumers also vary in their costs of mismatching denoted by y. A y-cost consumer has a reservation valuation of $r - y$ for the less preferred good. The distribution of costs y has a continuous density function $f_y(\cdot)$ and a di erentiable cumulative distribution $F_y(\cdot)$ on $[0; r]$ which is stochastically independent of the consumer's preference type[7]. y may however be correlated with the reservation valuation r for a given consumer. Thus the reservation valuation and cost of mismatching have the joint distribution $(r; y) \sim f(r; y)$ on the support $[0; r_{max}] \times [0; r]$. This distribution is assumed to be continuous everywhere. This distribution ensures that the valuation for the less preferred good is non-negative and is weakly less than

[6] The peak good is the good which has higher ex-post demand.

[7] While di erent consumers types could have a di erent distribution of mismatch costs, this would complicate the analysis without su ciently adding to the results.

the reserve valuation for the preferred good. Additionally, demand will be downward sloping in price. Finally, each consumer knows their own reservation value and cost of consuming the less preferred good in advance but the seller only knows the distribution of the valuations and costs.

Assumption 3. *Capacity Bounds*

$$_U + \quad _A > K \geq \frac{1}{2}$$

This condition ensures that while the peak good will need to be rationed if all consumers were to wait to purchase, every consumer can always choose to wait and purchase one of the two goods. Additionally, the marginal cost of both goods is normalized to zero[8].

Finally, as in Gale and Holmes (1992), the monopolist firm will commit to prices for both periods in advance. Let p_0 be the price for advance sales and p_1 be the price for sales at time $t = 1$. All sales at $t = 0$ are final and there is no secondary market for these goods. Since the monopolist commits to the pricing of the goods in advance, the goods' prices cannot be conditional on whether the good is peak or off peak. Since this paper examines the optimality of advanced pricing discounts and premiums, define $p_0 = p_1 +$ where < 0 denotes a discount for purchasing in advance and > 0 denotes having to pay a premium in advance.

4 Consumer Behavior

Consumers in this model make three choices in two time periods. Consumers will first decide whether to purchase in the advance period, $t = 0$, at price p_0 or postpone the purchase decision to until $t = 1$. Type-U consumers learn their preference between time $t = 0$ and time $t = 1$. If a consumer did not purchase in period zero he will try to purchase the preferred

$t = 1$ to purchase the good is,[9]

$$v_{1A}(r; y) = v_{1B}(r; y) = \begin{cases} \frac{1}{2}[\phi_p(r - p_1) + (1 - \phi_p)(r - p_1 - y)] + \frac{1}{2}(r - p_1) & \text{if } y \leq r - p_1 \\ \frac{1}{2}[\phi_p(r - p_1)] + \frac{1}{2}(r - p_1) & \text{if } y > r - p_1 \end{cases}$$

Simplifying yields,

$$v_{1A}(r; y) = v_{1B}(r; y) = \begin{cases} (r - p_1) - \frac{1 - \phi_p}{2} y & \text{if } y \leq r - p_1 \\ \frac{1 + \phi_p}{2}(r - p_1) & \text{if } y > r - p_1 \end{cases} \tag{2}$$

It is easily verified that $v_{1A}(r; y)$ and $v_{1B}(r; y)$ are continuous, weakly decreasing functions in y since both halves of the piecewise function take the same value at $y = r - p_1$. Since each section of the piecewise $v_{1A}(r; y)$ is linear, the only possible discontinuity occurs at $y = r - p_1$. Because both sides of the function take the same value, $\frac{1+\phi_p}{2}(r - p_1)$, at this point $v_{1A}(r; y)$ is continuous. For low values of y, $v_{1A}(r; y)$ is a negative sloped line and for high values of y the valuation function is constant. This ensure that $v_{1A}(r; y)$ is weakly decreasing.

Lemma 1. *$v_{1A}(r; y)$ and $v_{1B}(r; y)$ are continuous and weakly decreasing in y. $v_{1A}(r; y)$ and $v_{0A}(r; y)$ intersect at most once if $\in [0; \frac{1-\phi_p}{2}(r - p_1))$ and never intersect if $\notin [0; \frac{1-\phi_p}{2}(r - p_1))$*[10].

Proof. $v_{1A}(r; y)$ and $v_{1B}(r; y)$ are clearly continuous and weakly decreasing.

For the single crossing property, consider three cases: a discount, a small premium, and a large premium. (See Figure 1)

Case I, Discount: In the discount case the surplus for waiting is always below that for purchasing now because < 0 gives us $v_{1A}(r; 0) = r - p_1 \leq v_{0A}(r; 0) = r - p_1 -$. With

[9] This assumes that there is no excess demand for the off peak good. This is ensured by the capacity Assumption 3.

[10] In the case of $= \frac{1-\phi_p}{2}(r - p_1)$; $v_{1A}(r; y) = v_{0A}(r; y) \forall y \geq r - p_1$. The curves intersect but never cross. This corresponds to high-y consumers being indifferent between purchasing in advance and waiting. All consumers can be considered wait.

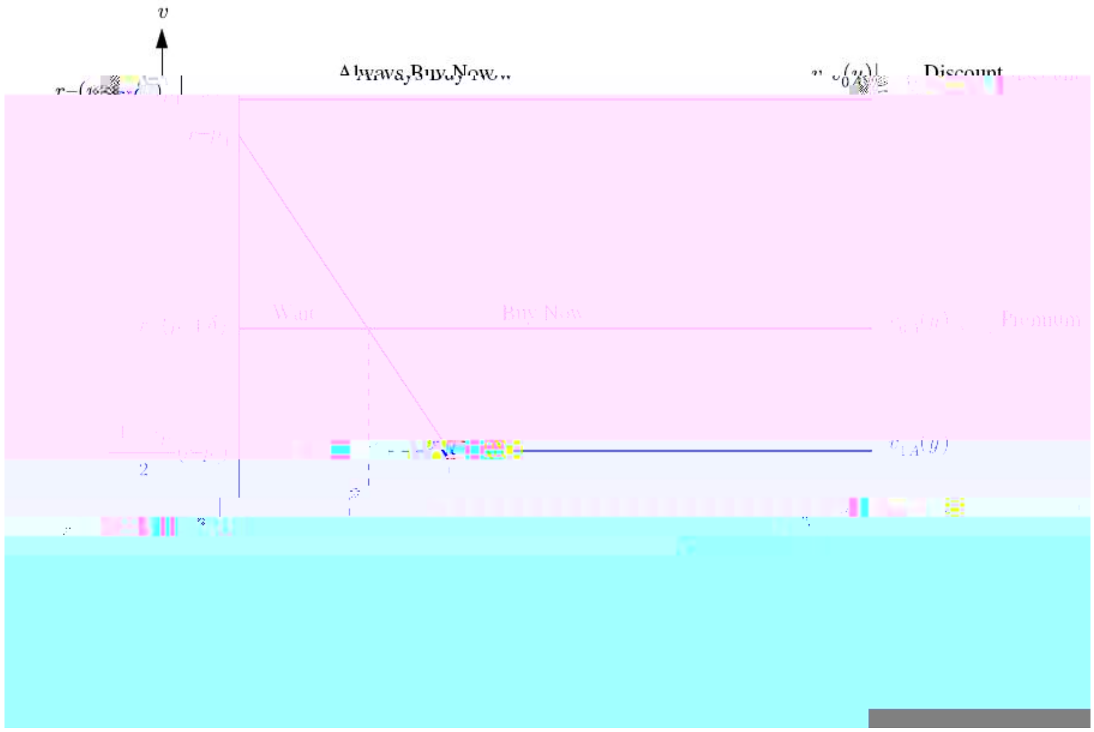


Figure 1: Expected Consumer Surplus for Type-A Consumers

$v_{1A}(r; y)$ being weakly decreasing in y, the surplus lines never cross.

Case II, Small Premium: In this case the consumer surplus from waiting starts above that for buying now but ends below. Examining the range $\in [0; \frac{1}{2}{}^{p}(r - p_1))$. $v_{1A}(r; 0) = r - p_1 \geq v_{0A}(r; 0) = r - p_1 -$ since is positive. $v_{1A}(r; r - p_1) = \frac{1}{2}{}^{p}(r - p_1) \geq v_{0A}(r; 0) = r - p_1 -$ since $\leq \frac{1}{2}{}^{p}(r - p_1)$. Since the surplus functions are continuous, the intermediate value theorem guarantees a crossing. Monotonicity ensures that it is a single crossing.

Case III, Large Premium: In this case the consumer surplus from waiting is always above that for purchasing now. For $> \frac{1}{2}{}^{p}(r - p_1)$, $\min v_{1A}(y) = r - p_1 - \quad > v_{0A}(y) = r - p_1 -$. The surplus functions will never cross. □

Next we can verify that $v_{1A}(r; y)$ and $v_{0A}(r; y)$ cross at most once.

Let y_A be the crossing point of $v_{1A}(r; y)$ and $v_{1B}(r; y)$ or 0 if they never cross. Solving

$v_{1A}(r;y_A) = v_{1B}(r;y_A)$ yields,

$$y_A(r) = \begin{cases} 0 & \text{if} \quad \in (-r;0) \\ \frac{2}{1 \quad p} & \text{if} \quad \in [0;\frac{1 \quad p}{2}(r-p_1)] \\ r & \text{if} \quad \in (\frac{1 \quad p}{2}(r-p_1);r) \end{cases} \tag{3}$$

.

If a discount is o ered for advanced purchases, all type-A consumers will purchase in advance resulting in $y_A(r) = 0$. Alternatively, for a large premium, $\geq \frac{1 \quad p}{2}(r_{max} - p_1)$, all type-$A$ customers will wait resulting in $y_A(r) = r$. A type-A consumer will purchase in adhappens half the time. Uncertain consumers are equally like to prefer either good[11]. This makes the expected surplus of a type-U consumer buying in advance v_{0U}.

$$v_{0U}(r; y) = r - p_0 - \frac{1}{2}y \tag{4}$$

Lemma 3. *$v_{0U}(r; y)$ is continuous and decreasing with respect to y.*

If a preference-uncertain consumer waits, he risk wanting the rationed peak good. Since each good is equally likely to be the peak good, half the time the uncertain consumers will end up wanting the peak good.

$$v_{1U}(r; y) = \begin{cases} \;_p(r - p_1) + \quad (1 - \quad _p)(r - p_1 - y) + (1 - \quad)(r - p_1) & \text{if } y \leq r - p_1 \\ \;_p(r - p_1) + (1 - \quad)(r - p_1) & \text{if } y > r - p_1 \end{cases}$$

Simplifying yields,

$$v_{1U}(r; y) = \begin{cases} (r - p_1) - \quad (1 - \quad _p)y & \text{if } y \leq r - p_1 \\ ((1 - \quad (1 - \quad _p))(r - p_1) & \text{if } y > r - p_1 \end{cases} \tag{5}$$

$v_{0U}(r; y)$ is downward sloping in y rather than constant as is $v_{0A}(r; y)$. In the case of a premium, this may lead to either a no crossing or double crossing depending on the slope of $v_{1U}(r; y)$. For an advanced-purchase discount a single crossing will occur. Regardless of the number of crossings in the premium case, the quantity of goods purchased in advance by type-U consumers will be decreasing and continuous with respect to (increasing as a larger discount is offered, see Figure 2 and Figure 3). To define this quantity it is useful to define the two potential crossing points as $y_U^H(r)$ and $y_U^L(r)$. If a discount is given then there is a single crossing. In this case define $y_U^H(r)$ as this crossing point and define $y_U^L(r) = 0$. If a

[11] If this were not the case then they would have a strict preference between the goods. Consumers who have a strict preference for one good in advance but may have that preference switch in the second time

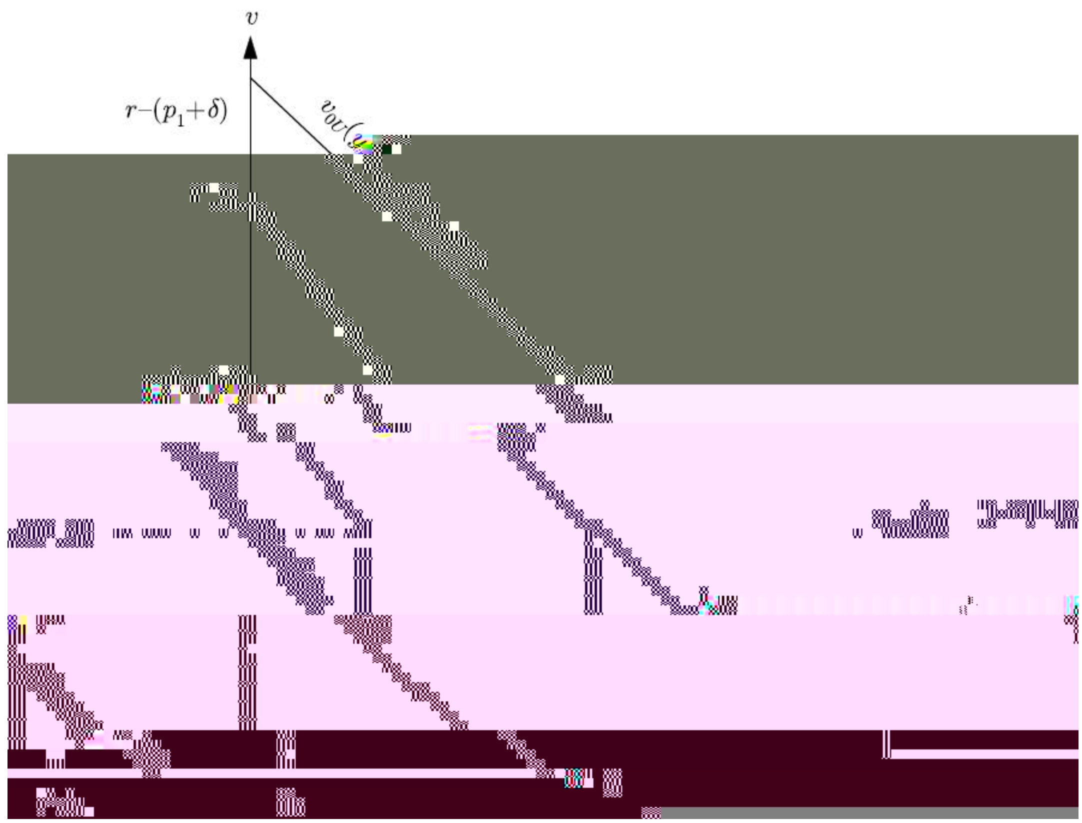
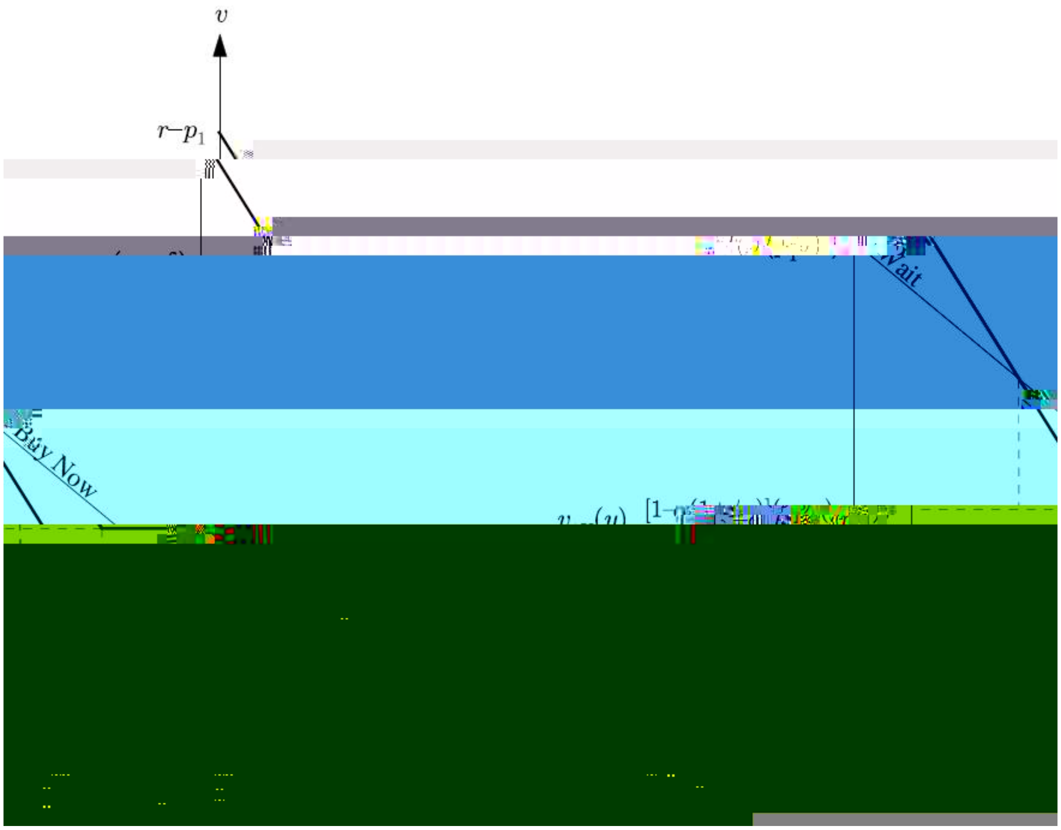


Figure 2: Expected Consumer Surplus for Type-U Consumers

premium is charged and there are two crossings, let $\bar{y}^H_U(r)$ be the larger of the two and $\bar{y}^L_U(r)$ be the smaller of the two. As the premium charged increases, $\bar{y}^L_U(r)$ increases to $r - p_1$ while $\bar{y}^H_U(r)$ falls to $r - p_1$. Alternatively there is no crossing if $v_{0U}(r; y)$ is steeper than $v_{1U}(r; y)$: $\frac{1}{2} > \ (1 - \ _p)$. In this case, no type-$U$ consumers will purchase in advance and all will wait if there is a premium charged so $\bar{y}^H_U(r) = \bar{y}^L_U(r)$. This yields

$$\bar{y}^L_U(r) = \begin{cases} 0 & \text{if } \frac{1}{2} > \ (1 - \ _p) \\ 0 & \text{if } \frac{1}{2} \leq \ (1 - \ _p), \ \leq 0 \\ \frac{}{(1 \ \ _p) \ \frac{1}{2}} & \text{if } \frac{1}{2} \leq \ (1 - \ _p), (\ (1 - \ _p) - \frac{1}{2})(r - p_1) > \ > 0 \\ r - p_1 & \text{if } \frac{1}{2} \leq \ (1 - \ _p), \ \geq (\ (1 - \ _p) - \frac{1}{2})(r - p_1) \end{cases} \tag{6}$$



v
r−p1
Buy Now
Wait

Lemma 4. *Q_{0U} is a function of , $_p$, and p_1 and is continuous everywhere.*

Proof. $\bar{y}_U^L$ is weakly increasing in and continuous in , $_p$, and p_1. $\bar{y}_U^H$ is continuous in , $_p$, and p_1

Proposition 1. *There exists a* $\ _p$ *such that* $\ _p = \frac{S_1^P(\ _p)}{D_1^P(\ _p)}$.

Proof. $\frac{S_1^P(\ _p)}{D_1^P(\ _p)}$ has only one discontinuity. It will be shown to be outside of the possible range of $\ _p$

of the size of the advanced-purchase discount or premium and show that either can occur depending on the relative size of each consumer type, the number of consumers who will prefer the peak good, and the capacity constraint. First, the monopolist's profit maximization problem will be defined. Next, properties of the demand function will be proven. Finally, using the demand properties, it will be shown that under different consumer characteristics a premium or a discount can prevail.

If all consumers are preference-uncertain, this model collapses to the Gale-Holmes model and an advanced-purchase discount will be optimal. Alternatively, if there are mostly preference-certain consumers in the market an advanced-purchase premium will prevail. The premium will be possible because the preference-uncertain consumers will have a lower expected valuation than the preference-certain consumers. This occurs because they have the risk of getting the wrong good when purchasing in advance. Preference-certain consumers always purchase the correct good in advance. The difference in valuations for advanced-purchases creates a risk of rationing in the second period. As in the simple example (Section 2) this gives the higher-valuation, preference-certain consumers an incentive to pay a premium and purchase in advance.

The monopolist's profit maximization problem is as follows:

$$\max_{(\ ;p_1)} \sum_{j=A;B;U}$$

$$(\frac{\partial Q_0}{\partial})(p_1 + \ ) + (\frac{\partial Q_1}{\partial})p_1 + Q_0 = 0 \quad (10)$$

Solving the rst order condition for yields:

$$= \frac{-(\frac{\partial Q_0}{\partial} + \frac{\partial Q_1}{\partial})p_1 - Q_0}{\frac{\partial Q_0}{\partial}} \quad (11)$$

As the premium charged increases, the quantity of advanced purchase must fall so $\frac{\partial Q_0}{\partial} \leq 0$[14].

Lemma 6. $\frac{\partial Q_0}{\partial} \leq 0$

Proof. $\tilde{y}_A$ is the intersection of $(r - p_1) - \frac{1 \ \ p}{2} y$ and the constant $r - p_1 -$ when the intersection is at $y \leq r - p_1$. $\tilde{y}_A = r$ for $\geq \frac{1 \ \ p}{2}(r - p_1)$. Thus

$$\tilde{y}_A(\ ) = \tilde{y}_B(\ ) = \begin{cases} \frac{2}{1 \ \ p} & \text{if} \ < \frac{1 \ \ p}{2}(r - p_1) \\ r & \text{if} \ \geq \frac{1 \ \ p}{2}(r - p_1) \end{cases} \quad (12)$$

$\tilde{y}_A(\ )$ is increasing in . $Q_{0A} = \ _A(1 - F(\tilde{y}_A))$ is decreasing in because $F(\cdot)$ is an increasing function of $\tilde{y}_A(\ )$.

As shown before, $\tilde{y}_U^L$ and $\tilde{y}_U^H$ converge to $r - p_1$ as $\rightarrow (\frac{1}{2} - \ (1 - \ _p))(r - p_1)$. Thus $Q_{0U} = \ _U(F(\tilde{y}_U^H) - F(\tilde{y}_U^L))$ is decreasing in because $F(\cdot)$ is increasing, $\tilde{y}_U^H$ is decreasing, and $\tilde{y}_U^L$ is increasing.

Since $Q_0 = Q_{0A} + Q_{0B} + Q_{0U}$ it must also be decreasing in . □

This means that for a premium to be charged for advanced purchases (> 0) it must be the case that $(\frac{\partial Q_0}{\partial} + \frac{\partial Q_1}{\partial})p_1 + Q_0 > 0$.

First, in examining $\frac{\partial Q_0}{\partial} + \frac{\partial Q_1}{\partial}$ it is useful to decompose the expression into $\frac{\partial Q_{0A}}{\partial} + \frac{\partial Q_{1A}}{\partial} + \frac{\partial Q_{0B}}{\partial} + \frac{\partial Q_{1B}}{\partial} + \frac{\partial Q_{0U}}{\partial} + \frac{\partial Q_{1U}}{\partial}$. It is shown in Lemma 7 that $\frac{\partial Q_{0A}}{\partial} + \frac{\partial Q_{1A}}{\partial} = 0$. The same is

[14] Intuitively as the premium charged increases fewer consumers will purchase in advance.

true for type-B consumers. The low-cost consumers are more likely to wait but they are the types that are willing to purchase their less preferred good.

Lemma 7. *As the premium charged increases, all preference-certain consumers that no longer purchase at $t = 0$ purchase at $t = 1$. Thus $\frac{\partial Q_{0A}}{\partial} + \frac{\partial Q_{1A}}{\partial} = 0$.*

Proof. As long as the premium is not too large[15], all type-A consumers who wait until $t = 1$ to purchase have $y < r - p_1$ since it is the low-y preference-certain consumers who wait (see Figure 1). For these relevant premiums ($< \frac{1 \ \ p}{2}(r - p_1)$), the consumers who no longer purchase in advance are willing to purchase the less preferred good. As such, all preference-certain consumers purchase and any decrease in Q_{0A} is matched by an increase in Q_{1A}. Thus $\frac{\partial Q_{0A}}{\partial} + \frac{\partial Q_{1A}}{\partial} = 0$. □

For type-U consumers, however, a decrease in Q_{0U} corresponds to a less than one-for-one increase in Q_{1U} because it is high-y consumers waiting and they won't purchase their less preferred good if their preferred good is sold out. Therefore $\frac{\partial Q_0 U}{\partial} + \frac{\partial Q_1 U}{\partial} \leq 0$.

Lemma 8. *As the premium charged increases, only some preference-uncertain consumers that no longer purchase in advance choose to purchase in the second period. Thus $\frac{\partial Q_{0U}}{\partial} + \frac{\partial Q_{1U}}{\partial} \leq 0$.*

Proof. For a premium, either all uncertain consumers waited in which case changing has no e ect: $\frac{\partial Q_0 U}{\partial} + \frac{\partial Q_1 U}{\partial} = 0$. Alternatively, it may be the case that some low-y consumers and some high-y consumers wait (see Figure 3). Here increasing leads to more of both consumer types waiting. The low-y types have $y < r - p_1$ and thus will purchase a less preferred good if need be. The high-y types have $y > r - p_1$ and thus will not purchase a less preferred good if need be. Since not all of the consumers who used to purchase in advance purchase now, $-\frac{\partial Q_0 U}{\partial} > \frac{\partial Q_1 U}{\partial}$. Thus, $\frac{\partial Q_0 U}{\partial} + \frac{\partial Q_1 U}{\partial} \leq 0$. □

[15] This is the same restriction as for the existence of *.

Finally, using these two properties of the demand functions it can be shown that it is possible to get a premium or a discount in equilibrium. The discount case is the limiting case of no preference-certain consumers and was proved in Gale and Holmes (1993). Due to the continuity of the problem, this result must hold in the neighborhood of having no

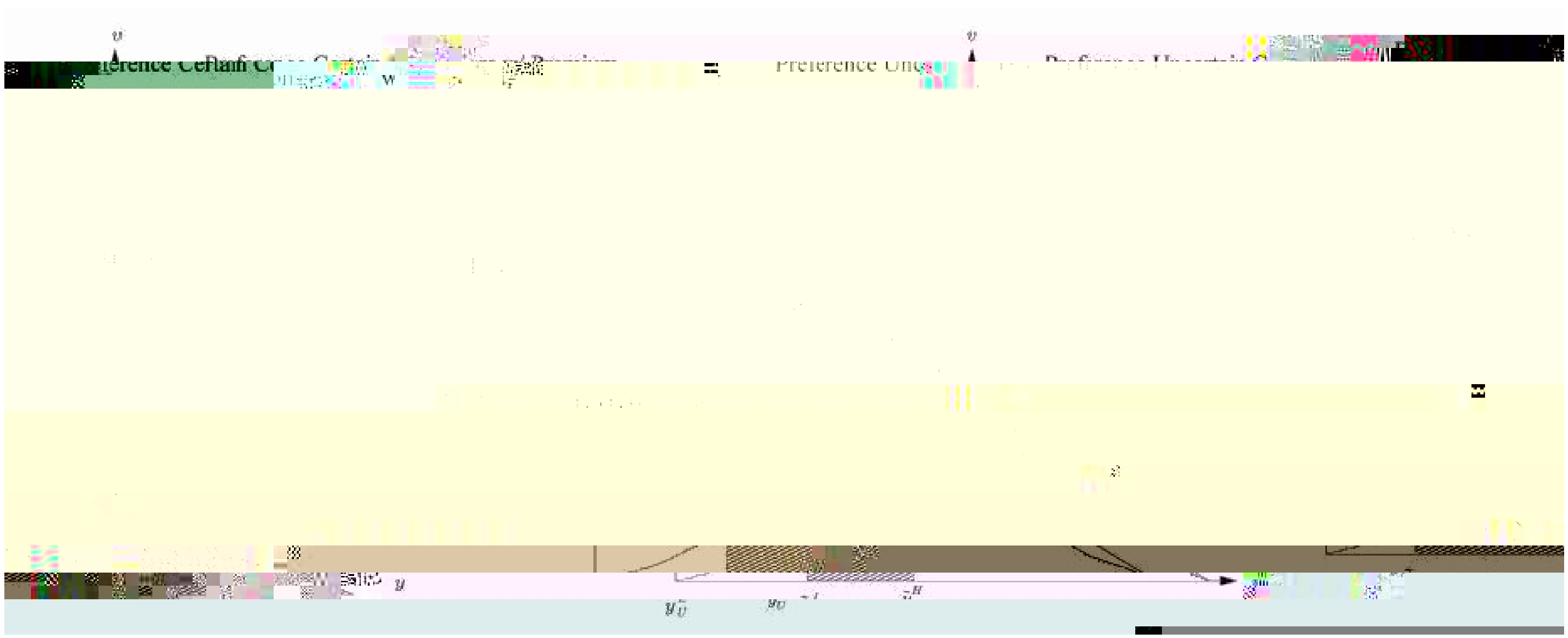


Figure 4: Effect of High-Cost Distribution

Fandango or waiting until they arrive at the theater to purchase the tickets. Fandango charges a premium of $1.00 per ticket purchased in advance. Most consumers that buy in advance do so either the same day as the show or a few days prior choosing between different show times and locations. These consumers typically know which showtime and location they will prefer. For popular shows however there is a risk of selling out. As such consumers who have strong preferences (i.e. high cost to getting the less preferred good) for a particular movie or showtime are willing to pay the surcharge to guarantee themselves a ticket.

The difference in advanced sales pricing behavior between the airline and movie industries can be explained by the model in two different but complementary ways. First, as espoused in the preceding paragraphs, the preference-certainty of the consumers in these markets is different. Alternatively, the distribution of costs of getting a less preferred good may be different between different markets[19]. Getting a less preferred flight may not impose a large cost for most travelers if the day of the flight is already scheduled for travel and other plans can be rearranged as the purchase is made weeks in advance. This results in few preference-certain consumers being willing to pay extra in advance to guarantee a seat on their preferred flight and advanced-purchase discounts being offered. In the market for movie tickets, because the purchase is typically made close to the day of the show, ending up with a less preferred time could cause one to have to change othntion consuming the wrong good. If most consumers have a high cost to consuming their less preferred good, a large discount is needed to induce advanced-purchases and thus price discriminate. In the premium case it is the high-cost, preference-certain consumers who purchase in advance, but there is a bound on the premium they are willing to pay. Therefore large premiums for advanced purchase do not typically occur.

Finally, this model can be used to help explain the typical movement of airline ight prices. As discussed above, prices a month or so in advance of a ight are typically lower than those closer to the departure date. However, as one gets within a few days of the ight's departure, if there are tickets remaining, the price on the tickets drops considerably. Per the advice of a travel website

> ...while it is potentially possible to get great deals at the very last minute, the

7 Conclusions

Based on the model presented here, consumer's knowledge of their future preferences and the costs associated with consuming the less preferred good are determining factors in whether a premium or discount will be associated with advanced sales of a capacity constrained good. Preference-certain consumers risk not getting their preferred good if they wait to consume and will thus be willing to pay a premium to purchase in advance. Preference-uncertain consumers risk purchasing the wrong good if they buy in advance. They will only purchase in advance when offered a discount sufficient to compensate them for this risk. The size of the costs associated with consuming the less preferred good naturally affects the size of the discount or premium. The model explains why in some cases, like airlines, a discount pricing scheme is used, while in others, like concerts, a premium can be charged.

There is still room for additional analysis to be done. First, the market should be opened up to competition. While a concert or movie theater may be a local monopoly, most airline routes are open to competition. Because the pricing behavior is primarily dependent on the consumer types, as in Dana (1992), price discrimination may be robust to competition. Second, there is the question as to what effect secondary markets will have on the allocation of tickets and ability to charge a premium or discount. Allowing resale of tickets may eliminate the ability to offer a discount because a third party could purchase in advance and resell them, undercutting the higher second period price. A premium may still persist since resale is not profitable. Future work will look at the effects of secondary markets on ticket sales. Finally, in some markets firms do not commit to a price path. Relaxing the commitment assumption in the future will yield additional insights into the intertemporal movement of prices.

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