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Theory of Social Transformation, Political Transition and Economic Growth

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Theory of Social Transformation, Political Transition and Economic Growth

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Abstract

In this paper, I develop a model of sociopolitical transition that links sociopolitical transformational process of countries to the dynamic process of output per capita and economic growth. Social polarization breeds discriminatory practices regarding government redistribution. This brings about ine¢ cient allocation of resources away from production to political power struggle leading to poor economic outcomes. However, the model shows that social integrative processes may correct this ine¢ ciency over time depending on the degree of social fractionalization, the level of social distance between the groups, the level of production technology, etc. Even though the model predicts long-run convergence of growth rates and output per capita across countries, it shows possible prolonged divergence of these economic variables.

Keywords: economic growth, fractionalization, integration

JEL Classi…cation: O41, O43

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1 Introduction

After the Second World War, economists started to show greater interest in …nding answers to what causes the large gaps among countries of the world in terms of economic growth and output per capita. This took a more interesting turn in the 1950s and 1960s when neoclassical economists like Solow (1956), Cass (1965) and Koopman (1965) published models predicting eventual convergence of economic growth rates and GDP per capita among countries of the world. Yet, as time passed by, few signs, if at all, emerged that showed that this prediction

I argue in this paper that social fractionalization in terms of ethnicity, race, religion, etc. breeds discriminatory practices thereby creating political tensions. This results in ine¢ cient use of economic resources to struggle for political power in order to take control of government machinery for the purpose of being in charge of the government's redistribution mechanism. That is, I show that ethnic/racial or religious fragmentation ultimately leads to economic ine¢ ciency and thus poor economic outcomes. However, I show that social dynamics may minimize this ine¢ ciency over time thereby enhancing economic performance as countries undergo sociopolitical transformation. And since this process may di¤er from one country to another, gaps will emerge among countries in terms of output per capita and economic growth, especially at the initial stage of the sociopolitical process.

The economy in this model is populated by two groups of people. The groups are de…ned along ethnic, racial or religious lines. I assume that the government formed by a group is not di¤erent from the group as a whole in terms of its objectives. Like Pham (2005), van Long and Shimomura (2004), Corneo and Jeanne (2001), Rauscher (1997), Fershtmam et al. (1996), etc. individuals in this model derive util-

thus have political power because the group is able to more e¤ectively advance its sociocultural or religious values or ideology when in power than when in the opposition. Power struggle between the groups may be so intense at the initial stage of the country's formation that legal or democratic framework established to dictate smooth power transition may not work because of presence of incentives to deviate from established rules. That is, the intensity of political power struggle at the initial stage may be relatively high, and may thus take violent forms. This is the source of the economic ine¢ ciency at the initial stage. However, as the groups become more socially integrated, this ine¢ ciency will diminish over time thereby enhancing economic growth. Nevertheless, the rate of social integration may di¤er from one country to

fragmentation to economic growth and output per capita (see, for instance, Alesina and Drazen (1991), Alesina and Spolaore (1997)). That is, these models tend to link social fragmentation to issues like public

in this literature generally concentrate on the nature of the production function and how it impacts the dynamics of per-capita output, while implicitly assuming similar social and political environments for countries. That is, these models fail to answer the fundamental question of how sociopolitical evolutionary process of countries a¤ects the dynamic behavior of output per capita.

The second strand of literature that this paper relates to is the literature on social con‡icts (see, for instance, Grossman (1991), Acemoglu and Robinson (2001), Roemer (1995), Tornell and Velasco (1992)). Grossman develops a theory of insurrections that treats insurrection and its deterrence as economic activities that compete with production of goods. This model adopts a similar stand by arguing that allocating resources for political power struggle or political con‡icts is pareto ine¢ cient, and it decreases resources available for production. However, this paper goes beyond this idea by showing how social integration helps minimize the amount of resources ine¢ ciently allocated towards political power struggle. Acemoglu and Robinson model the complications created by the existence of di¤erent social groups in a country as the country undergoes political transition. The social groups in Acemoglu and Robinson's model are the rich who dislike democracy because of its redistributive e¤ect and the poor who want democracy. However, the social groups in this model are ethinc, racial or religious in nature, and their objective is not either to democratize or not, but they struggle for political power to advance their sociocultural or religious values or ideologies with the goal of attaining higher social status, since that is an input into individual utility function.

Third, this paper relates to the literature that links social fragmen-

tation to economic performance of countries (see, for example, Alesina and Drazen (1991), Alesina and Spolaore (1997), Alesina, Baqir and Easterly (1999)). As I argued above, these models tend to link social fragmentation to issues like public goods provision and macroeconomic stabilization. However, this paper takes a more direct approach in linking social fragmentation to the dynamics of output per-capital and economic growth by showing how social dynamic process relaxes the political complications associated with social divisions. That is, this paper shows how social integrative process reallocates resources from ine¢ cient use (political power struggle or even political con‡icts) to e¢ cient use (production).

3 The Model

3.1 The Setting

Consider an economy in which there are two groups of people: groups X and Z. The groups are de..ned along ethnic, racial or religious lines. Let N and N respectively represent the population sizes of X and Z at time t. Also,Alrme

e¢ ciency requires that at every t, (h m

ciocultural values of a group, while a and b respectively measure the importance individuals attach to consumption and social status (as measured by the level of implemented sociocultural values). I assert that as a result of cultural indoctrination, people of di¤erent cultural backgrounds tend to feel that their cultural or religious values or philosophy is superior to others, at least at the initial stage of a country's formation. This, in fact, is the cause of the feeling of ethnic/racial or religious superiority. This feeling of superiority is generally greater the more di¤erent the groups are in terms of social characteristics like language, religion, race/ethnicity, and others. As a result of this, if the country is so ethnically/racially or religiously divided and fragmented, members of group X would feel more socially elevated if the government is controlled by group X. This is due to the fact that a group is able to more e¤ectively implement its sociocultural or religious values or philosophy when in power than when in the opposition. That is, if X is in power $V \geq V$ (and vice versa, if Z is in power). Let the level of promoted sociocultural or religious values or philosophy of a group be equivalent to the level of government spending in favor of that group. This means that the government formed by group X, for instance, favors members of group X more than members of group Z. Thus, the government of X discriminates against members of group Z. I further assume that the government does this by adopting fair taxation, but uses unfair lump-sum transfers.

Let be the per-capita lump-sum tax of the government. For now, suppose that $N = N_{\substack{x \\ tx \\ t=}}$

government's transfers to groups X and Z at every t be respectively as follows:

$$N_X G_X = \quad = \quad T N_X$$

(2:a)

$$N_Z G_Z = (1 - \quad) \quad = (1 - \quad) T N_Z$$

Dividing the two equations in (2:a) by $N_X = N_Z = N$, per-capita transfers are

$$G_X = V_X = \quad T$$

(2:b)

$$G_Z = V_Z = (1 - \quad) T$$

Where is the fraction of total government tax revenue that is transferred to group X. With the present assumption that $N_X = N_Z$, is also the fraction of tax revenue from x and z (T) that is transferred to x. Now, as long as the social groups discriminate in their own favor when they are in power, $> \frac{1}{2}$. Note that if in power, the government formed by Z

(4)

$U = aC + b(G -)$

The two equations in (4) indicate that the size of is utility reducing to z because the government formed by X transfers some income away from z to x through redistribution of tax revenue because of the climate of discrimination that results from each group's desire to promote its sociocultural or religious values at the expense of the other group.

3.2 Social Integration

Let individuals attach importance to the welfare of relatives. This means that as intermarriges between the groups increase and the groups become socially integrated through family links, discrimination against the rival group becomes costly to members of the governing group. This implies that social integration generated by inter-group marriages decreases the degree of discrimination[5]. Now, let s be the proportion of all marriages that are intermarriages at time t. This means that $1 - s = s$ is the proportion of marriages between people from the same social group. As it is expected, let us assume that $s_0 > s_0$. Since social integration decreases the degree of discrimination, let the fraction (of tax revenue) transferred in lump sum to the representative member of the rival group by the governing group at time t $(1 -)$ be positively related to the proportion of intermarriages in the society (s) as follows:

[5] Also, if we assume that members of the government formed by one group are probabilistically chosen from the group, then as intermarriages and thus members of the group who trace their parentage from the other group increase, we will have the degree of discrimination to decrease over time.

$$1 - \quad = f(s \quad) \Rightarrow \quad = 1 - f(s \quad) \qquad (5)$$

Equation (5) implies that to know how evolves over time, we should know how s evolves over time. The vector $S = (s \quad s \quad)$ evolves according the Markov process. Consider the following Markov transition matrix:

$$P = \begin{matrix} r & q \\ q & r \end{matrix} \quad r; q \geq 0;$$

Where r is the probability that a person from group i will marry from group i, while q is the probability that a person from group i will marry from group j. Let us suppose here that the society is patrilineal (or matrilineal, as the choice doesn't matter here), meaning that children belong to their fathers' lineage.

As a characteristic of the Markov chain, let P be a stochastic matrix. That is, $r + q = 1$. As it is expected, letA

Now, given that $S_0 = (s_0 \quad s_0)$ and $P = \begin{matrix} r & q \\ q & r \end{matrix} = \begin{matrix} r & 1-r \\ 1-r & r \end{matrix}$,

by Markov chain[6],

$$S = s \quad s = s_0 \quad s_0 \begin{matrix} r & 1-r \\ 1-r & r \end{matrix} \qquad (6)$$

$\begin{matrix} r & 1-r \\ 1-r & r \end{matrix}$ can be expanded as follows[7]:

$$\begin{matrix} r & 1-r \\ 1-r & r \end{matrix} = \begin{matrix} \frac{1}{2} + \frac{1}{2}(2r-1) & \frac{1}{2} - \frac{1}{2}(2r-1) \\ \frac{1}{2} - \frac{1}{2}(2r-1) & \frac{1}{2} + \frac{1}{2}(2r-1) \end{matrix}$$

Equation (6) can therefore be re-written as

$$S = s \quad s = s_0 \quad s_0 \begin{matrix} \frac{1}{2} + \frac{1}{2}(2r-1) & \frac{1}{2} - \frac{1}{2}(2r-1) \\ \frac{1}{2} - \frac{1}{2}(2r-1) & \frac{1}{2} + \frac{1}{2}(2r-1) \end{matrix} \qquad (7)$$

For simplicity, let us assume that at $t = 0$, groups X and Z are such that $s_0 \quad s_0 = 1 \quad 0$.

$= 1 - f\left(\frac{1}{2} - \frac{1}{2}(2r - 1)\right)$ (9)

Let and s

large, this struggle for power may take violent forms such as coups, insurrections or wars because of what is at stake in terms of utility lost. This means that, with large at the initial stage, the groups, especially the minority group, may …nd it attractive to deviate from established rules dictating smooth and democratic transitions and thus employ extralegal means to acquire or hold on to political power in order to enjoy the large . I will return to this point later.

Suppose that to engage in struggle for political power all members of a group contribute the same amount of the endowed resource H. Recall that $m\ ;\ i = x; z$ is the portion of H each member of a group allocates towards struggle for political power at time t. I assume that the probability of a successful takeover of political power (or a successful defense of political power) of groups Z and X are respectively given by

$$p \quad = 1 - p \quad = \frac{}{+} = \frac{(1+)\ _0}{(1+)\ _0 +(1+)\ _0} = \frac{_0}{_0 + _0} \tag{13}$$

$$p \quad = 1 - p \quad = \frac{}{+} = \frac{(1+)\ _0}{(1+)\ _0 +(1+)\ _0} = \frac{_0}{_0 + _0} \tag{14}$$

Equations (13) and (14) show that the probability of a successful struggle for power not only depends on a group's military resources, but also on the military resources of the rival group. Also, we can see that the larger the size of a group, the greater the chances of the group acquiring or holding on to political power. Now let $R = \frac{}{} = \frac{_0}{_0}$. This means that by multiplying both the numerators and the denominators of (13) by R and (14) by $\frac{1}{}$ we get

$$p \quad = 1 - p \quad = \frac{}{+} \tag{15}$$

$$p = 1 - p = \frac{}{\frac{1}{} + } \qquad (16)$$

In this case where I have assumed that $N = N$, $R = \frac{1}{} = 1$. I will consider the general case where N may not be equal to N in section 3.4.

Equilibrium m and h

At every t, members of X and Z decide how much of H they want to individually allocate towards the production of y (h) and how much they want to allocate towards the struggle to either acquire or defend political power in order to enjoy (m). Remember that at every t, the probability that a group will be in power is p. This means that at every t, the expected utility of $i = x; z$ is $EU = p[aC + bG] + (1 - p)[aC + b(G -)]$. Note that, $G = G$ if group i is in power and $G = G -$ if group i is not in power. For this reason, to choose m and h, x and z solve the following optimization problem:

$$(A)\ \text{Max}_{=0}^{1} \quad p[aC + bG] + (1 - p)[aC + b(G -)] = \text{Max}_{=0}^{1} \quad aC + bG - p\,b \quad , \quad i \neq j$$

Subject to

$$\text{i)}\ H = m + h \Rightarrow m = H - h$$

$$\text{ii)}\ C = y -$$

$$\text{iii)}\ y = h$$

Where is the discount factor. The above optimization problem implies that choices made in each period a¤ect only that period's pay-o¤s. Because of this, optimization problem (A) is a series of single

periods optimization problems. Substituting the constraints (and the expressions for p and), at every t, x and z solve the following problem choosing h:

$$(B)\quad \text{Max}\quad a(\quad h\ -\quad)+bG-\frac{(\qquad)}{(\qquad)+(\qquad)}\,bT(2r-1)\quad,\ i\neq j$$

The …rst order conditions for interior solution at every t are

$$x:\quad a\quad -\frac{(\qquad)\quad(2\quad 1)}{[(\qquad)+(\qquad)]^2}=0 \qquad (17)$$

$$z:\quad a\quad -\frac{(\qquad)\quad(2\quad 1)}{[(\qquad)+(\qquad)]^2}=0 \qquad (18)$$

Solving (17) and (18) simultaneously yields the following solutions:

$$h\quad =h\quad =h\quad =H-\tfrac{1}{4}—(2r-1) \qquad (19)$$

$$m\quad =m\quad =m\quad =\tfrac{1}{4}—(2r-1) \qquad (20)$$

And substituting (19) into the production function, I derive the dynamic process of output per capita as follows:

$$y\quad =y\quad =y\quad =\quad H-\tfrac{1}{4}—(2r-1) \qquad (21)$$

We can see from these results that the intensity of political power struggle as shown by equation (20), and the size of per capita output as shown by equation (21) depend on the size of productive technology ( ), how socially di¤erent the groups are (r), time (t), etc.

Implications and Discussion

To appreciate the above results, let us consider the following implications:

a)

ai:

these resources to productive use, thereby increasing h

same time, other things remaining the same, the country with the less

we hold the other factors constant. This means that the country may permanently be characterized by con‡icts and wars with no chance of the country tasting of real economic growth and development. To taste of economic growth and development in this extreme case, the only solution may be to divide the country along the lines of the social groupings, if possible. However, we shall see in the next section that economic development may not be a problem even in the extreme case if one of the two groups is so small in size.

d)

di: $\frac{\partial \tau^*}{\partial m} = \frac{(2 \quad 1)}{4} \geq 0$

dii: $\frac{\partial \tau^*}{\partial \theta} = -\frac{(2 \quad 1)}{4} \leq 0$

Another interesting implication is that, as shown by implications di and dii, the size of the government revenue is positively related to the intensity of political power struggle (m

funds to a government formed by one social group fuels social and political conflicts, and therefore leads to a continued inefficient allocation of economic resources toward struggle for political power, thereby negatively impacting the performance of the economy. This means that, at the bilateral and multilateral levels, the best economic assistance many of the newly independent poor nations can get is not economic aid that ends up in the hands of a government that discriminates or is perceived to discriminate, but it is measures that bring about social cohesion and genuine sociopolitical reforms, since this will induce efficient allocation of domestic resources. Additionally, this relationship explains why many poor nations in the developing world with large deposits of natural resources like oil and gas deposits do very poorly economically. That is, large sums of revenues from these resources to the government stimulates very intensive political power struggle (political conflicts) leading to very large economic inefficiencies and thus very poor economic performance.

e)

ei: $\frac{\partial \theta^*}{\partial} = -\frac{(2 \quad 1)}{4 \quad ^2} \leq 0$

eii: $\frac{\partial \theta^*}{\partial} = H \quad \geq 0$

And finally, the level of production technology is negatively related to the amount of resources inefficiently allocated to political power struggle. The explanation for this is that, the greater the level of production technology, the greater the opportunity cost of political conflicts and hence the smaller the intensity of such conflicts. This means that if other factors remain the same, this model predicts that the advancements in production technology in modern times should make the

newly independent nations have relatively shorter time of sociopolitical transformational period, and also relatively smaller intensity of power struggle as compared to the transformational process of the countries in the developed world that were formed a long time ago when production technology was not that advanced.

3.4 The General Case Where N_t^X May not be Equal to N_t^Z

The analysis so far assumes that N = N . In this section, I relax this restricting assumption. That is, I consider the general case where N may be di¤erent from N . If N is di¤erent from N , the idea that, if, for instance, X is in power, the government's per-capita transfers are G = T and G = (1–)T will lead to

(22), the di¤erence in per-capita government transfers of equation (3) is now

$$= G \; - G \; = \frac{}{} \; - \frac{1}{} \; = \frac{(\; + \;)}{} - \frac{(1 \;)(\; + \;)}{} \qquad (23)$$

Since $N \; = (1 + n) \; N_0$, (23) becomes

$$= \frac{(\; _0 + \; _0)}{_0} - \frac{(1 \;)(\; _0 + \; _0)}{_0} = \; _0 - \; _0 = (\; - \;) \; _0 = \; _0 \qquad (24)$$

Where $= \frac{}{_0}$, $= \frac{1}{_0}$ and $= \; - \; . \; 0$

$$z: \quad (C_2) \quad \mathrm{Max} \quad a(\quad h \quad - \quad) + bG \quad - \quad \frac{(\qquad)}{\frac{1}{=}(\qquad)+(}$$

more per-capita resources allocated towards political power struggle, and will thus have lower output per capita as compared to another country where the population distribution is, say, 70% for one group and 30% for the other group[8]. $\frac{}{(1+\ )^2} = \frac{\overline{\quad}}{1+\overline{\quad}^2}$

empirical literature that studies the relationship between social fragmentation and economic growth and development (see, for instance, Easterly and Levine (1997), Collier (2000), Alesina et al. (2003), Alesina and Ferrara (2005), etc.). The ethnic fractionalization (EF) measure used in this literature is a Her...ndahl index de...ned as $EF = 1 - \sum_{i=1}^{k} s_i^2$, where s_i is the ratio of group i to the total population and k is the number of the ethnic groups. EF is the probability that two individuals selected at random belong to two di¤erent ethnic groups. In fact, $\frac{1}{(1+\mu)^2} = \frac{1}{2}(1 - \sum_{i=1}^{k} s_i^2)$, which implies that these measures are not any di¤erent, except that $\frac{1}{(1+\mu)^2}$ assumes the existence of only two social groups. At ...rst, the assumption of only two social groups may appear too strict. However, it is not that strict in practice. The reason is that if there are three of more social groups in a country, there becomes coalition formations at the political front. This means that additional social groups may not necessarily add to political tensions. In fact, so many social groups may even reduce political tensions thereby minimizing the negative impact of social fractionalization. This means that, as was recognized by Alesina and Ferrara (2005), the EF tends to overstate the negative e¤ects associated with social fractionalization as the number of the social groups increases. Recognizing this, Posner (2004), in his measure of ethnic fractionalization, took into accounts actual political coalitions. However, the problem with this approach is that political alignments tend to switch around over time and are thus not permanent.

4. Conclusion

We have seen from this model that the social and political environments in which economies operate immeasurably a¤ect economic outcomes. This is because, as we saw from the analyses, social polarization creates political tensions, which brings about unhealthy power struggle, at least at the initial stage of the sociopolitical process leading to ine¢ cient allocation of economic resources towards political power struggle, which results in poor economic performance. However, over time as the society becomes more and more integrated through intergroup marriages, this political tension diminishes, leading to more e¢ cient allocation of resources towards production away from political power struggle thereby improving economic outcome. Through these processes, I have shown that growth rate of per-capita output is generally higher at the initial stage than in the limits. However, fundamental di¤erences in terms of the level of social distance between the groups, the relative sizes of the social groups, etc. a¤ect the rate at which each country can grow and catch up with the already developed world. And, with the exception of the extreme case where the social groups have no chance of integrating (which may lead to the country disintegrating into pieces), economic growth will happen, even though it may happen at slower rate and the catch-up take longer time.

One policy recommendation of this model is that to help a country come out of economic challenges and thus speed up economic growth, we may not merely want to provide ..nancial aid, but we may have to provide measures that will speed up social integration and social harmonization so as to bring about a more e¢ cient use of domestic resources.

It is clear that the model in this paper is silent about economic

processes like physical capital accumulation, human capital accumulation, …nancial sector development, etc. while these processes may impact economic outcome. For this reason, it may be more interesting to incorporate these processes into the model. However, if the appropriate sociopolitical environments exist, I do not see why these economic processes will di¤er from one country to another, especially if we assume that employment of these resources exhibit diminishing marginal productivity.

We can deduce from the analysis so far that the main reason why other models that seek to explain cross-country growth di¤erences fail to fully explain the di¤erences in growth rates, especially among countries in the developing world, is that these models implicitly assume that all countries have similar, if not identical, social and political environments. But as we saw in the analyses, countries do di¤er greatly in terms of sociopolitical en1(a)10(l)-32il ant, onad(i)6(n)12(e)8(s)89-32inatmsurlda
apd p263()10(u)11()11(p)11(n)18()9(sz9(s)-i260(f)-39)12(e)8(s)8(er)17(s)i267(t)-100(t26(t)8

APPENDIX

I show in this appendix that 8 Td632t□

$$P = \begin{matrix} r & 1-r \\ 1-r & r \end{matrix} = \begin{matrix} \frac{1}{2}+\frac{1}{2}(2r-1) & \frac{1}{2}-\frac{1}{2}(2r-1) \\ \frac{1}{2}-\frac{1}{2}(2r-1) & \frac{1}{2}+\frac{1}{2}(2r-1) \end{matrix}$$

Let us decompose P into matrixes of eigenvalues and eigenvectors as follows. Let 3 1 7 5 = 436 w 0 l18 7.97 Tf 12.104Td[85790 T36 r r r t□ tt t =t□ =t

Solving (A5) for the characteristic roots, I get

$$= \frac{2 \quad \sqrt{4^{\,2} \quad 8 \; +4}}{2}$$

$$\Rightarrow \quad = 1 \text{ or } \quad = 2r - 1 \qquad \text{(A6)}$$

From (A6) if $= 1$, we can solve for the elements of l_1 in $P l_1 = 1 l_1$ as follows:

$$(1 - r) l_{11} = (1 - r) l_{12} \Rightarrow l_{11} = l_{12} \qquad \text{(A7)}$$

And if $= 2r - 1$, we can solve for the elements of l_2 in $P l_2 = (2r - 1) l_2$ as follows:

$$-l_{22} = l_{21} \qquad \text{(A8)}$$

Based on (A7) and (A8), let $l_{11} = l_{12} = 1$, and let $l_{21} = 1$ and $l_{22} = -1$. This means that

$$L = \begin{matrix} 1 & 1 \\ 1 & -1 \end{matrix}, \quad = \begin{matrix} 1 & 0 \\ 0 & 2r - 1 \end{matrix} \text{ and } L^{\,1} = -\tfrac{1}{2} \begin{matrix} -1 & -1 \\ -1 & 1 \end{matrix}$$

First, let us verify that $P = L \; L^{\,1}$ as follows:

$$-\tfrac{1}{2} \begin{matrix} 1 & 1 \\ 1 & -1 \end{matrix} \begin{matrix} 1 & 0 \\ 0 & 2r - 1 \end{matrix} \begin{matrix} -1 & -1 \\ -1 & 1 \end{matrix} = \tfrac{1}{2} \begin{matrix} 1 & 2r - 1 \\ 1 & 1 - 2r \end{matrix} \begin{matrix} 1 & 1 \\ 1 & -1 \end{matrix} = \begin{matrix} r & 1 - r \\ 1 - r & r \end{matrix} = P$$

And $P = (L \; L^{\,1}) = L \quad L^{\,1}$ can be calculated as follows:

$$P = \tfrac{1}{2} \begin{matrix} 1 & 1 \\ 1 & -1 \end{matrix} \begin{matrix} 1 & 0 \\ 0 & (2r - 1) \end{matrix} \begin{matrix} 1 & 1 \\ 1 & -1 \end{matrix} = \begin{matrix} \frac{1}{2} + \frac{1}{2}(2r - 1) & \frac{1}{2} - \frac{1}{2}(2r - 1) \\ \frac{1}{2} - \frac{1}{2}(2r - 1) & \frac{1}{2} + \frac{1}{2}(2r - 1) \end{matrix}$$

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