

# DISCUSSION PAPERS IN ECONOMICS

Working Paper No. 05-05

Companion Paper to "Tax Competition and the Creation  
of Redundant Products": Proof of Proposition 2

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COMPANION PAPER TO  
“ TAX COMPETITION AND  
THE CREATION OF REDUNDANT PRODUCTS”:  
PROOF OF PROPOSITION 2

by

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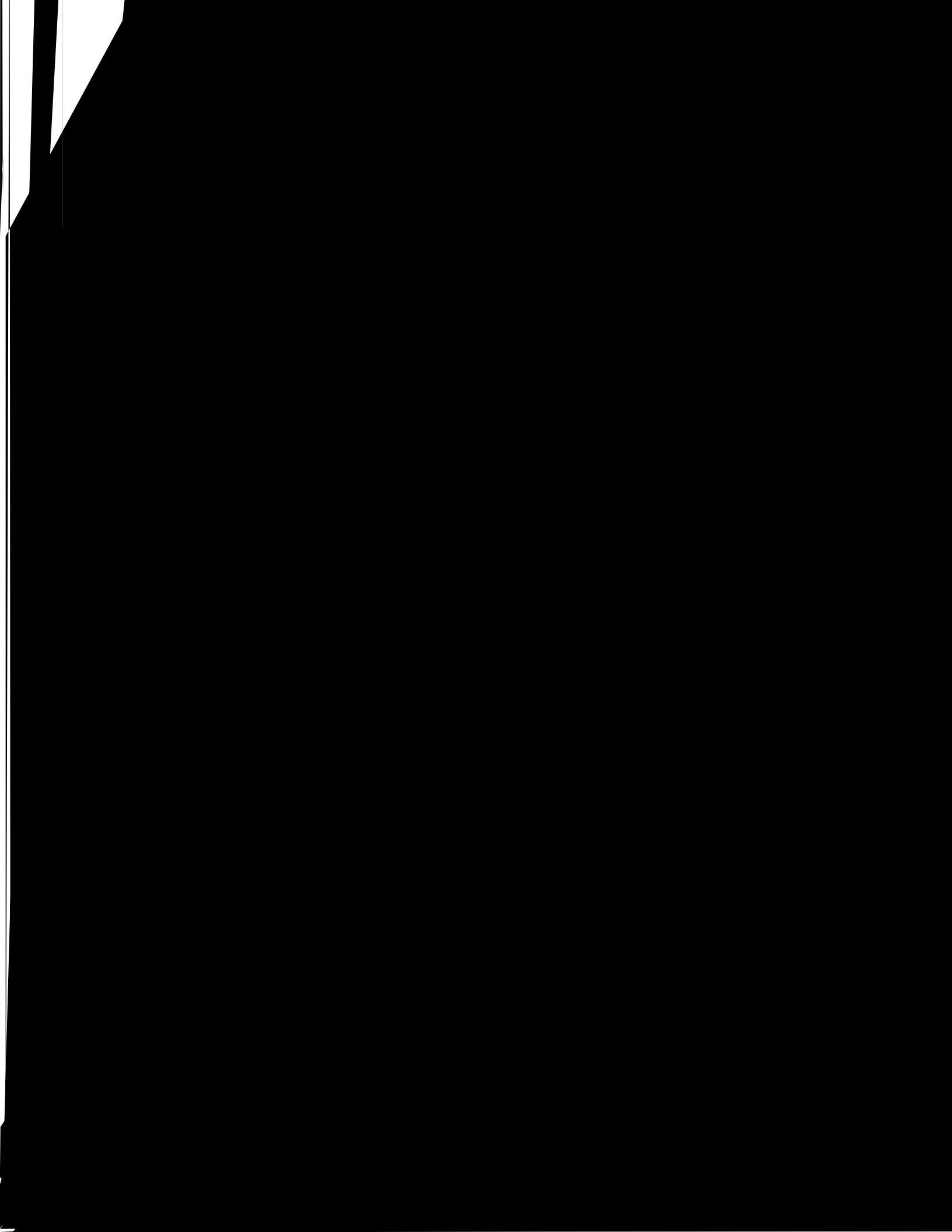
Acknowledgments: I am grateful for financial support from the Canadian Social Science and Humanities Research Council.

This working paper provides the full proof of Proposition 2 in “Tax Competition and the Creation of Redundant Products” (*University of Colorado Working Paper #05-3*).

This paper shows that, if Region 2’s strategy is ( \_\_\_\_\_ ), the tax

$$R^{1*} = \left( \delta + \frac{T}{4} \right) Q \bar{w}.$$





where  $CS$  denotes “complementary slackness”. Try  $A > 0$ . From Equation (2):

.

### 3. TAX REVENUE UNDER ALL ALTERNATIVE STRATEGIES.

To show that                      and                      are the best response, I must consider the tax



Figure: possible tax rates combinations



An individual in Region 2 faces the same price for an imported assembled and an own kit

if  $p + t_a + \nu \leq p_a$  or, setting  $p_a = p + t_a + \nu$ , if

CASES A1: *the inclusive price line of an imported assembled cigarette intersects ABC on AB.*

There are three possible subcases which are characterized by the intersection of the inclusive price line of imported kits in Region 2 with the envelope EFBC:

Case A1.(i) the intersection is on EF;

Case A1.(ii) the intersection is on FB;

Case A1.(iii) the intersection is on BC.

Note that, if the inclusive price line of an imported kit lies below C at  $w = \bar{w}$ ,

*p*

. Setting

, this becomes

. Region 1 is selling kits in Reg

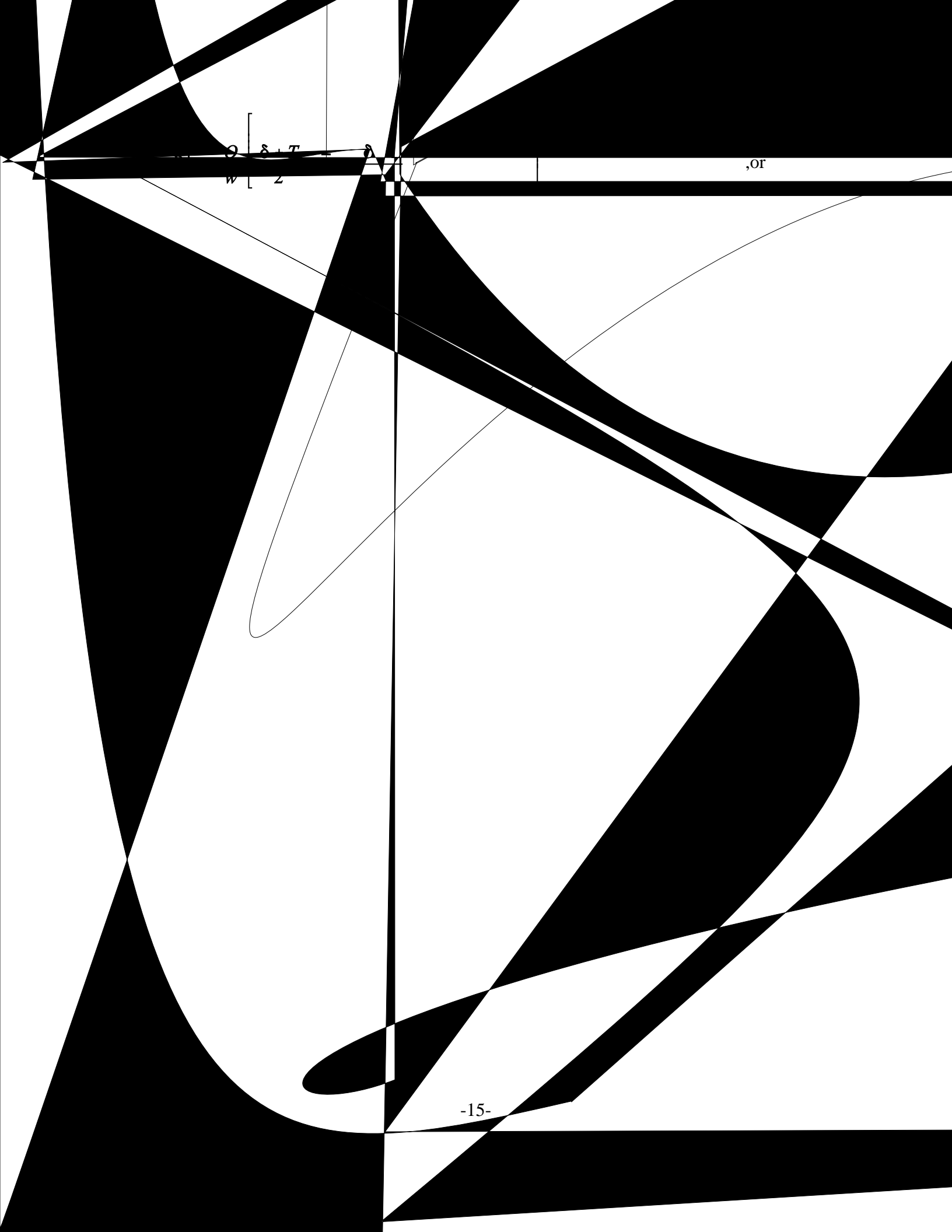
CASE A1.(i)

s.t.  $t_a^1 \leq t_k^2 = \delta \bar{w}$ ;

$\delta \bar{w}$

s i n g

$$t_a^1 = \frac{2\delta}{\dots} .$$



O S T A  
W Z

,or



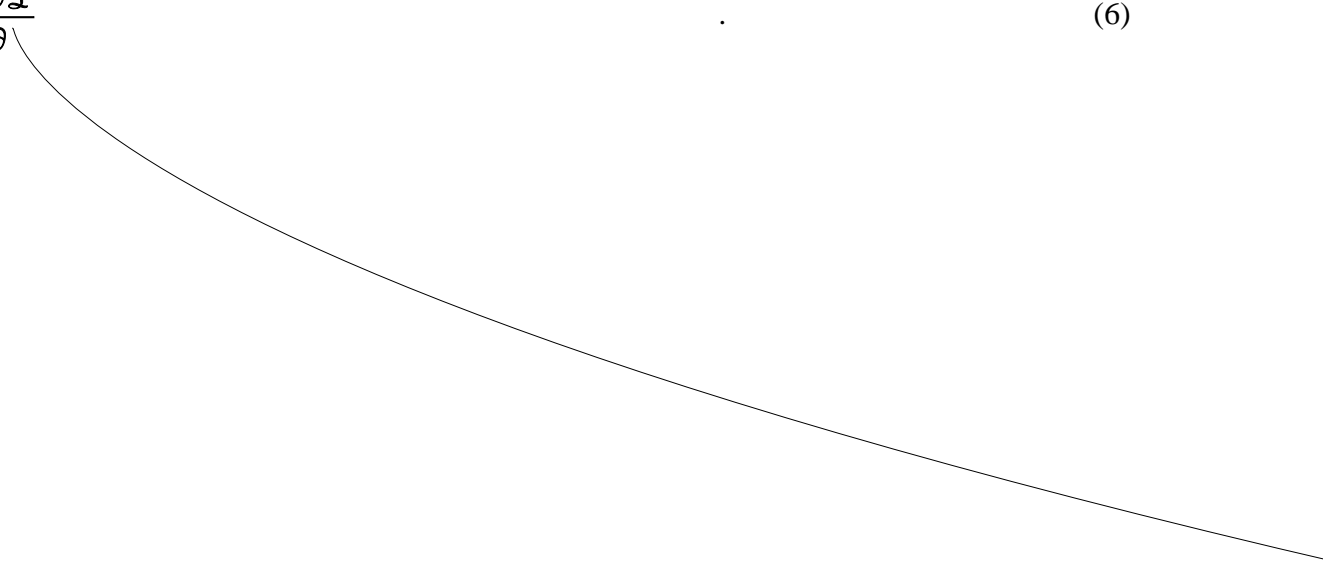


$$\frac{t_a^1 - t^1}{\quad} ;$$



$\frac{\partial \mathcal{L}}{\partial \theta}$

(6)



In this case

$$R^1 = \frac{\delta ( \quad )}{\quad} .$$

I now consider the alternative subcase when Region 1 sells only kits domestically:

$$\bar{w} \leq \frac{t^1}{\text{————}}$$



*imported kit intersects EFBC on BC*

shown in the figure below

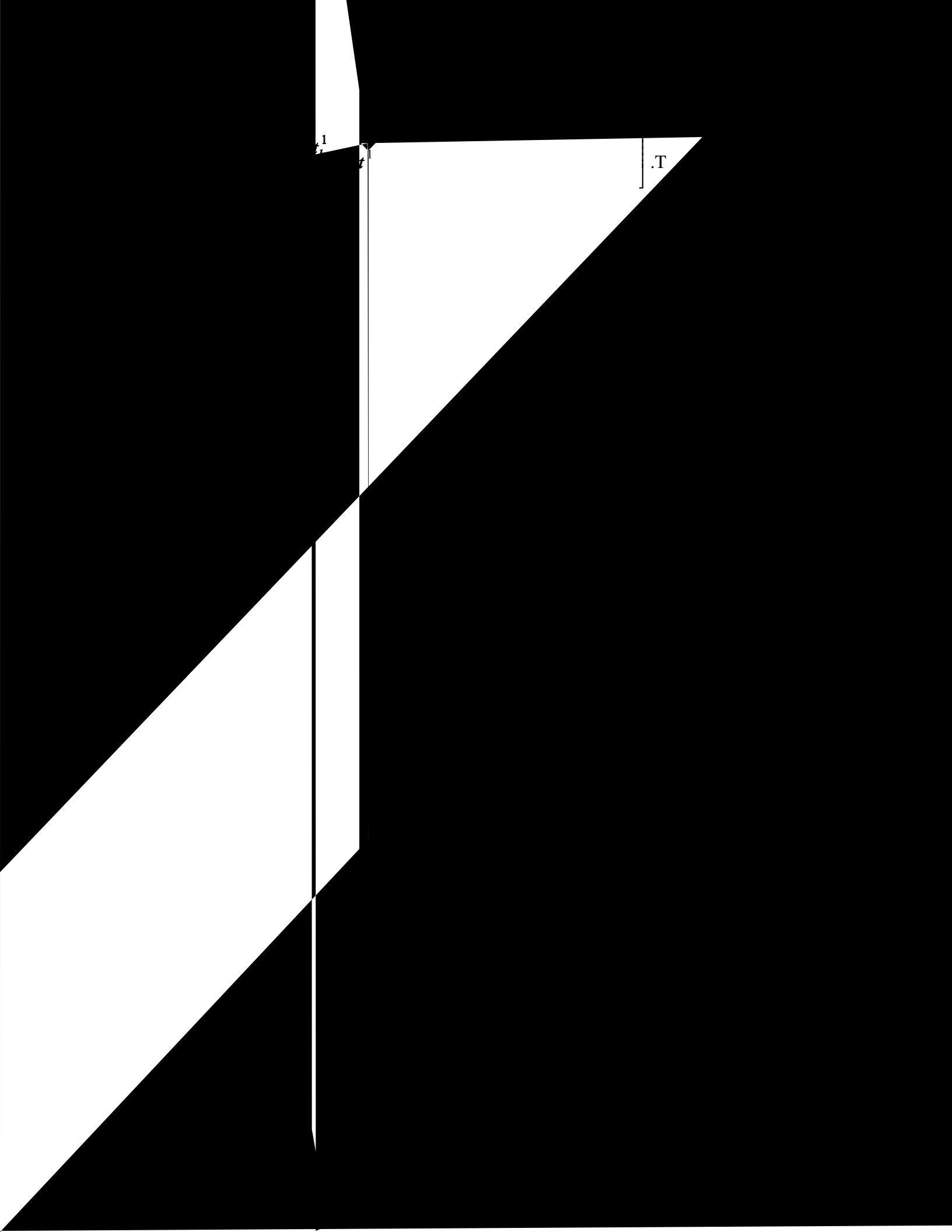


er only kits are sold in Region 1:

$\leq \text{---} .I$

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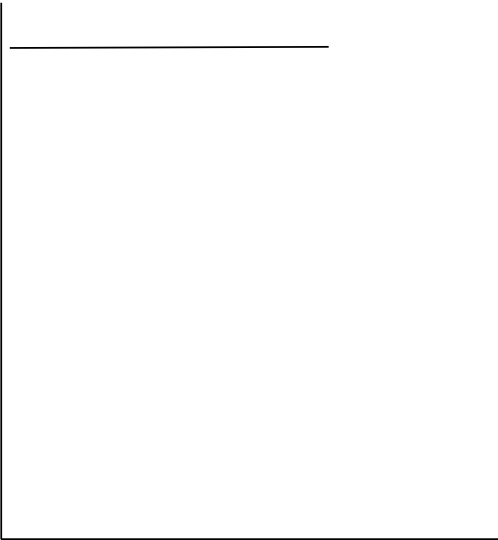
(4)





Try  $A > 0$  and

CASES A2: *the inclusive price line of an imported assembled cigarette intersects ABC on BC*







CASE A2.(i): *the inclusive price line an imported kits intersects* ~~GHTD~~ ~~W(-32-)~~ ~~18.6-~~

$$\text{s.t. } \frac{\bar{w}}{2} \leq \frac{(\delta + \frac{T}{2})\bar{w} - t_a^1}{\delta};$$

$$\frac{(\delta + \frac{T}{2})\bar{w} - t_a^1}{\delta} \leq \bar{w}.$$

Note that the restriction  $t^1$  is implied by and is therefore omitted.

o

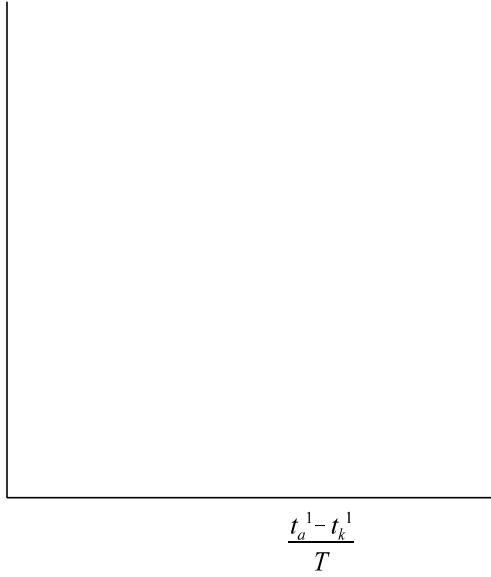
CS

;

(2)

CASE A2.(ii): *the inclusive price line of an imported kit intersects  $GHC$  on  $HC$*

The inclusive price line of an imported kit intersects the  $GHC$  on the  $HC$  axis. This is shown in the diagram below.



do the former subcase first. Region 1's problem is:s.t.

;

$R$

The Lagrangean becomes:

—

CS ; (4)





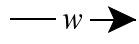


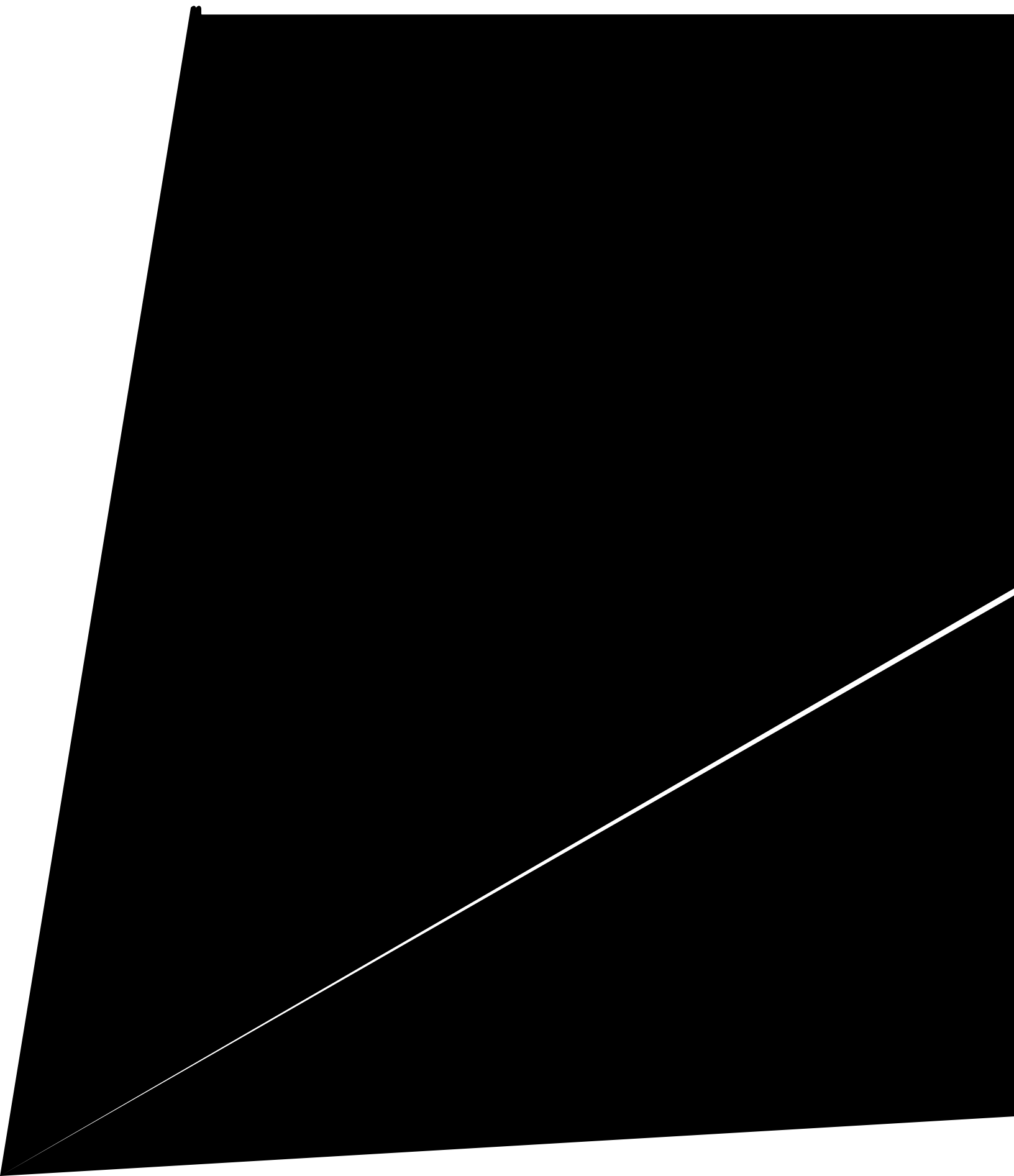
individual faces the same inclusive price for an imported kit and an own assembled if

$$p + t_k^1 + wT + w\delta = \quad \text{or, setting} \quad , \text{ if} \quad .$$

21

$$\frac{\bar{w}}{2}$$









CASE B2: *the inclusive price line of an imported kit intersects ABC on BC*

The inclusive price lines for this case are shown below:

*OR* whether only kits are sold in Region 1:

- .



3.3 CASE C:  $t_k^1 \leq t$  .



CASE C1: *the inclusive price line of an imported kit intersects ABC on AB*

The inclusive price lines for this case are shown in the figure below:

—







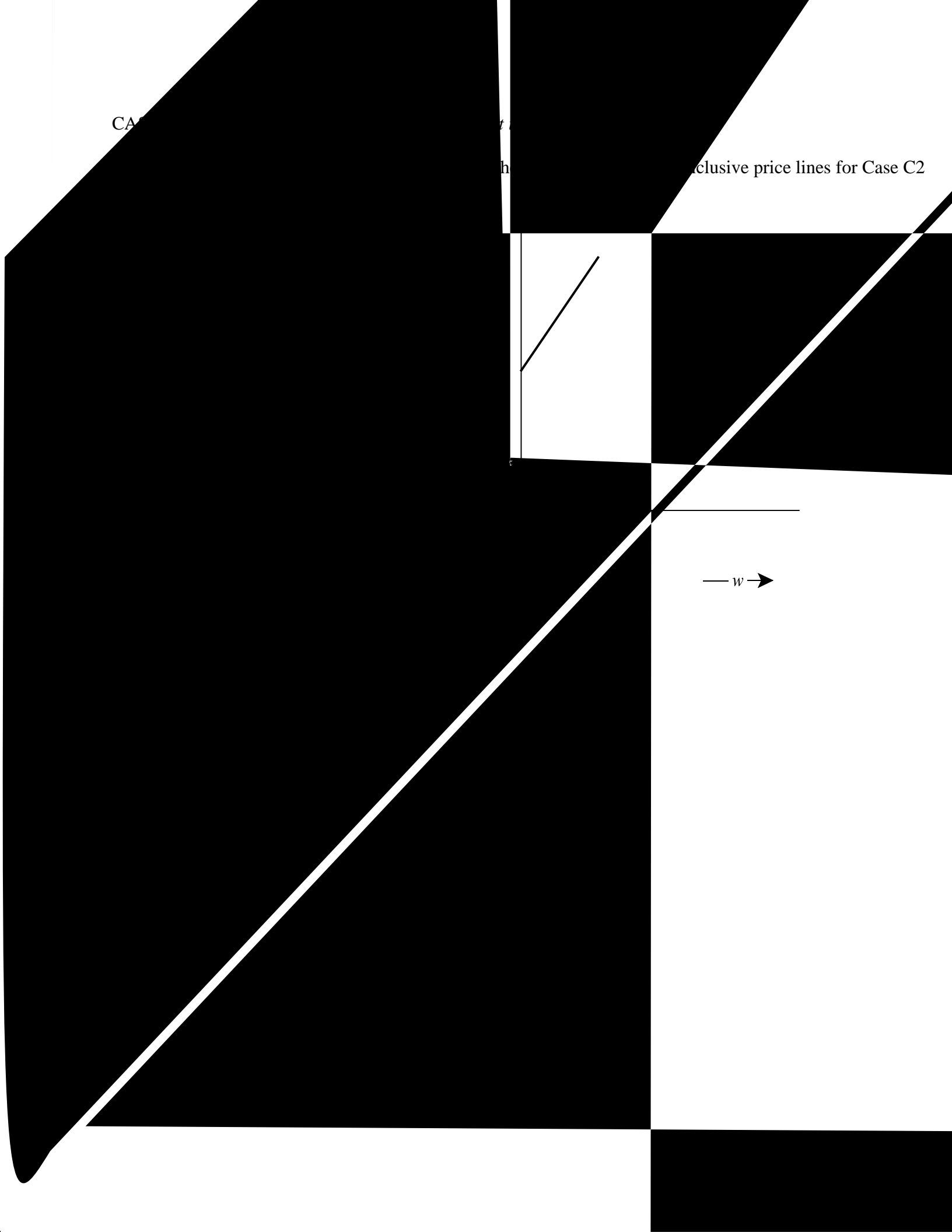
CASE

of

h

clusive price lines for Case C2

$w$  →





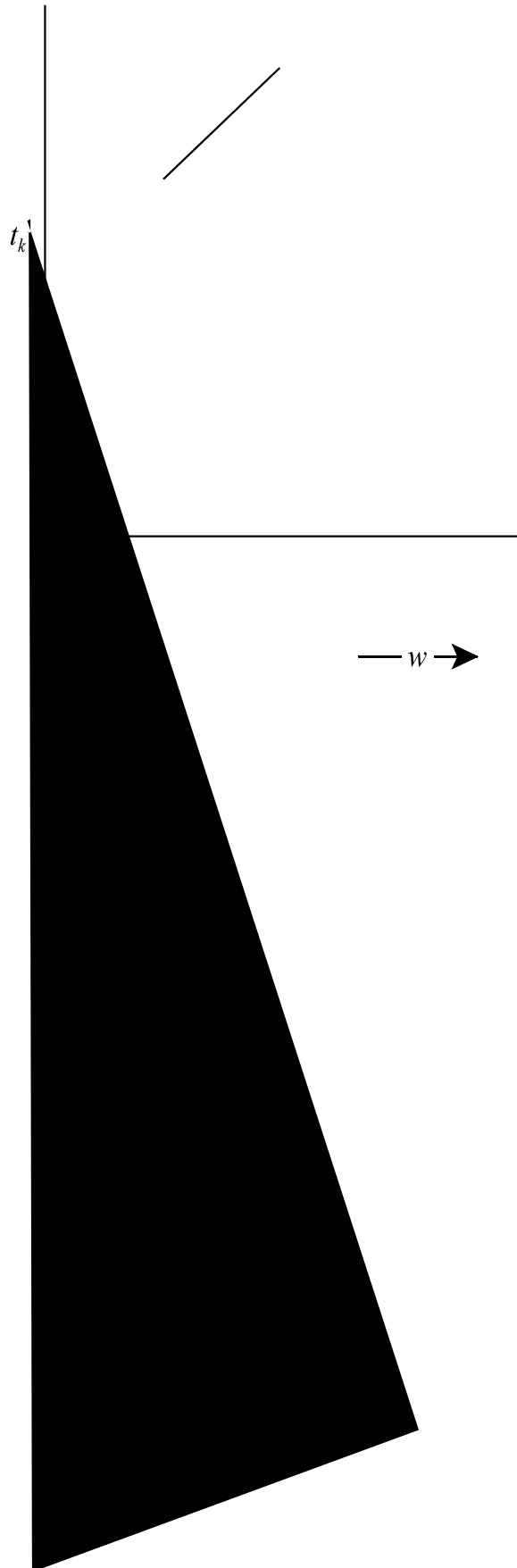




the same price for an own assembled and an own kit  $p + t_a =$  of 11

CASE D1: *the inclusive price line of an own assembled intersects ABC on AB*

The inclusive price lines in Region 1 are shown in the figure below



$$t_a^1 \leq (\delta + \frac{T}{2})\bar{w}.$$

Setting  $f(w) = 1/\bar{w}$  and integrating, the tax revenue in Region 1 is:

$$R^1 = \underline{\underline{Q}}$$

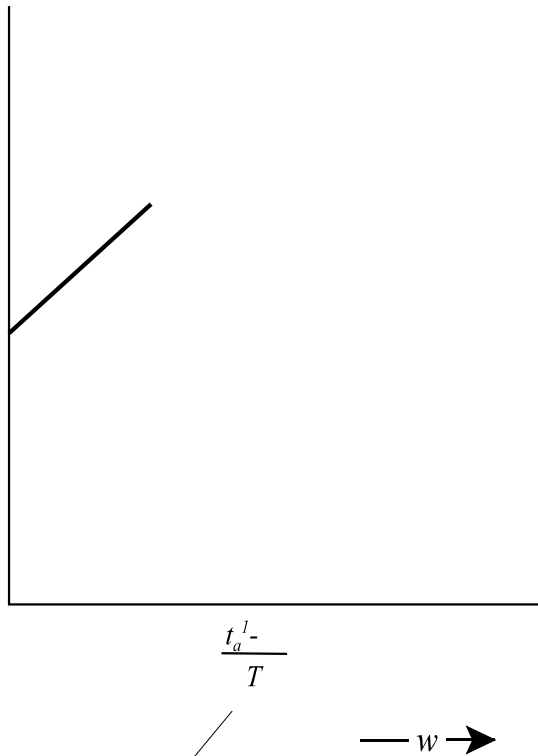
.The Lagrangean is:

∂

as required;

CASE D2: *the inclusive price line of an own assembled ABC on BC*

The inclusive price lines are shown in the figure below:



$$\frac{t_k^1 - \delta \bar{w}}{\delta} \leq \frac{t_a^1 - t_k^1}{T} .$$

Setting  $f$  and integrating, tax revenue in Region 1 is:

$$\frac{\partial \mathcal{L}}{\partial B} = \left( \delta + \frac{T}{2} \right) \bar{w} \quad ; \quad (4)$$







CASE E1: *the inclusive price li*

The inclusive price lines for thi

$t_a$

$t_k$

$t_k^2$

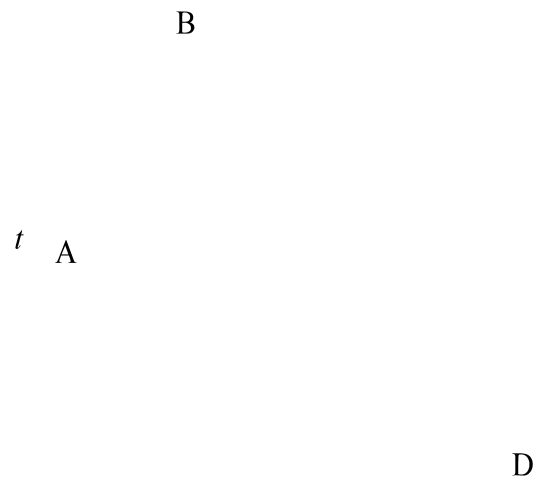




7  
( )  
2

CASE E2: *the inclusive price line of own assembled cigarette intersects ABC on BC*

The inclusive price lines in Region 1 are shown in the figure below:







$$\frac{\partial \mathcal{L}}{\partial t_a^1} = \frac{Q}{w} \quad ; \quad (2)$$



3.6 CASE F:  $t_k^2$  .

imported kit if  $p + t_a^1 = p + t_k^2 + w'$  or setting . if

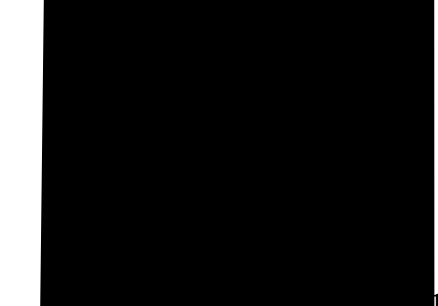
CASES F1: *inclusive price line of an own assembled cigarette intersects ABC on AB*

$t_k$

CASE F1.(i):

$$t_k^2 = \delta \bar{w}, \text{ if } w = (t_a^1 -$$

. An individual in Region 1 faces the same inclusive price of



W

]





Try  $A > 0$ ,  $B = 0$ ,  $C = 0$  and  $D = 0$ .

From Equation (3):

$$1 = (\delta + \frac{T}{2}) \dots \text{Equation (2)} \dots \text{Equation (1) as *ind}$$

CASE F1.(ii): *the inclusive price line of own kit lies above ADE* .

The inclusive price lines for this case are shown in the figure below:

$$t_k^2 + wT^+$$

$$\frac{t_a^1 - \delta \bar{w}}{T + \delta} \leq \frac{\bar{w}}{T + \delta} . \text{Setting } t_a^1 = \delta \bar{w} \text{, tax2.5venue in Region 1 is}$$

And tax revenue in Region 1 is:

$$R^1 = \frac{Q}{\bar{w}} \left( \delta + \frac{T}{2} \right) \bar{w} \left[ \bar{w} - \frac{(\delta + \frac{T}{2})\bar{w} - \delta}{\bar{w}} \right]$$

CASES F2:

Case F2.(i): the intersection is on AB;

Case F2.(ii): the intersection is on BD

Case F2.(iii): the intersection lies above ABDE.<sup>2</sup>

These are now considered in turn

CASE F2.(i) *the inclusive price line of an own kits intersects ABDE on AB*

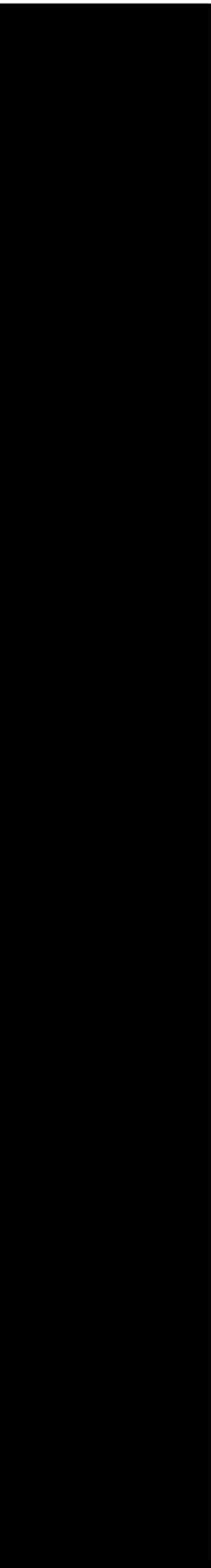
The inclusive prices in Region 1 are shown in the figure below:

*t*



-88-s.t.

;



+  $B$  (

.The Kuhn-Tucker



The alternative case is that  $t_a^1$  is set so high that no own assembled product is sold in Region 1, or  $\bar{w} \leq (t^1$  . However, given that this constraint does not bind in the case

CASE F2.(ii): the

In this case, revenue maximization by Region 1 requires that  $(t_a^1 - t_k^1) / T \leq \bar{w}$ . If this constraint were to bind,  $\partial t_a^1 / \partial w = -T / (t_a^1 - t_k^1) \bar{w}$ . If this constraint were not to bind,  $\partial t_a^1 / \partial w = 0$ . The total effect of a change in  $w$  on  $t_a^1$  is  $\partial t_a^1 / \partial w = -T / (t_a^1 - t_k^1) \bar{w}$ .

The Lagrangean is:





CASE F2.(iii): *the inclusive price line of an own kit lies above ABDE.*

The inclusive price lines in Region 1 are shown in the figure below:

$$\max t_a^1$$

Try  $A > 0$ .

From Equatio

$$t_a^1 =$$

## ENDNOTES

1. Note that the inclusive price line of an own kit is upward sloping but the price line of an assembled product is horizontal. Therefore if the inclusive price line of an own kit does not intersect on AB, it does not intersect ABC
2. Note that the inclusive price line of an own kit is upward sloping but the price line of an assembled product is horizontal. Therefore if the inclusive price line of own kits does not intersect on AB or BD, it does not intersect ABDE