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Community Income Distributions in a Metropolitan Area

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1. INTRODUCTION

A long literature in local public economics discusses the way households of different incomes distribute themselves across a metropolitan area. In Tiebout's (1956) model of fiscal competition, the primary characteristic of a jurisdiction is the public service it provides. A household chooses the jurisdiction in which to reside by trading off the public service provided by a jurisdiction against the tax it levies. If a household's public service demand increases with his income, households with higher income choose jurisdictions which provide higher public service levels, or there is sorting by income of households between jurisdictions (McGuire (1974), Berglas (1976a and 1976b), Wooders (1978)). In Tiebout's original model, jurisdictions are costlessly formed on a featureless plain and their boundaries may be adjusted to match the land demand of the households choosing to live in the jurisdiction. The conclusion of sorting by income is robust if the assumption of adjustable jurisdictional er incom ranked by median income, Chelsea is the poorest and Weston is the wealthiest.

Chelsea and Weston had median incomes in 1980 of $11 201 and $46 646 distance from the metropolitan center increases (Wheaton (1977)). In either case, there is a monotonic income gradient from the metropolitan center.

Like the Tiebout model, the Alonso-Mills-Muth model is a poor fit with the data. Firstly, the income elasticity of commuting costs is almost certain to exceed the income elasticity of land demand but the poor tend to live disproportionately in the central cities. As Glaeser, Kahn and Rappaport (©, 2000) report:

> "Theory suggests that the income elasticity of commuting cost per mile should be close to one. The marginal cost of an extra mile spent commuting includes both time and cash costs, but generally cash costs per mile are small relative to time costs. Valuing time at either the wage rate...implies a unitary income elasticity of commuting costs. ...Our objective now is to estimate the income elasticity of demand for land...our results show a quite consistent pattern where the elasticity of the demand for space with respect to income lies between 0.1 and 0.4. If these elasticities are correct, then the Alonso-Mills-Muth theory can only explain [the observed] sorting if the income elasticity of commuting time were lower than 0.3, which seems implausible."

Secondly, the income gradient is not monotonic: Glaeser, Kahn and Rappaport (©, 2000) write:

> "[We discuss] the income -distance relationship for four older metropolitan areas (New York, Chicago, Philadelphia and Boston). In these cities (and in most other older cities) there is a clear U-shaped pattern. The census tracts closest to the city center are often among the

the US this boundary typically occurs around 4 miles from the metropolitan center. Finally, income decreases in the suburb as the location moves away from the center.

Other authors have noted the failure of the canonical models to explain the empirically relevant case of income mixing between jurisdictions. Epple and Platt (1998) show that jurisdictional income distributions can overlap if households differ in two dimensions - in their income and in a parameter which reflects their preference for the public service. The preference parameter is distributed independently of income so that a jurisdiction providing a medium public service level is chosen both by a high-income household with a low preference for the public service and by a low-income household with a high preference for the public service. In our model households also differ in two ways - by their income and by their preference for the public service - but the preference parameter is perfectly correlated with income so that at a fundamental level households differ only in the single dimension of income or income is a sufficient statistic to describe a household.

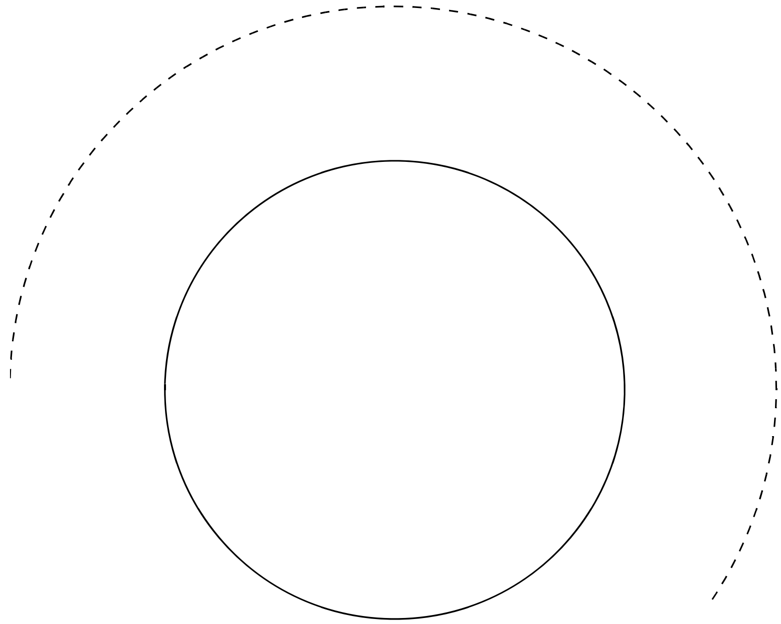
LeRoy and Sonstelie (1983) use commuting considerations to explain why some rich households live outside of some poor households in a metropolitan area. They have two income classes each with its own commuting cost, and two modes of transport which are labeled “car” and “bus”. Car travel is faster but more expensive, and in consequence it is used only by households living further out. They therefore create an income profile as the locations moves away from the metropolitan center as: rich households using the bus live on the land closest to the metropolitan center, then poor households using the bus, then rich households using the car and then poor households using the car. Although the final income gradient in LeRoy and Sonstelie is similar to our income gradient, there are important differences. In LeRoy and

Sonstelie, there are no jurisdictions and no public services (except possibly the bus). In turn this implies that house prices are continuous in the metropolitan area whereas in our model house prices change discontinuously as the location moves across the jurisdictional boundary between the central city and the suburb.

Finally, Nechyba (2000) has a model in which there is income-mixing between jurisdictions as a consequence of jurisdictions having different housing stocks. Some high-income households choose to live in the jurisdiction of low average- income because the large houses in that jurisdiction are relatively cheap. In his model the housing stock in each jurisdiction is exogenous and the reason for the assortment of house sizes is unexplained. In contrast, in our model all households buy the same lot size.

2. THE MODEL

*2.1 Spatial overview*F



$X < B$: there is undeveloped land at the edge of the city;

$X = B$: there is no undeveloped land in the city.

The city is surrounded by a suburb, labeled *s*. The outer jurisdictional boundary of the suburb is sufficiently distant that all households live in the city or in the suburb; the outer limit of development in the suburb is a circle of radius *Y*. Our interest is in how households of differing incomes distribute themselves across the metropolitan area.

2.2 Basic Analytic Structure

A household has an endowed income *M* and obtains utility *U* from consuming a privately-provided good *c* and a public service *g*. The privately-provided good is the numeraire good. In this paper we make two simplifications. Firstly, the household's demand for lot size, *a*, is assumed to be exogenous so that housing *per se* does not enter the utility function[2]: the non-land components of housing are included as part of the private good. Secondly, we consider the utility function to have consumer surplus form. In particular, we assume that a household *i* of endowed income M_i has a utility function of form

$$U^i = c + \beta_i$$

cost of the public service to low-income households and thereby encourages them to move into [redacted] *sdiction*

[redacted] hieves utility *W(M).* His bid-rent [redacted] diction *j* is *R(s, j; M)* and is [redacted]

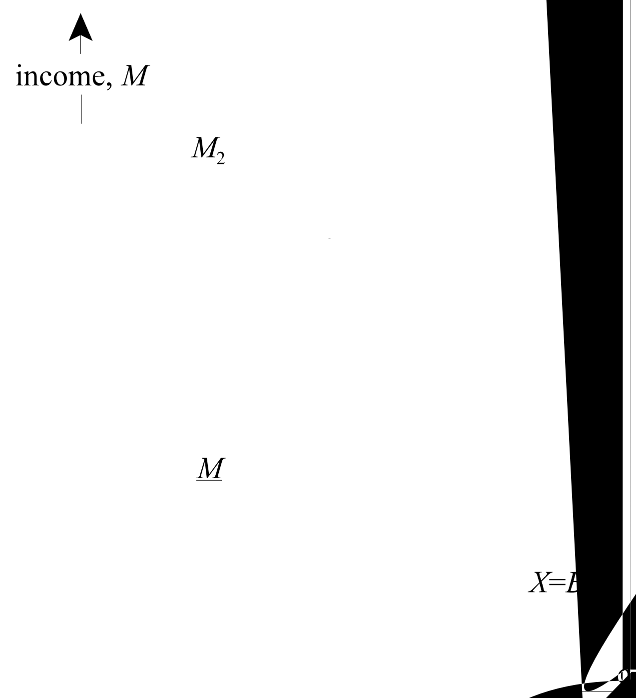
[redacted] (g_j),

from the city's center has ir gsBT8(the e)-cente

2.4 Strict income sorting between jurisdictions

For the sake of completeness and in order to provide the contrast with the equilibrium with income-mixing, we state below that, in our structure which is laid out more fully in the following subsections, there is always an equilibrium with strict income sorting:

LEMMA B: there is always a sorting equilibrium: households with endowed income in the range $[\underline{M},$ *reside in the suburb and households with endowed income in the range* *reside* [illegible]ted income profiles of the two jurisdictions [illegible]ustrated in Figure 3 where, for ease of [illegible]ation, we focus on the case in which there is [illegible]eveloped city land.



equilibrium are presented in the Appendix. For ease of presentation and because it seems the empirically relevant case, we restrict our attention to the case in which there is no undeveloped city land (*X* = *B).*

If households of income *M* locate in the city at distance *x(M)* from the city's center and in the suburb at distance *y(M)* from the city's center, the rent level at each location must be such that the households achieve the same utility at either location, or (using the assumed utility function)

(4 suburban329nt premium equals the net benefit of the higher suburban3public service less the

Differentiating Equation (4) with respect to *M*, and using Equation (3):

. (7)

g

PROOF: See Appendix A.

2.6 Closing the model

The reservation price of land is r_0: at the limit of development,

$$r(Y) = r_0 \,. \tag{12}$$

The difference in the public service in the two jurisdictions suggests that the rent changes discontinuously across the jurisdictional boundary. In the case under consideration there is no undeveloped land in the city so that on the city side of the jurisdictional boundary the rent may exceed r_0, or[9]

$$X = B : \lim_{x \to B^-} r(x) \geq \tag{13}$$

There are

Because the inequalities are strictly satisfied, continuity implies that an equilibrium, in which the city and suburb income ranges overlap, continues to exist if there is a small change in the model’s parameter values.

Our next observation concerns the distribution of the households between the city and suburb in the range of overlap. All households with incomes below M_1 live in the city, or ″$(M: M < M_1) = 1$. Equation (6) shows that in general ″$(M_1) < 1$; therefore ″(M) is in general discontinuous at M_1 . In contrast, all households with incomes above M_2 live in the suburb, or ″$(M: M > M_2) = 0$. $x(M_2) = 0$ and hence Equation (6) shows ″$(M_2) = 0$, or ″(M) is continuous at M_2. The likely case is that ″(M) decreases monotonically from ″(M_1) to ″(M_2),or the city contains a decreasing share of the higher income households in the range of overlap. This is a consequence of the geometry of a monocentric metropolitan area: as x decreases, circular elements in the city become smaller at a faster rate than circular elements in the suburb.

Having established the existence of an equilibrium with income-mixing, we now revisit the three quotations of the Introduction. Firstly, the equilibrium does have the property that median city income is less than median suburban income but the highest income in the city exceeds the lowest income in the suburb. Secondly, our model has an elasticity of land demand (zero) which is much less than the income elasticity of commuting cost (unity). Thirdly, refering to Figure 3, the income distribution predicted in the equilibrium is approximately U-shaped: higher income households live close to the city's center, income then decreases but it jumps up again at the city's boundary. Finally, income falls away in the outer suburb.

4. BOUNDARY FIXEDNESS

In the equilibrium with income mixing, households adjacent to the jurisdictional boundary at B are *not* indifferent as to the jurisdiction in which they live. Referring to Figure 3, on the city

5. CONCLUSION

In this paper we have placed the model of fiscal competition inside a spatial model of a central city surrounded by a suburb. In doing so, we have produced a model in which there are equilibria with the property that, although the city has lower median income, the highest income in the city exceeds the lowest income in the suburb. The model thereby explains some stylized facts which the canonical models of Tiebout and Alonso-Mills-Muth are unable to explain. We are currently working on testing the model using Connecticut data.

In addition, the model explains the fixedness of jurisdiction boundaries and why there is no presumption that boundaries shift to expand jurisdictions with high rents. This explains the findings of Epple and Romer (1989).

The model has two limitations. Firstly, it assumes that all households have a fixed demand for land and the normality of the public service has been introduced by making a household's taste depend on its endowed income. We are currently addressing this limitation by reworking the model, making land demand endogenous and giving all households the same Cobb-Douglas utility function. Secondly, the model does not explain why the equilibrium with income-mixing

APPENDIX A: PROOF OF LEMMAS A AND C

b a e h t

PROOF OF LEMMA A:

The proof is sketched - a full proof is available from the authors on request. An equilibrium is constructed by considering an initial assignment as: The limit of city development is set at the jurisdictional boundary, $X = B$; the associated value of Y is calculated by setting land demand

s a eh t $_{0}$, d en a X X

APPENDIX B: ALGORITHM USED TO CALCULATE EQUILIBRIUM

FOR THE EXAMPLE USED TO ESTABLISH THE THEOREM

The metropolitan income distribution is uniform on the range [_ so that

[redacted] : land demand supply equals land demand, or

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1. A possible reason for the difference between the older cities and the new cities is that the newer cities have more decentralized employment (as noted by Glaeser et al. (2000)) .

2. The assumption of a fixed housing size greatly simplifies the analysis as it makes constant the population of each jurisdiction and ensures that, within each jurisdiction, the rich households live closer to the central business district. In addition, it avoids well-known existence problems for traditional stratified local public finance equilibria (as in Rose-Ackerman (1979) and Epple, Filimon and Romer (1984, 1993)) and it allows us to focus thereby on the existence of the equilibrium with income-mixing.

3. We want to stress that, because household *i*'s income is exogenous, it's taste parameter $\beta_i \equiv \beta(M_i)$ is exogenous.

4. For convenience of presentation, the public service is assumed to show constant returns to jurisdiction size. Because each jurisdiction contains a fixed number of households, no results change if the service is a local public good.

5. See Fujita (1989, Chap. 4) for a more general development. See Beckman (1969) and Montesano (1972) for the development of an urban model with an income distribution.

6. The importance of using the absolute value in this way is stressed by Montesano (1972).

7. We do not believe that the myopic assumption is important. What is important is that the suburb votes a different public service than the city. See Epple and Romer (1986) for non-myopic voting in a model with redistributive local governments.

8. A household with income *M* is located at *s(M)*. His desired public service is:

,

9. If there [redacted] oped land at the periphery of the city, the rent at t [redacted]
r_0:

$$X \text{ [redacted] }) = r_0 \ .$$

10. If the [redacted] eloped land in the city,

[redacted] .

[redacted]