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Technology Life-Cycles and Endogenous Growth

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Technology Life-Cycles and Endogenous Growth

Abstract

1. Introduction

$$t \quad i \quad t \quad \frac{\mu \quad \P_{\circledR}}{t \quad t}$$

2.2. Individuals

6

2.3. The Technology and Potential versus Actual Productivity

t t

t

7

t

$$t \quad \frac{t \quad t \quad t}{\bar{\circledR}}$$

t

6

7 Á z

i t

t+1 t i

2.4. Adoption of New Technologies and Next Generation Machines

2

$$\hat{A}_t^j;\ z_t^j;\ q_t^j;\ L_t^j \qquad t \quad t^{j\ \circledR} \quad t^{j\ 1\ i\ \circledR} \quad i \quad t\ t^{j} \quad i \quad t\ t^{j}$$

t t

8 2

$$\left[\ \mu \quad i \ \frac{\quad}{\quad i} \quad \P \ \frac{t^{j}\ t^{j\ \circledR}}{t^{j\ 1\ i\ \circledR}}\ \right]^{\frac{\quad}{i\ \circledR}} \qquad t^{j}$$

i

t

Lemma 1: $\frac{@q_t^j}{@(t\ i\ v)}$ $\frac{@q_t^j}{@x_t}$

Proof:

$$\frac{t^{j}}{t} \quad \frac{\quad}{i} \quad \frac{t^{j}}{i^{\ 2}}$$

$$\frac{t^{j}}{i} \quad \frac{\quad}{i} \quad \frac{t^{j}}{t^{2}}$$

$$\dot{A}_t \; \_ \; \circledR \; \dot{\imath} \; \frac{t\,\dot{\imath}}{t} \; \dot{\imath} \; \frac{\dot{\imath}}{\dot{\imath}}$$

Proof:

$\dot{\imath}$

Lemma 3: $8_{t,} \; \acute{} \; \dot{\imath} \; \; \bar{} \; \dot{\imath}$

$$\frac{x_t}{\dot{\imath}} \qquad \frac{x_t}{t}$$

$$8_{t,\,t} \acute{} \; f \; \dot{\imath} \; \dot{\imath} \; \dot{\imath} \; \dot{\imath} \; \dot{\imath} \; g$$

$$\frac{\dot{A}_t}{\dot{\imath}} \qquad \frac{\dot{A}_t}{t} \qquad \frac{\dot{A}_t}{}$$

Proof: 8 t ¸ $\overset{x}{t}$ $\overset{3}{\overline{\quad ® \quad}}$

exp

Lemma 4: $8_{t\,,}$

$$\frac{x_t}{x_t}$$

$8_{t\,,\ t}$

$$\frac{Á_t}{Á_t}$$

Proof: $x_t \quad x_t \ i \quad x_t \quad 8_{t\,,}$

$$\frac{x_t}{x_t} \quad i \quad \frac{x_t}{i \quad x_t}^{3} \quad x_{t;v} \ i \ -'$$

$$Á_t \quad Á_t \ i \quad Á_t \quad 8_{t\,,\ t;v}$$

$$\frac{Á_t}{Á_t} \quad i \quad \frac{Á_t}{i \quad Á_t}^{3} \quad Á_t \ i \ -'$$

¤

2.5. Equilibrium R&D E®ort in Inventions versus Innovations

10 $Á_t$

x_t

$Á_t \quad x_t$

µ ¤ ¶°

¤ $\frac{¤_t}{¤_t}$ ¤ .

10 8)8)8 ; 8

¤
t

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¤;n ¤
t t

( )

Á Á;n x;n
t i x t Á t i x t i Á;n x;n
t t t z t t t

! Á;n , ! x;n
t t

Á Á;n Á x;n x
t i x t t t t t i t t t t

11 ¤ ¤ ¤ R 1 ¤;j
t t i t 0 t

Á
t

Á v Á
t t t

x Á x
t t t

Proposition 1:

8

Á
Á;n t i x Á
t t t

x
x;n t i Á x
t t t

Á;n x;n Á Á;n Á;n
t t t t t

x x;n x;n
t t t

Proof:

¤

¤ ¤ ¤;n ¤
t t t

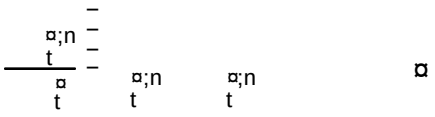
12 13

12

! ¤ I

13

Proposition 2: 8



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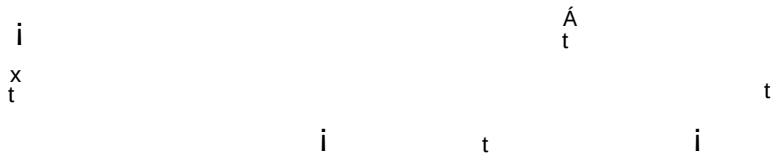


Á

x

0

Á x

i

t i

6. Appendix

2

$$8_t \cdot \quad i$$

$$\frac{Á_t \; x_t}{i} \qquad \frac{Á_t \; x_t}{t}$$

$$8_t \qquad i \quad \frac{@(I_t^{Á}=I_t^{x})}{@(t\ ¡\ v)} \qquad \frac{@(I_t^{Á}=I_t^{x})}{@x_t} \quad ,$$

□

Figure 1

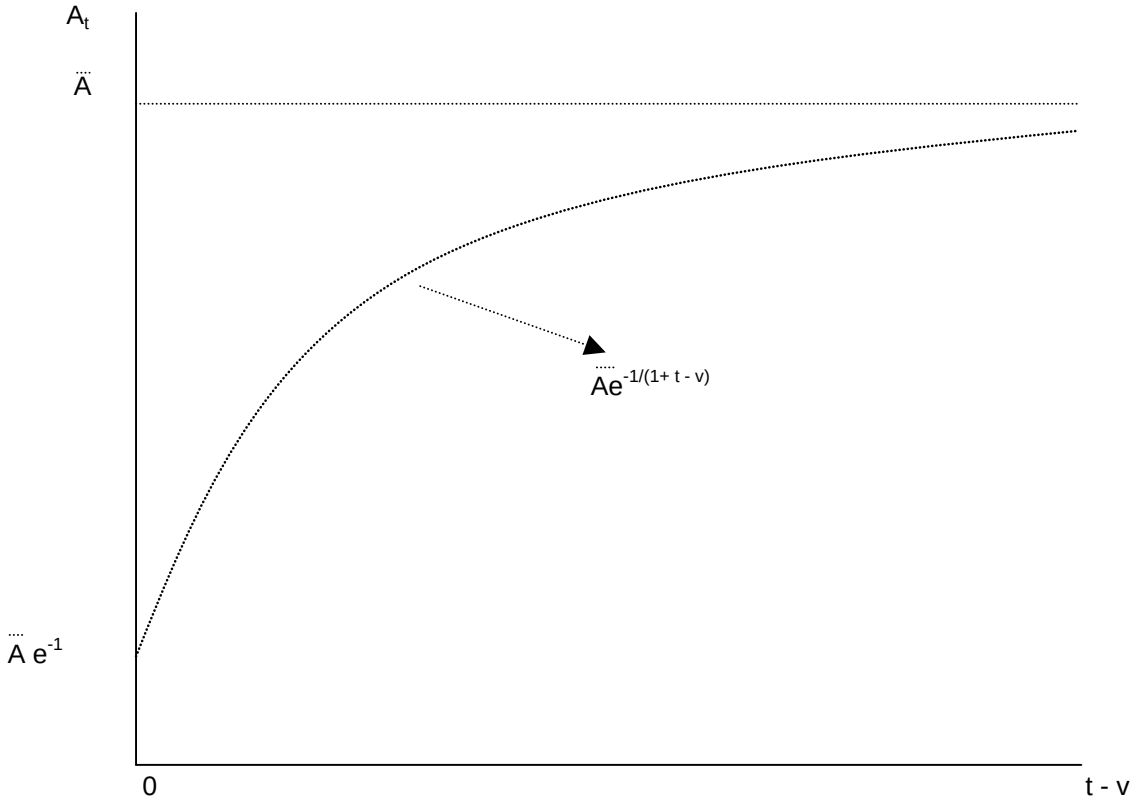


Figure 2

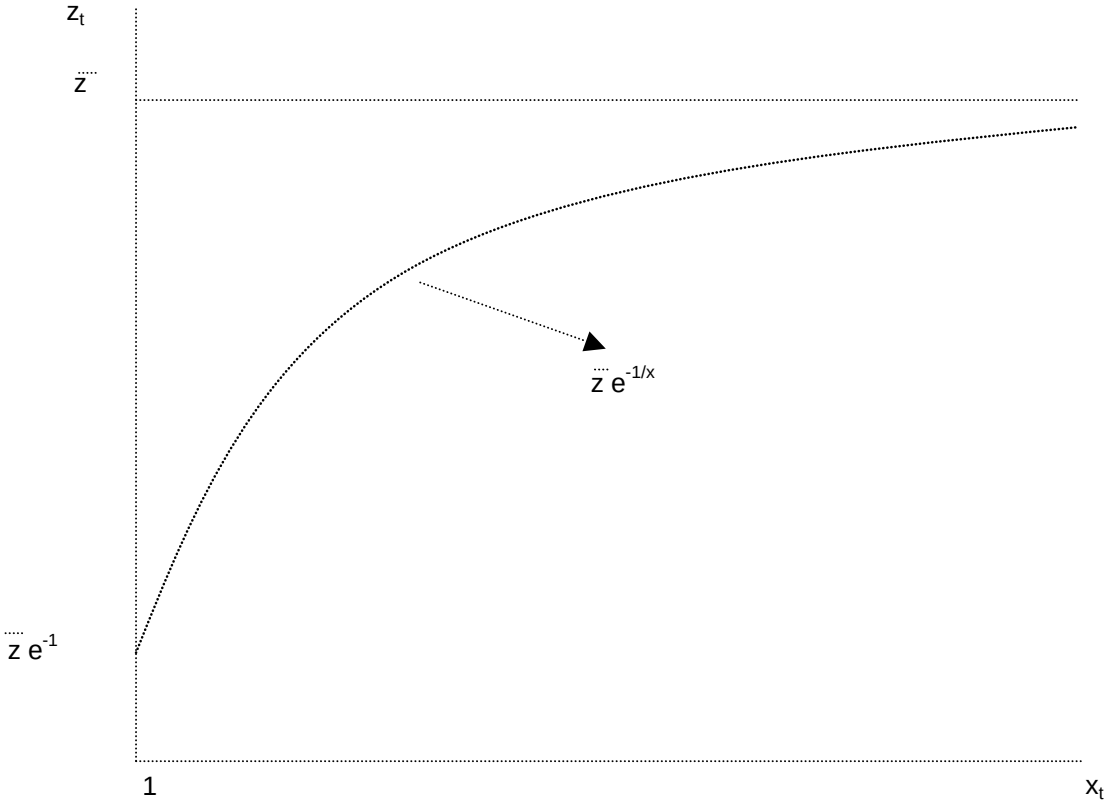


Figure 3

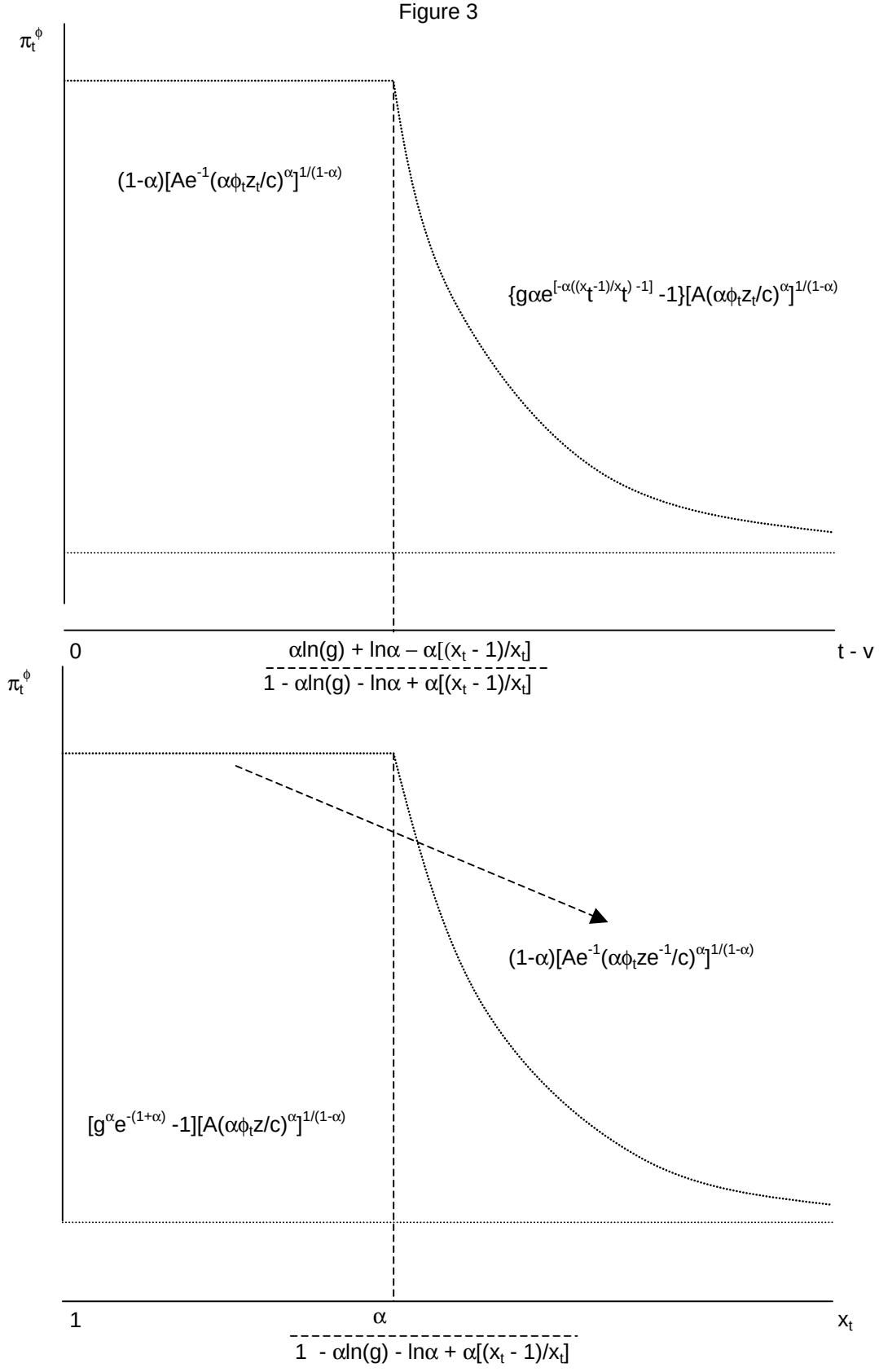
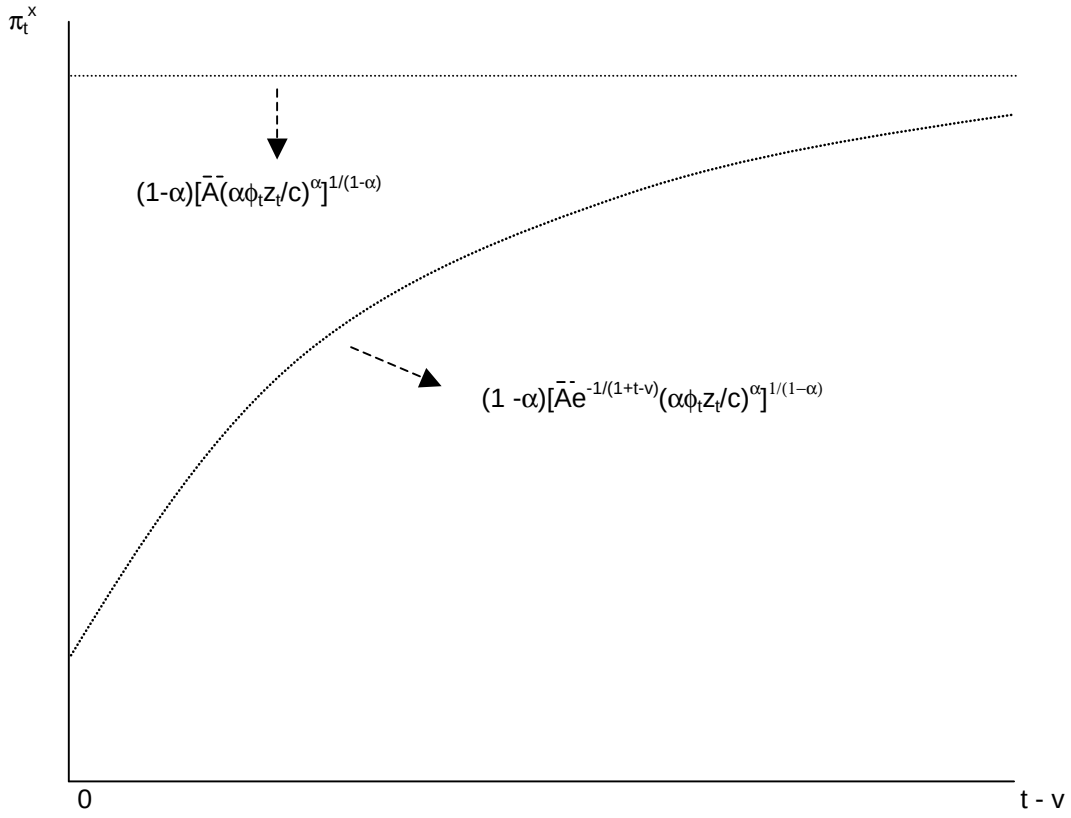

π_t^{ϕ}
$(1-\alpha)[Ae^{-1}(\alpha\phi_t z_t/c)^{\alpha}]^{1/(1-\alpha)}$
$\{g\alpha e^{[-\alpha((x_t-1)/x_t)-1]}-1\}[A(\alpha\phi_t z_t/c)^{\alpha}]^{1/(1-\alpha)}$
0
$\frac{\alpha\ln(g)+\ln\alpha-\alpha[(x_t-1)/x_t]}{1-\alpha\ln(g)-\ln\alpha+\alpha[(x_t-1)/x_t]}$
t - v
π_t^{ϕ}
$(1-\alpha)[Ae^{-1}(\alpha\phi_t z e^{-1}/c)^{\alpha}]^{1/(1-\alpha)}$
$[g^{\alpha}e^{-(1+\alpha)}-1][A(\alpha\phi_t z/c)^{\alpha}]^{1/(1-\alpha)}$
1
$\frac{\alpha}{1-\alpha\ln(g)-\ln\alpha+\alpha[(x_t-1)/x_t]}$
x_t

Figure 4


π_t^x
$(1-\alpha)[\bar{\bar{A}}(\alpha\phi_t z_t/c)^{\alpha}]^{1/(1-\alpha)}$
$(1-\alpha)[\bar{\bar{A}}e^{-1/(1+t-v)}(\alpha\phi_t z_t/c)^{\alpha}]^{1/(1-\alpha)}$
0
t - v

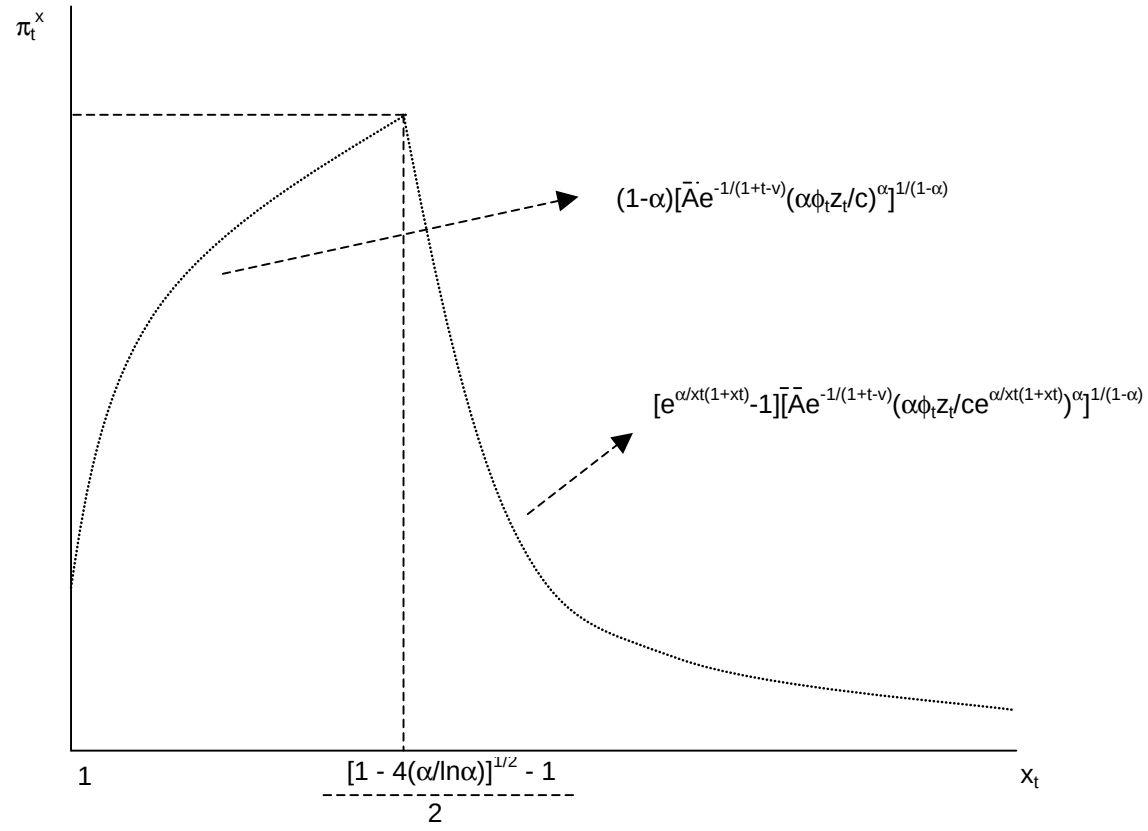

π_t^x
$(1-\alpha)[\bar{\bar{A}}e^{-1/(1+t-v)}(\alpha\phi_t z_t/c)^{\alpha}]^{1/(1-\alpha)}$
$[e^{\alpha/xt(1+xt)}-1][\bar{\bar{A}}e^{-1/(1+t-v)}(\alpha\phi_t z_t/ce^{\alpha/xt(1+xt)})^{\alpha}]^{1/(1-\alpha)}$
1
$\frac{[1-4(\alpha/\ln\alpha)]^{1/2}-1}{2}$
x_t

Figure 5:

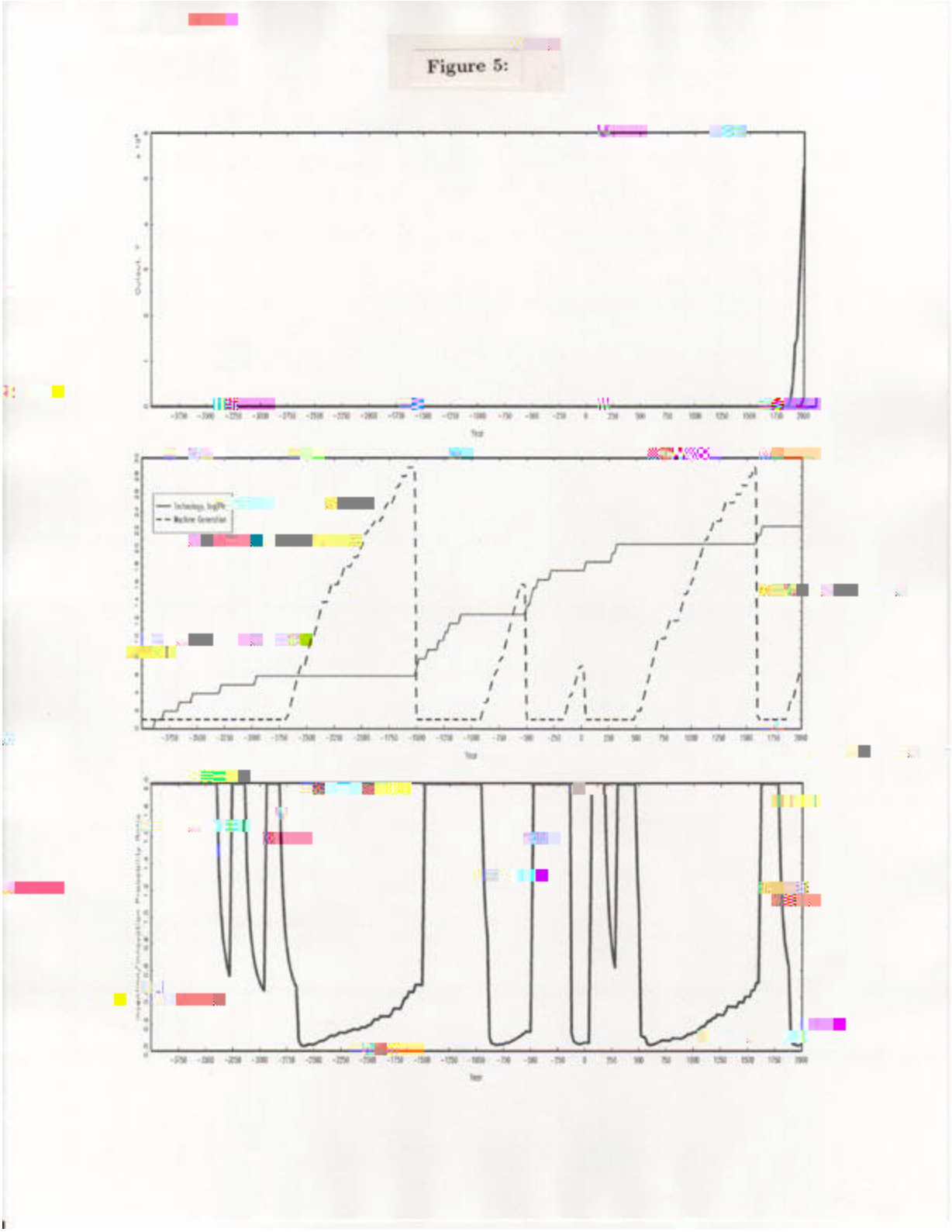


Figure 6:

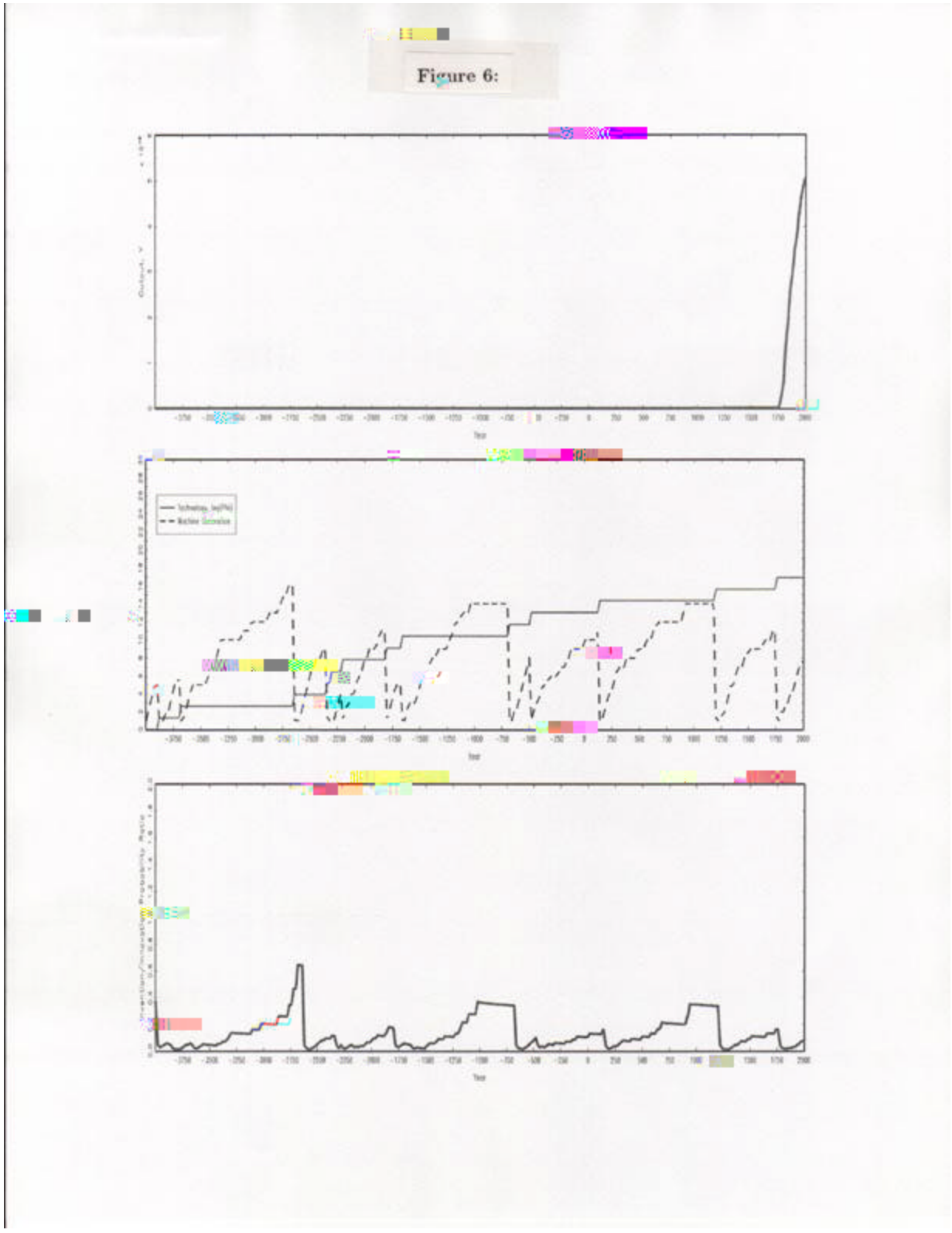


Table 1:

Á		
x		
0	¡ 20	¡ 20