DISCUSSION PAPERS IN ECONOMICS

Working Paper No. 00-01

Does Evolution Solve the Hold-up Problem?

Tore Ellingsen
Department of Economics, Stockholm School of Economics
Stockholm, Sweden

Jack Robles
Department of Economics, University of Colorado at Boulder
Boulder, Colorado

January 2000

Center for Economic Analysis
Department of Economics

University of Colorado at Boulder
Boulder, Colorado 80309

Does Evolution Solve the Hold{up Problem?

1 Introduction

[illegible]

2 Investment and Bargaining

[illegible]

[illegible] $D_A \models \{V\ I() - x; V\ I()\}$ [illegible]

[illegible] B [illegible] A [illegible]

[illegible] A [illegible] D [illegible]

[illegible] B [illegible] A [illegible]

[illegible] $I; y\ x()$ [illegible] A [illegible] $y, D \rightarrow D$ [illegible]

[illegible]

[illegible] A [illegible] $I \models I^*$ I

3 Evolution

... A ... B ... *population* ... N ... $t \in \{1, 2, \ldots\}$ ... A ... B ... *beliefs* ... A ... B ... B ... A ... B ... x ...

Assumption 1 *(i) The pie division is small: ... (ii) The population is large: ...*

adaptation

random mutation.

Θ

basin of attraction

μ *B* μ

μ′ μ μ′ *single mutation neighborhood* μ μ′ ∈ *M* μ μ′ μ

X *mutation connected set*

μ₁ μ*ₙ* ⊂ *X* μ₂, …, μ*ₙ*₋₁

k ; …; *n*; *M* μ ∩ *BX*μ *n*

$$x^L = \max\{x \in D_B(I^*) \mid (V(I^*) - x)\frac{N-1}{N} - I^* \geq V(I) - I\}.$$

$$x^M = \max\{x \in D_B(I^*) \mid V(I^*) - x - I^* > V(I) - I\}.$$

$x^M \leq x^L \leq x^M.$

$$\begin{aligned} V^* - x^M (N-1)/N - I^* &= V(I^*) - x^M - I^* - (V(I^*) - x^M)/N \\ &\quad - (N-1)/N \\ &> V(I^*) - x^M - I^* - x^M/N \\ &\geq V(I^*) - x^M - I^* \\ &\geq V(I) - I; \end{aligned}$$

$(N-1)/N$

x^L

Proposition 3 *Let agents bargain according to the Nash demand game. The outcome ‰ is locally stable if and only if ‰ = $\{I^*; V(I^*) - x; x\}$, where $x \leq x^L$.*

Theorem 1 *An equilibrium μ is stochastically stable if and only if no other equilibrium has lower stochastic potential.*

$r\ x_{()} > r\ x_{()}$ $x > x^L$

$x^M > x^{NBS}$

$r\ x^L_{()} \geq r\ x^L - -_{()};$

$$- \longrightarrow\ - \longrightarrow ::: \longrightarrow x^L - - \longrightarrow x^L;$$

$r\ x^L_{()} \geq r\ x^L - -_{()}$

$$x^L \longrightarrow - \longrightarrow\ - \longrightarrow ::: \longrightarrow x^L - -:$$

Appendix: Proofs

Lemma 1 *Let* $z_1 < z_2 ::: < z$ *be demands in* $D\ I()$ *for some* $I \in \Psi$. *Assume that the set of demands following* I *for agents in the relevant population is* $\{z_l\}_l$

Lemma 4 *Let* μ' *(*$\text{‰}\,\mu'() = \{I'; y'; x'()\}$*) be an equilibrium. If* $I \angle I'$ *and* $y - I \geq y' - I'$*, then the population can get from* μ' *to an equilibrium* μ *with* $\text{‰}\,\mu() = \{I; y; x()\}$ *through a sequence of single mutation transitions.*

$I; V \; I () - - ()$

$\{ I^*; y; x \}$ $x \leq x^L$

μ_1

‰ μ_1 ‰ μ $I \angle I^* \; V \; I - - - I \leq V \; I - - - I <$ $\Omega \; \Omega$

$V \; I^* () - x^L () \; N - i = N - I^* \leq V \; I^* () - x () \; N - i = N - I^*$

$A \; x () \; \mu i$

[illegible] A [illegible] $(I; V - -; -)$ [illegible] B [illegible] $x^L \in \{x^M; x^M - -\}$:

Lemma 5 *The number of mutations required to get from an equilibrium with outcome $(I^*; y; x)$ with $x \leq x^L$ to an equilibrium with outcome $(I; V - -; -)$ is $r(x) = \min\{r \mid r > N(1 - \frac{\hat{V} - \delta - \hat{I} + I^*}{V^* -})\}$.*

[illegible] r [illegible] B [illegible]

$$\frac{N - r}{N}(V^* - x) - I^* < V - I - -; \quad (\ )$$

[illegible] r [illegible] □

Lemma 6

(i) If µ is an equilibrium with outcome $(I^*; y; x)$ and $x < \min\{x^M; x^{NBS}\}$; the easiest transition away from ‰ µ ( ) Sqbb2655 b29 Te(0 82TΩ0.9482 2 2 2F7283m)-343(with)-38326

$x > x^L$ ... $x = x^L = x^M$ ... $x^M < x^{NBS}$; ... $r(x) \geq r(x)$ ... $V^* - x^M - - \leq V - - - I$ □

$x = x^M > x^{NBS}$ ... $(I; V - -; -)$ ... $(I^*; y -; x - -)$ ...

... $(I^*; y; x)$ ... $y = V^* - x$ ... $(I^*; y'; x')$ ...

Lemma 7 *From an outcome $(I^*; y; x)$ the easiest transition in which investment is at all times e–cient, but which ends with difierent demands, is to an outcome $(I^*; y'; x')$ where $x' = x - -; x$ –;–; or $V^* - -$.*

□

Lemma 8

(i) Moving from x to $x - -$ takes $N(-\frac{-\delta}{})$ mutations to pop A.

(ii) Moving from x to x – takes $N(-\frac{V^* - -\delta}{V^* -})$ mutations to pop B.

(iii) Moving from x to – takes $N(\frac{V^* -}{V^* - \delta})$ mutations to pop B.

(iv) Moving from x to $V^* - -$ takes $N(\frac{}{V^* - \delta})$ mutations to pop A.

... N □

Lemma 9

(i) If $- < x < V^* - -$, then moving from x to $x - -$ takes fewer mutations than moving from x to –, and moving from x to x – takes fewer mutations than moving from x to $V^* - -$.

(ii) If x ⊢ *– then moving from x to*

[illegible] □

[illegible]

Lemma 11 *Let surplus be divided by the ultimatum game. The component with the subgame perfect outcome,* $(l^H, V^H - x^{\max}(l^H), x^{\max}(l^H))$, *is a subset of the unique locally stable set.*

[illegible]

□

Lemma 12 *Let surplus be divided by the ultimatum game. Agents in population A receive a payofi of at least* $V^H - l^H - x^{\max}(l^H)$ *in every equilibrium.*

[illegible] □

Lemma 13 *Let surplus be divided by the 'ultimatum' game. If* $V(l) - l - x \geq V^H - l^H - x^{\max}(l^H)$, *then there exists an equilibrium* μ *such that* $\mu \in \Theta^L$ *and* ‰$(\mu) = (l, V(l) - x, x)$.

6 References

Journal of Economic Theory

Economic Journal

Review of Economic Studies

Games and Economic Behavior

Quarterly Journal of Economics

American Economic Review

Theoretical Population Biology

Games and Economic Behavior

Journal of Political Economy

Econometrica

Journal of Law, Economics and Organization

Markets and Hierarchies: Analysis and Antitrust Implications,

Econometrica

Journal of Economic Theory

Review of Economic Studies