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Does Evolution Solve the Hold-up Problem?

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ll r f r p p l r k r g p p g g m r b l p
 p b m l ll r l p k r p p b f b r g p p g
 g m b p q a l r m b b r k r p p b r p
 b p j a p b k r g p p g m b b p r l
 r p g p f r r b b k b f ll p g a p
 • \W b f b r g p p g g m b m l l g m r f a l r r'
 • \W b f m l b r l p p b p g m r f p'

l p p p r p g m l p l b m l r b l p
p p l f r g m r f l r p f r m r g p p g
r m p b l p b r r f g m r f
p m p b l p l r m l r g p p g g m
b p g g m r f a l r m p m l p l m m g m p
b b b p r p m l r r b r l m b r k p g
r p r r l r p g m r f p b g m r b r k p g
l m l f b l r l m b p r b l p r
p m r b m g r b p b r n g r p r b l p p r ll
b b l r l p p p l b p r b l m k r ll p
p m p b l b m b ll l m p b r
m p l W n g b p p m p l l
l b g b r m l f r g p p g g m b r l
g g b g p r l r p l m g b r f b p b r p p
p p p p g m r f p b W l p m l
k m p m l l p a b m p b
b r b p b p p p g m r f p r p m b
l b l l n g r f b p g p r l f r b
r g p p g g m r r p r m p l p r
p l f b b l r l m n g b r p p r r g p p g g m
p m p p m p
b l r p l l p r b r l p g p r p l
p r p n g b p f r r p p b r p b b
p r l r b r f b r l b p r p k
b b r n g r p r l b ll p m r g
f r ll b r r p l l r p b b p m p p r
k p b r b p r r p p p f r l a l r m
r b m m k b f r r p p r g m p b b r n g
r p r b l r a r b r p b r p l b r b p
b r n g r p r b l p r p b p a l r m b b m k
b r l l f p m p r p k f r b p r p m

2 Investment and Bargaining

$\mathcal{W} = \{ \omega \in \mathbb{R}^N \mid \omega_i \geq 0, \sum_{i=1}^N \omega_i = 1 \}$

$\Psi = \{ \psi = (\psi_0, \psi_1, \dots, \psi_N) \mid \psi_i \geq 0, \sum_{i=0}^N \psi_i = 1 \}$

$V = \{ v \in \mathbb{R}^N \mid v_i \geq 0, \sum_{i=1}^N v_i = 1 \}$

$\mathcal{W} \times \Psi \times V$

$\mathcal{S} \text{ gm } \mathcal{R}^f \text{ p m } r \text{ ll } p \text{ m } p \text{ b } b \text{ m } p \text{ gm}$
 $p \text{ l } m \text{ l } p \text{ m } p \text{ b } l \text{ m } m \text{ gm } b \text{ p } l \text{ b}$
 $p \text{ l } p \text{ r } \text{ ll } r \text{ p } b \text{ p } l \text{ b } r \text{ r } p \text{ f } l \text{ p } r$
 $l \text{ p } \text{ (} \text{ gm } \mathcal{R}^f \text{ p}$

3 Evolution

$\text{b } b \text{ r } g \text{ r } r \text{ p l } p \text{ r } p \text{ r } b \text{ m } b \text{ l } g \text{ p } \text{ (}$
 $p \text{ f } b \text{ b } r \text{ g } p \text{ r } m \text{ r } r \text{ r}$
 $b \text{ f } b \text{ l } p \text{ p } g \text{ m } p \text{ r } r \text{ p } p \text{ g}$
 $\text{ (} \text{) } p \text{ r } l \text{ b } p \text{ l } \text{ (} \text{) } p \text{ p } g \text{ l } \text{) } b \text{ p } p$
 $p \text{ f } r \text{ p } g \text{ m } l \text{ k } \text{) } p \text{ s } m \text{ l } p \text{ l } \text{) } p \text{ l}$
 $b \text{ r } \text{ (} r \text{ m } r \text{ k } b \text{ r } \text{)}$
 $r \text{ b } l \text{ r } r \text{ l } A \text{ p } B \text{ l } b \text{ r } \text{ population } \text{ (} N \text{) } b$
 $r \text{ t} \in \{l; m\} \text{ r } l \text{ m } p \text{ p } \text{ (} g \text{ p } p \text{ l } p \text{ A } p$
 $B \text{ p } p \text{ l } b \text{ p } m \text{ p } m \text{ r } g \text{ p } p \text{ g } g \text{ m } b \text{ f } r \text{ g } k$
 $b \text{ m } g \text{ p } l \text{ b } l \text{ beliefs } b \text{ r } p \text{ p } \text{ (} p \text{ g}$
 $p \text{ l } p \text{ r } r \text{ b } p \text{ r } g \text{ r } l \text{ r } b \text{ r } p \text{ ll } g \text{ p}$
 $b \text{ r } l \text{ (} \text{ " } \text{) } p \text{ l } r \text{ A } l \text{ (} p \text{ r } p \text{ g } l \text{ r } B \text{ m } p$
 $p \text{ l } \frac{3}{4} \text{) } p \text{ l } r \text{ B } l \text{ (} l \text{ r } A \text{ m } p \text{ b } \text{ "$
 $p \text{ } \frac{3}{4} \text{ r } r \text{) } l \text{ r } p \text{ p } b \text{ (} l \text{ m } p \text{ p } p \text{ b}$
 $r \text{ p } p \text{ g } p \text{ b } p \text{ m } p \text{ l } \text{ (} r \text{ l } r \text{ p } g \text{ b}$
 $l \text{ m } m \text{ g } m \text{ } \frac{3}{4} \text{ l } p \text{ p } l \text{ r } B \text{ m } p \text{ x}$
 $\text{ \textbackslash W } \text{ p } \text{ (} p \text{ l } \text{ [} b \text{ p } l \text{ m } p$

Assumption 1 (i) The pie division is small: $V \text{ l } \text{) } > \text{ -}$. (ii) The population is large: $V \text{ l } \text{) }^*$

$f: \mathbb{R}^n \rightarrow \mathbb{R}^n$ is a continuous map. Let $\mu \in \mathbb{R}^n$ and $B(\mu, \epsilon)$ be a neighborhood of μ . The basin of attraction of μ is the set of points $x \in \mathbb{R}^n$ such that $f^k(x) \rightarrow \mu$ as $k \rightarrow \infty$. A single mutation is a point $\mu' \in B(\mu, \epsilon)$. A mutation connected set is a set of points $\mu_1, \mu_2, \dots, \mu_{n-1}$ such that $\mu_i \in B(\mu_{i-1}, \epsilon)$ for $i = 2, \dots, n-1$. The set $M(\mu) \cap B(\mu, \epsilon)$ is the set of points $\mu_1, \mu_2, \dots, \mu_{n-1}$ such that $\mu_i \in B(\mu_{i-1}, \epsilon)$ for $i = 2, \dots, n-1$.

$\{x \in D_B \mid V(x) \geq V(I^*) - \frac{N-i}{N} \cdot (I^* - I)\}$

$$x^L \leftarrow \{x \in D_B \mid V(x) \geq V(I^*) - \frac{N-i}{N} \cdot (I^* - I)\}$$

$\{x \in D_B \mid V(x) \geq V(I^*) - \frac{N-i}{N} \cdot (I^* - I)\}$

$$x^M \leftarrow \{x \in D_B \mid V(x) \geq V(I^*) - \frac{N-i}{N} \cdot (I^* - I)\}$$

$x^M \leq x^L \leq x^M$

$$\begin{aligned}
 V^* - x^M &= V(I^*) - x^M - I^* - V(I^*) - x^M = N - N - i = N - i \\
 &> V(I^*) - x^M - I^* - \dots - x^M = N \\
 &\geq V(I^*) - x^M - I^* \\
 &\geq V(I^*) - I^*
 \end{aligned}$$

$\{x \in D_B \mid V(x) \geq V(I^*) - \frac{N-i}{N} \cdot (I^* - I)\}$

Proposition 3 *Let agents bargain according to the Nash demand game. The outcome x_0 is locally stable if and only if $x_0 \in \{I^*; V(I^*) - x; x\}$, where $x \leq x^L$.*

$\{x \in D_B \mid V(x) \geq V(I^*) - \frac{N-i}{N} \cdot (I^* - I)\}$

$$\begin{array}{c}
\begin{array}{c}
p \ b \ b \ r \ X \rangle > r \ X \rangle \ b \ p \ r \ X > X^L \\
p \ r \ p \ p \ p \ p \ r \ p \ r \ b \ X^M > X^{NBS} \ p \ \rangle \ \parallel \ r \\
b \ p \ l \ \{ \ b \ b \ p \ p \ g \ p \ p \ p \ g \ l \ \} \ b \ r \ \{ \\
r \ X^L \rangle \geq r \ X^L - \rangle \ b \ p \ p \ p \ p \ r \ p \ r \ g \ p \ \}
\end{array} \\
\longrightarrow \longrightarrow \dots \longrightarrow X^L \longrightarrow X^L; \\
b \ l \ \{ \ r \ X^L \rangle \geq r \ X^L - \rangle \ b \ p \ p \ p \ p \ r \ p \ r \ g \ p \\
X^L \longrightarrow \longrightarrow \dots \longrightarrow X^L \longrightarrow
\end{array}$$

p p p b p b r p p p p p l l r p⁸
l s p b r g p p g p r p g b r l f b
l p p g p p /^H b b V /^H - /^H

$\mathbb{W} \setminus \{z_i\} \cap \mathbb{D} \cap \mathbb{Q} \neq \emptyset$
 $\mathbb{W} \setminus \{z_i\} \cap \mathbb{D} \cap \mathbb{Q} \neq \emptyset$
 $\mathbb{W} \setminus \{z_i\} \cap \mathbb{D} \cap \mathbb{Q} \neq \emptyset$
 $\mathbb{W} \setminus \{z_i\} \cap \mathbb{D} \cap \mathbb{Q} \neq \emptyset$

Appendix: Proofs

$\mathbb{W} \setminus \{z_i\} \cap \mathbb{D} \cap \mathbb{Q} \neq \emptyset$
 $\mathbb{W} \setminus \{z_i\} \cap \mathbb{D} \cap \mathbb{Q} \neq \emptyset$
 $\mathbb{W} \setminus \{z_i\} \cap \mathbb{D} \cap \mathbb{Q} \neq \emptyset$

Lemma 1 Let $z_1 < z_2 < \dots < z_n$ be demands in $\mathbb{D} \setminus \{z_i\}$ for some $i \in \Psi$. Assume that the set of demands following i for agents in the relevant population is $\{z_i\}_i$

p p p l p l ll g p p b r l p b p k
 l b p p V l z p p p g p l ll p p k
 p p z p b p p b r p p r r
 b b p r b p p b Q p r pg sp M p N
 p r l y r p p x b r l r p
 f ll pg l sp b p p r r b r r g pg l r
 ll b r f r b l p p g l r p Q pg p \square
 r b p r r r p
 \backslash W r b p r r r p
 r f f r p p b Q p pg p sp pg
 p b pg b l f $\%$

(ii) If x_{\downarrow} - then moving from x to

$x^{NBS} < x \leq x^L$ $\mu; \mu'_{\neq \delta}$ $x < x^{NBS}$ $\mu; \mu'_{\neq \delta}$ $x < x^{NBS}$
 $x^{NBS} < x \leq x^L$ $\mu; \mu'_{\neq \delta}$ $x < x^{NBS}$ $\mu; \mu'_{\neq \delta}$ $x < x^{NBS}$

A complex diagram showing a game tree with nodes and branches, representing a signaling game or ultimatum game. The tree starts with a node where a player chooses between two actions, leading to different information sets for the other player. The game concludes with a payoff vector (x, y) .

Lemma 11 *Let surplus be divided by the ultimatum game. The component with the subgame perfect outcome, $(I^H; V^H - x^{\max}, I^H)$, is a subset of the unique locally stable set.*

$\mu \in T \mu^H$ $\mu^H \in T \mu^H$ $\{I^H; V^H - x^{\max}, I^H\}$ $\mu^H \in T \mu^H$ $\{I^H; V^H - x^{\max}, I^H\}$
 $\mu \in T \mu^H$ $\mu^H \in T \mu^H$ $\{I^H; V^H - x^{\max}, I^H\}$ $\mu^H \in T \mu^H$ $\{I^H; V^H - x^{\max}, I^H\}$

Lemma 12 *Let surplus be divided by the ultimatum game. Agents in population A receive a payoff of at least $V^H - I^H - x^{\max}$ in every equilibrium.*

$\mu \in T \mu^H$ $\mu^H \in T \mu^H$ $\{I^H; V^H - x^{\max}, I^H\}$ $\mu^H \in T \mu^H$ $\{I^H; V^H - x^{\max}, I^H\}$

Lemma 13 *Let surplus be divided by the 'ultimatum' game. If $V - I - x \geq V^H - I^H - x^{\max}$, then there exists an equilibrium μ such that $\mu \in \Theta^L$ and $\mu \in T \mu^H$.*

r f m m f r m m l p □
 b f r f m g b m A b g r b p
 b g p b b l a l r m b p b r r g l r p b b b
 m r r p b l m m r m p b r b p
 g l r p r r p b l r b f b r l m m
 r f f r p r m m l l p l k b b b
 p g l ll l b b s m l p l b p m g l b
 b ll l □

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