DISCUSSION PAPERS IN ECONOMICS

Working Paper No. 00-01

Does Evolution Solve the Hold-up Problem?

Tore Ellingsen *Department of Economics, Stockholm School of Economics Stockholm, Sweden*

Jack Robles *Department of Economics, University of Colorado at Boulder Boulder, Colorado*

January 2000

Center for Economic Analysis Department of Economics

University of Colorado at Boulder Boulder, Colorado 80309

© 2000 Tore Ellingsen, Jack Robles

Does Evolution Solve the Hold{up Problem?

1 Introduction

 $p \cdot p \cdot p \cdot q \cdot 1 \cdot p b$ –specific $p \cdot q \cdot p \cdot 1 \cdot p$ in $q \cdot p \cdot q \cdot p$

 $\exists \mathbf{r} \langle \mathbf{r} - p \mathbf{p} \rangle = \mathbf{r} - \mathbf{r} \mathbf{e}^r$ g pregative ga \mathbf{r} in solution $\mathcal{P} = \mathcal{P} + \mathcal{P} = \mathbb{I}$ in backward induction. Thus, if the bargaining induction $\mathcal{P} = \mathcal{P} + \mathcal{P}$ \mathfrak{so} by $\mathfrak{g}_{\mathfrak{m}}$ and $\mathfrak{g}_{\mathfrak{m}}$ becomes backward induction, there is no $\mathfrak{g}_{\mathfrak{m}}$ backward induction, the $\mathfrak{g}_{\mathfrak{m}}$ was that sunce that sunk contributions contributed the \mathfrak{p} is the natural measurement of \mathfrak{p} $\mathbf{s'}$ point $\mathbf{s'}$ of our papers, which as $\mathbf{s'}$ \bullet ψ \rightarrow ϵ if the bargain is multiple subgame perfect equilibria? \bullet $\frac{1}{\sqrt{k}}$ \leftarrow \mathbb{P} if \mathbb{P} if \mathbb{P} if \mathbb{P} if \mathbb{P} if \mathbb{P}

 $\{1,9,9\}$ and $\{1,9,1,9\}$ (1993), here and $\{1,9,9\}$ (1999), $\{1,9,9\}$ may not always favor subgame perfect equilibria. Evidence from bargaining experiments has also cast of subset of subset \mathbf{r} of subset \mathbf{r}' of subset \mathbf{r}' \mathbb{P} and \mathbb{P} and with P_{α} and α via α is the perfect perfect p which investor can only a proposal matrix \mathbf{r}_i or \mathbf{r}_i the trading by the trading by the trading by the trading \mathbf{r}_i partner. For partners in subgame perfection, the starkest \mathbb{P}^1 possible example of a hold–up problem: The investor should accept any offer, p mathemather bor p g or p bor $\frac{1}{2}$. The trading partner should be trading partners in $\frac{11}{2}$ the whole surplus, and consequently the investor should make $\mathbb{P}^{\mathcal{P}}$ P investment. While this outcome is stochastically stable, so is almost any other outcome, including the efficient investment level.

 A are two specific examples of \mathbb{R}^n are the results of \mathbb{R}^n and \mathbb{R}^n are resul suggest that a general principle might be at work: When there is no ten- \mathcal{S}_n between \mathcal{S}_n and \mathcal{S}_n and \mathcal{S}_n by \mathcal{S}_n in the only only only only \mathcal{S}_n f picks picks some efficient outcome, but also selects a unique such outcomes a unique such outcome. On the p Δ other hend, when p is perfectively and subgame perfection and Δ tic stability has little cutting power. If this is indeed a general feature, the \mathbf{f} rganing games we study represent opposite extremes, and evolutionary represent opposite extremes, and evolutionary \mathbf{f} analysis of the hold–up problem using other non–cooperative bargaining games

 $\begin{array}{ccccc}\np & p & p & p & p\n\end{array}$
 $\begin{array}{ccc}\np & p & p & p\n\end{array}$ \mathbb{R}^1 or planelysis is even a rationary, the results can be given as results can be given a rationary of \mathbb{R}^1 \mathcal{P} interpretation \mathcal{P} induction \mathcal{P} induction \mathcal{P} induction. The reason when \mathcal{P} $p \in \mathcal{C}$ is a share of the surplus that is sufficient to cover surplus tha costs is that the trading partner believes that it will not pay to be more greedy. $A_{\mathbf{L}} \mathbf{a}^{\mathbf{r}}$ and $\mathbf{b} \mathbf{a}^{\mathbf{r}}$ are only the investment was under-the investment was undertaken. Either the investor expected coordination on a favorable equilibrium $\mathbf{a}^* \mathbf{b}$ and $\mathbf{b}^* \mathbf{c}^*$ and $\mathbf{b}^* \mathbf{c}^*$ argument satisfy the trading sample satisfy $\mathbf{b}^* \mathbf{c}^*$. The trading sample sample sample satisfy $\mathbf{b}^* \mathbf{c}^*$, $\mathbf{b}^* \mathbf{c}^*$, $\mathbf{b}^* \$ \mathbf{p} and \mathbf{p} showled action showled action with \mathbf{p} and \mathbf{p} action \mathbf{p} $\frac{1}{2}$ $b = c - 1$ leven of plent $p = c - p$ and $p = p$

2 Investment and Bargaining

 $\mathbf{F} \bullet \mathbf{r} = \mathbf{F} \bullet \mathbf{A} \bullet \mathbf{B} \bullet \mathbf{I} = \mathbf{F} \bullet \mathbf{B} \bullet \mathbf{I}$ \Box investment in investment I for $P = \Psi_{i} = \{ \begin{array}{c} \Box \Box \end{array} \{ 0; I_1; ...; I_{N} \underline{} \end{array} \}$ \mathbf{F} investment corresponding \mathbf{F} in \mathbf{F} $\left\Vert \left\Vert \left\langle \mathbf{r}\right\rangle \right\Vert _{M}=\left\Vert \left\langle \mathbf{r}\right\r$ is \mathbb{I} er; it contains \mathbb{I} contains $D_{A_{\{i\}}} = \{V | I_{\{i\}} - X; V | I_{\{i\}}\}$, where the the \mathbf{r} is equivalent to accepting the second of \mathbf{r} and the second element is \mathbf{r} and the second element is \mathbf{r} and the second element is \mathbf{r} and \mathbf{r} and \mathbf{r} and \mathbf{r} and \mathbf{r} and e_{α} in r is not of demonstrating it. However, the smaller set of demands for α is not a is not α is not a is not α is not a i an important difference between the two games. It would not matter much if \mathbb{I} if A to choose any demand in \mathbb{P} to choose any demand in \mathbb{P} $\mathbb{P}\left(\mathbb{P}\right)$ bargaining is the two bargaining $\mathbb{P}\left(\mathbb{P}\right)$ is that $\mathbb{P}\left(\mathbb{P}\right)$ \mathbb{F}_p is parameter at player \mathbb{F}_p of \mathbb{F}_p and \mathbb{F}_p is a pure strategy for the \mathbb{F}_p ${\mathfrak p}$, a $I: {\mathcal Y} \times_{\mathcal V} \backslash A$ and ${\mathfrak p} \subset {\mathfrak p}$ $P \subsetneq P \subsetneq P$ \mathcal{L} to the evolution of the evolution \mathcal{L} and \mathcal{L} p erfect \mathbf{a} is \mathbf{a} , when the investment decision is followed by the \mathbf{b} $\rho \circ \mathfrak{so}$ is a variety of the induced is a variety of \mathfrak{so} is a variety of \mathfrak{so} . In particular, \mathfrak{so} $\text{tr} \left[\begin{array}{ccc} \mathbf{r} & \mathbf{r} & \mathbf{r} \end{array} \right] = \mathbf{r} \math$ $\int p \cdot p!$ A $\int b \cdot r \cdot g \cdot l = l^* l$

 S game \mathbb{R} perfection and \mathbb{R} and \mathbb{R} in the Nash demand \mathbb{R} but only admits low investment in the ultimatum game. As shown below, the \mathbb{R}^n is radically different when \mathbb{R}^n and \mathbb{R}^n apply the criterion of evolution of \mathbb{R}^n stability instead of subgame perfection.

3 Evolution

 $\mathcal{F} \subset \mathbb{P}$ and \mathcal{F} and $\mathcal{$ \mathbf{b} by strategies are rational. Instead, it to answer the more equals to answer the more equals to answer the more equals of \mathbf{r}_i and \mathbf{r}_i and \mathbf{r}_i and \mathbf{r}_i and \mathbf{r}_i and \mathbf{r}_i and $\mathbf{r}_$ $\mathbf{y} = \mathbb{P}\left(\begin{array}{ccc} \mathbf{b} & \mathbf{b} & \mathbf{c}^T & \mathbf{g} & \mathbb{P} \end{array}\right)$. The survive computation of \mathbf{c}^T T bestudy of stochastic evolution in games was pioneered by T , T $(1-\gamma)^{-p}$ and $(1-\beta)^{-p}$ and $(1-\gamma)^{-p}$ and $(1-\gamma)^{-p}$ and $(1-\gamma)^{-p}$ to extensive form $\begin{array}{ccc} 1 & 0 \end{array}$ due $\begin{array}{ccc} 1 & 0 \end{array}$ and $\begin{array}{ccc} 1 & 0 \end{array}$ $\mathbf{t} \bullet \mathbf{r} \math$ $\bullet^r \bullet l \bullet^r l \bullet p \circ B l \bullet^r \bullet population \cdot N \bullet b$ \mathbf{p}^r $t \in \{j; j:::r\}$ \mathbf{p}^r of agents in population of a generalized \mathbf{p} and \mathbf{p} B meet and play the investment cum bargaining cum bargaining B meet \mathcal{S} is the set of strategies is in the set of strategies is a contribution of strategies is a contribution of B is a contribution of B is a \mathbb{R}^n and \mathbb{R}^n are also hold beliefs about the function of \mathbb{R}^n $p \equiv 1$ $p \cdot r \cdot r \cdot b$ by $p \cdot r \cdot r \cdot d \cdot r \cdot b \cdot r \cdot p \cdot p \cdot g \cdot p$ $\mathbf{b} \bullet$ \mathbf{r} denote \mathbf{r} is \mathbf{r} and \mathbf{r} at \mathbf{r} and $p \leftarrow 1$ $\frac{y}{4} \left| I_{1} \right\rangle$ $p \leftarrow 1$ or $B \leftarrow 1$ denote player about player A p \mathscr{U} are probability distributions on the set of possible demonstrations on the set of possible dem a^r ppgp passion the investment a^r pg is divided according to the invest $\lfloor\, \mathbb{P}\rfloor$ $\lfloor\, \mathbb{P}\rfloor$

Assumption 1 (i) The pie division is small: V I_{λ} > -. (ii) The population is large: V I^*

 $\mathbb{P}\left[\begin{array}{ccc} \mathbb{P} & \mathbb{P} & \mathbb{P} & \mathbb{P} & \mathbb{P} \ \mathbb{P} & \mathbb{P} & \mathbb{P} & \mathbb{P} \end{array} \right]$

the two games space.) With each state space ϕ is an associated with μ there is an associated with μ there is an associated with μ the μ $\bullet^r = \frac{1}{6}$ of $\bullet^r = \frac{1}{6}$ of $\bullet^r = \frac{1}{2}$. $B \subset \mathbb{R}^n$ in the strategies evolve in the strategies of \mathbb{R}^n adaptation to the strategies of the top \mathbb{R}^n current environment environmentation. And $\mathbf{p} = \mathbf{r} \cdot \mathbf{p} + \epsilon \cdot \mathbf{p}$ long \mathbb{R} in i.i.d. chance of \mathbb{R} i.i.d. chance of \mathbb{R} his beliefs and strategy. This is called an updating draw. An updating agent observes the serves of the \mathbf{r} parameter \mathbf{r} based on the \mathbf{r} $\mathbb{P} \mathbb{P} \stackrel{\circ}{\longrightarrow} \mathbb{P} \mathbb{P} \stackrel{\circ}{\longrightarrow} \mathbb{P} \mathbb{P} \longrightarrow \mathbb{P} \mathbb{P} \mathbb{P} \stackrel{\circ}{\longrightarrow} \mathbb{P} \mathbb{P} \stackrel{\circ}{\longrightarrow} \mathbb{P} \longrightarrow \mathbb{P} \stackrel{\circ}{\longrightarrow} \mathbb{P} \longrightarrow \mathbb{P$ \mathcal{A} and \mathcal{A} about \mathcal{A} about \mathcal{A} about \mathcal{A} rather than absorbing sets \mathcal{A} \mathbf{f}^r prophenomenon on \mathbf{f}^r be the set of \mathbf{f}^r of \mathbf{f}^r $\mathcal{L}(\mathbf{a}^{\mathrm{T}}, \mathbf{g}^{\mathrm{T}}, \mathbf{p} \mathbf{g}^{\mathrm{T}}, \mathbf{p}^{\mathrm{T}}, \mathbf{p}^{\mathrm{T}})$ results, we need to be more precisely precisely assumed to be more precisely assumed to be more precisely assumed to be more precisely assumed to be m \mathbf{a}^r one defined by definition of attraction of \mathbf{a} and \mathbf{a} of \mathbf{a} μ p. B μ), is the set of the set of states μ is the population can get ρ μ to μ with p is \mathbb{R}^n in the single mutation. Similarly, we say that μ is in the single mutation neighborhood $\langle \mu-p-\mu'\in M\ \mu\setminus \langle \mu'\ \ {\scriptstyle{{\cal P}}\ \ \mu}\qquad {\bf r}_c\qquad {\scriptstyle{{\cal P}}\ {\scriptstyle{{\cal Q}}\ \ {\scriptstyle{{\cal P}}\ \ {\scriptstyle{{\cal P}}}}\qquad {\scriptstyle{{\cal P}}\ \ \ {\scriptstyle{{\cal P}}\ \ {\scriptstyle{{\cal P}}}}\qquad {\scriptstyle{{\cal P}}\ \ {\scriptstyle{{\cal P}}\ \ {\scriptstyle{{\cal P}}}}\qquad {\scriptstyle{{\cal P}}\ \ {\scriptstyle{{\cal P}}\ \ {\scriptstyle{{\cal P}}}}\qquad {\scriptstyle{{\cal P}}\ \ {\scriptstyle{{\cal P}}\ \ {\scriptstyle{{$ \mathbb{P} \mathbb{P} if \mathbb{Q} and \mathbb{P} if X is mutation connected set if if \mathbb{P} if \mathbb{Q} $\mu_1 \mu_n \subset X$, there exists some ordering of the remaining of the remaining $\mu_1 \mu_{n-1}$, μ_2 \mathbf{b} \mathbf{b} \mathbf{c} \mathbf{d} \mathbf{r} \mathbf{d} \mathbf{g} \mathbf{g} do better than a land agents in population and all other agents in population and all definitions in population population A update the their strategy will also strategy \mathbb{P}^1 and \mathbb{P}^2 are \mathbb{P}^1 in \mathbb{P}^2 and \mathbb{P}^1 While all locally stable outcomes have investment I∗, there is some scope f' variation is division of the equilibrium division of surplus. We show that the largest \mathbb{R}^n \mathbb{C} agent B which is consistent with its constant \mathbb{C} of \mathbb{C} and \mathbb{C} stability is consistent with local stability is \mathbb{C}

$$
x^{L} \underset{I}{\longrightarrow} \mathbb{P} \quad \{ x \in D_B \mid I^* \backslash \} \quad V \quad I^* \underset{I}{\longrightarrow} -X \underset{N}{\backslash} \frac{N - I}{N} - I^* \geq V \quad I \underset{I}{\backslash} - I - -\}.
$$

 \mathbb{P} of \mathbb{P} and $\lim_{n\to\infty} \mathbb{P}$ of χ^L of \mathbb{P} and \mathbb{P} observe that the largest demand that the largest demand that the largest demand \mathbb{P} agent B could make \mathbb{R} and \mathbb{R} and \mathbb{R} and \mathbb{R} and \mathbb{R} and \mathbb{R} would not give \mathbb{R} and \mathbb{R} agent and \mathbb{F}_q by choose a less efficient in \mathbb{F}_q $g = \lim_{N \to \infty} \frac{N}{N} \lim_{N \to \infty} \frac{1}{N}$

$$
x^{M} \underset{\mathbf{a}^{P}}{\mathbb{P}} \{X \in D_{B} \mid l^{*} \setminus l^{*} \setminus -X-l^{*} > V \mid l_{\setminus} - - -l \}.
$$
\n
$$
\underset{\mathbf{a}^{P}}{\mathbb{P}} \underset{\mathbf{b} \in \{1, \ldots, P\}}{\mathbb{P}} \underset{\mathbf{a}^{P}}{\mathbb{P}} \underset{\mathbf{a}^{P}}{\mathbb{P}} \underset{\mathbf{a}^{P}}{\mathbb{P}} \mathbb{P} \qquad \mathbf{b} \qquad \mathbf{b} \qquad \mathbf{a}^{M} - \leq x^{L} \leq x^{M} \leq x^{M}.
$$
\n
$$
\overset{\mathbf{a}^{P}}{\mathbb{P}} \underset{\mathbf{a}^{P} \in \{1, \ldots, P\}}{\mathbb{P}} \mathbb{P} \qquad \mathbf{b} \qquad \mathbf{b} \qquad \mathbf{a}^{M} - \leq x^{L} \leq x^{M}.
$$

$$
V^* - x^M - \sqrt{N} - \sqrt{N} - I^* = V I^* \sqrt{N} - X^M - I^* - V I^* \sqrt{N} - \sqrt{N}
$$

 \mathbb{R}^n is due to \mathbb{R}^n is due to Assumption 1. Intuitively, the \mathbb{R}^n $\mathbf{f}^T \mathbf{p}^T \mathbf{$ will not cause agents in population A to change their investment away from \mathbf{b} effect level.

Proposition 3 Let agents bargain according to the Nash demand game. The outcome ‰ is locally stable if and only if ‰ $\frac{1}{x}$ { I^* ; V I^* ₁ \rightarrow x; X ₁}, where $x \leq x^L$.

Note how, in this case, local stability identifies a much smaller set of outcomes than did subgame perfection. In particular, subgame perfection allowed inefficient investment, whereas local stability does not. Let us now refine the set of locally stable outcomes and consider the smaller set of stochastically stable outcomes. Although stochastic stability is easy enough to define, the computation of stochastically stable equilibria is a bit more demanding. It basically requires counting the number of mutations needed to move from one equilibrium to another. The equilibria which are most easily reached from all other equilibria (in terms of requiring fewest mutations) are stochastically stable. To articulate this idea precisely, we need

a couple of additional definitions. Recall that Θ denotes the set of equilibria. $\Gamma \not\!\! \perp \mu \not\!\! \perp \psi$ be the minimum number of ζ in the move from and ζ in the move from an ζ e_{α} of \tilde{p} μ and \tilde{p} and \tilde{p} and \tilde{p} and \tilde{p} as the graph \tilde{p} and \tilde{p} and of vertices, one vertex for each equilibrium, with a directed edge from every \bullet^r of to every other to resistance \bullet^r cost \setminus of the edge \downarrow is $\mu \rightarrow \mu'$ is μ ; μ' , \setminus μ {tree, Γ, is a collection of every vertex in μ , μ $\phi \bullet$ p_{α} q_{α} and p_{α} , and p_{α} are are no cycles. The resistance of p_{α} \langle of Γ is possible resistances of all the edges in the edge in the tree. the stochastic potential $\langle p, p \in \mathbb{R}^n \mid p \in \mathbb{R}^n$ is the minimum resistance over all μ -trees. The key to checking when μ to checking when μ is stochastically stable is stochastically stable in p and p q and p and p and p .

Theorem 1 An equilibrium μ is stochastically stable if and only if no other equilibrium has lower stochastic potential.

 $P(\begin{array}{cccc} b & p \end{array})$ is probability to continuous to continuous the proof of Proposition 4) that it sufficients to con- $\mathbf{s}^r = \mathbf{s}^r - \mathbf{b} - \mathbf{s}^r$ that are much simpler than the those description, $\mathbf{s}^r = \mathbf{b} - \mathbf{s}^r$ and the those definition, $\mathbf{s}^r = \mathbf{s}^r - \mathbf{s}^r = \mathbf{s}^r - \mathbf{s}^r = \mathbf{s}^r - \mathbf{s}^r = \mathbf{s}^r - \mathbf{s}^r = \mathbf{s}^r - \mathbf{s}^$ p and p is a regular regu \mathbb{P} of \mathbb{P} gauge trees, one can ignore the canonical trees, one can ignore experiment with \mathbb{P} Notice also that from any equilibrium one may arrive at an equilibrium within a locally stable set through a sequence of one mutation transitions, and one may move around within the locally stable set in the same manner. Hence, it \mathbb{R} suffices to construct the construction of \mathbb{R} of \mathbb{R} or \mathbb{R} stable sets (represented by locally stable stability stable stable s \mathbb{P} as vertices. $\mathcal{S}_\mathcal{D}$ all locally stable outcomes under Nash demand bargaining $\mathcal{S}_\mathcal{D}$ property that I = I[∗] and that y = V I∗) − x, these outcomes can be fully $\mathbf{b} \mathbf{c}$ depends by \mathbf{c} demand, \mathbf{c} are two ways in which are two ways in which are two ways in which a transition of \mathbf{c} \mathbb{P}^1 is a direct outcomes may occur. First, there may occur transition, during which investment is maintained at the efficient level. In \mathbf{b} and \mathbf{p} and $V \, I^*_{\lambda \Gamma}$; then the easy of Γ is the outcome Γ is to the outcome Γ is to the outcome Γ $x > X^{NBS}$; the easiest transition is to such as $x - -: X \rightarrow Y$ and $x - \rightarrow Y$ \mathbf{f} in the angle of \mathbf{f} and $\mathbb{E}[\mathbb{$ having a sufficiently large portion of population B increase their demands to be such an extent that effects investigated investment investment investment in \mathbb{P}^1 A \rightarrow D \rightarrow D outcome). If the interest appendix and dominate dependent of the \mathbb{P}^1 $\mathbf{b} = \mathbb{I}_q$ polymeric and \mathbf{r} population to the top to the top to this ineffect outcome, and after which after whic $s_{\mathbf{a}}$, and the resistance of \mathbf{a} in the resistance mutations (the resistances of which we can ignore). $\int_{a}^{a} g \phi \phi = \int_{a}^{a} p f' / V / \int_{a}^{a} f^{-} f(x) \phi(x) dx$ σ' the transition from the σ' ; V Γ' _i − χ ; χ _i γ' Γ ; γ' Γ Γ γ Γ γ Γ γ

we can show that ˆr x) > r x) whenever x>x^L (see Appendix), it is easy to construct a minimum resistance tree. The case x^M > xNBS essentially reduces to the analysis of the Nash demand game in Young (1993b). Otherwise, if r x^L) ≥ r x^L − –); the minimum resistance tree is given by – −→ – −→ ::: −→ x^L − – −→ x^L; while if ˆr x^L) ≥ r x^L − –), the minimum resistance tree is given by x^L −→ – −→ – −→ ::: −→ x^L − –:

 p investment in the must be investment in any stable p in any stable equilibrium. S uppose now that bargaining is conducted according to the rules of the rules \Box program pinstead. I^H be such that $V \, I^H$ _{$\searrow - I^H$}

 p and α is the the the theorem tends to α $\mathbf{b} \cdot \mathbf{p} = \mathbf{p}$ particles $\mathbf{p} = \mathbf{p}$ and $\mathbf{p} = \mathbf{p}$ and $\mathbf{p} = \mathbf{p}$ p in possibility from renew p in \mathbb{R}^n is split with p $\mathbb{P} \hspace{2mm} P$ is the current paper suggests that the current paper suggests that the current paper suggests that the current paper support of \mathbb{P} σ P $\frac{1}{2}$

Appendix: Proofs

Lemma 1 Let $z_1 < z_2: := < z$ be demands in D I \backslash for some I $\in \Psi$. Assume that the set of demands following I for agents in the relevant population is ${z_l}_l$

 p population is made, in the first population in the first population update, the first population p l benev l_{λ} – q not maps p is not made, ρ denote the set to demand and can not real not real notation dissapears and can not real can not real can not real ρ which contradicts the assumption that ω is a ω is ω is ω is ω is an absorbing ω can is a best response and \mathcal{F} is a best \mathcal{F} is a best (behavioral) response and \mathcal{F} is a best (behavioral) response and \mathcal{F} is a best (behavioral) response and \mathcal{F} $f\colon \mathbb{R}^n$ increase the played must be the played must be the played must be increased must \Box bordor by \Box population is \Box in equilibrium and \Box is a singleton. \Box $\sum_{\text{VUV}} \bullet^{\text{r}} \bullet^{\text{r}}$ $P \left(\left\{ \left\{ \begin{array}{ccc} \mathbf{r} & \mathbf{p} & \mathbf{p$

not change off–path beliefs, ‰

 B is an increase to change \mathbb{R} in the \mathbb{R} revealed to \mathbb{R} in the \mathbb{R} revealed revealed in \mathbb{R} in the \mathbb{R} revealed in \mathbb{R} in \mathbb{R} in \mathbb{R} in \mathbb{R} in \mathbb{R} in \mathbb{R} in $(\mathbf{b} \mathbf{v} \quad \mathbf{v} \quad \mathbf{r} \quad \mathbf{g} \mathbf{p} \quad \text{1} \quad \mathbf{p} \mathbf{g} \quad \text{1}, \text{y})$, and $\mathbf{p} \quad \mathbf{p} \quad \mathbf{g} \mathbf{p} \quad \mathbf{p} \quad \text{1} \quad \mathbf{p} \mathbf{A}$ hap an incentive to change to change to change to change to change in the strategy of the stra \mathbb{R}^p player agent playing \mathbb{R}^p repeating the process we are at the this process we are th e is matrix μ in the Lemma \Box

 ${\rm Lemma~4}$ Let μ^{\prime} (‰ $\mu^{\prime}_{\le \lambda} = \{$ $I^{\prime};$ ${\sf y}^{\prime};$ ${\sf x}^{\prime},$ $\},$ be an equilibrium. If I ${\not\perp}$ I' and y — I ≥ y — I', then the population can get from µ' to an equilibrium µ with ‰ μ_{λ} { I ; y; x, } through a sequence of single mutation transitions.

Proof: In state µ⁰ let agents in population B drift to believe with certainty that any agent in population A that invests I will demand y. This implies that for \exists in \exists A mutate to I , I y β is beliefs. If β is all period let I is all agents in A update, they are the $\lim_{n\to\infty} x$ in $\lim_{n\to\infty} x$ in $\lim_{n\to\infty} x$ in $\lim_{n\to\infty} x$ $I_y - I > y - I'$ best provided if Y ₎ to which the $I = b$ $l=$ P.g.b. p p $_{\mathbf{a}}$ \perp or p μ = b ‰ μ $_{\mathbf{b}}$ = { I; x; y $_{\mathbf{b}}$ c' y−I = y−I' b p \Box agents in population A are player at a best response and we are at a new areas and we are at a new areas \Box $e_{\mathbf{y}}$ of p μ_1 which $\mu_{\mathbf{1}}$ is μ_1 in μ_1 in μ_1 is μ_1 in μ_1 in μ_1 is μ_1 in μ_1 in μ_1 is μ_1 in μ_1 is μ_1 in μ_1 is μ_1 in μ_1 is μ_1 is μ_1 is μ_1

 $\begin{array}{ccccccc}\n\mathfrak{g} & \mathfrak{g} & \mathfr$ \mathbb{P}^r (i) \mathbb{P}^r of the population \mathbb{P}^r and \mathbb{P}^r and \mathbb{P}^r the population can get \mathbb{P}^r f ron and f outcome not satisfying the \mathbb{P}_q satisfying the P acterization to an equilibrium with does satisfy it does satisfy it. The contract of \mathcal{A}^r is does satisfy it, the contract of \mathcal{A}^r is does satisfy it. a pequence of single mutation that the population transition \mathbb{R}^n p order p and $\log b$ deform an outcome satisfying the characterization p two simultaneous mutations. Step μ we may consider μ $s \mapsto \mathcal{L} \circ \mathcal{L} \circ$ V I^*_{λ}) − – $I^* > V$ I_{λ} – $X - I$; so population λ λ λ λ λ μ μ get P_{α} is P_{α} if μ^L by \mathcal{W}_{α} μ^L \rightarrow $\{\ l^*, l^* - -; \cdot, \cdot \}$. If μ^* μ^* \rightarrow $x > x^L$ μ V I) – – I \geq V I \rightarrow – X – I, then population can get the population can get the population can get the population can get the population can get p e_{α} is proprieted with μ with μ and μ if μ is μ and μ a then gets the population to an equilibrium µ^L with ‰ µ^L) = { I∗; V [∗] − –; –)} $\mathbb{P} \parallel \mathbb{P} \text{ for } \Phi \rightarrow I_{\perp}$ I^* $x > x^L$ p V I_{λ} - - - 1 < V I_{λ} - $x - 1$

-[(st3 1 Tn)411

to get it ng pp pp and investment of I, which will lead them to play them to play them to play them to play the $\mathbb{P}_\mathbf{q} \setminus \mathbf{p}$ is a above, and \mathbf{p} in application of Lemma 4 completes the Lemma 4 completes the Lemma 4 completes the Lemma 4 complete the Lemma 4 complete the Lemma 4 complete the Lemma 4 complete the Le \bullet^r ($\mathcal{S}=\cup_{i,j}\in\mathbb{P}$, $\mathbb{P}=\mathbb{P}$, $\mu\in\mathbb{P}$ so $\mu_{i,j}=\{f^{*};y;x_{i}\}\in\mathbb{P}$, and $x\leq x^{L}$, we must \mathbb{R} by $\mathbb{R}^n \times \mathbb{R}$ with \mathbb{R}^n with \mathbb{R}^n with \mathbb{R}^n $\frac{1}{2}$ $\frac{1}{2}$ V I∗) − x^L)(N − 1)=N − I[∗] ≤ V I∗) − x)(N − 1)=N − I∗. Hence, agents $P = \frac{1}{\rho} \sum_i A_i \mu_i$ $\mathcal{I}^{\mathcal{I}}_{\mathcal{I}}\prod_{i=1}^{n} \sum_{j=1}^{n} \prod_{j=1}^{n} \prod_{j=1}^{n}$ x)

of mutations for a transition from a locally stable outcome to an outcome with inefficient investment (for comparison). Now, clearly, an agent in population A will only change his investment if he thinks that he is going to get something better, and the best outcome that he could expect with an inefficient investment is (I; V − –; –): Hence, the question becomes: how many agents in population B have to mutatte to a higher demand to make the above outcome better than maintaining efficient investment? Recall that x^L ∈ {x^M; x^M − –}:

Lemma 5 The number of mutations required to get from an equilibrium with outcome I^* ; y; $x\setminus$ with $x \leq x^L$ to an equilibrium with outcome $I: V - \rightarrow \rightarrow V$ $r x_{\bigcup_{i=1}^n}$ p{r|r > N $\frac{r}{n} - \frac{\hat{v} - \delta - \hat{I} + I^*}{V^* - \iota}$ \mathbf{p} in the number \mathbf{p} and \mathbf{p} investments \mathbf{p} and \mathbf{p} mutate to a higher demand of \mathbb{R}^n must be large enough so that \mathbb{R}^n $N - r$ $\frac{1}{N}$ $V^* - X_1 - I^* < V - I - -;$ \mathcal{P} ince updating a generally actions on the notation on the notation on \mathcal{P} are not alleged the not alleged the notations of \mathcal{P} playing a best response. Solving for r yields the desired expression. ✷

Lemma 6

(i) If μ is an equilibrium with outcome I^* ; y ; x_{γ} and $x < \mathbb{P} \{X^M$; $x^{NBS}\}$; the easiest transition away from ‰ µ_\S**Slo\SoSba0Ge**082T=0.9482 2 2 2F7283m)-343(with)-3832e

From Proposition 3, we do not need to worry about x>x^L, so all we need to check is the case where x = xL = x^M. If x^M < xNBS; then we know that r x) ≥ r x), because V [∗] − x^M − – ≤ V − – − I ✷ If x = x^M > xNBS then we can not say any more than that the easiest transition is to an equilibrium with an outcome of either (I; V −–; –) or (I∗; y –; x−–). We already knew this, but since either one of these transitions gets us easily to another locally stable set, and allows an easy construction of a tree around the Nash bargaining solution and efficient investment, it really does not matter. Next, we turn to the analysis of stochastic stability. We first consider the number of mutations required to make a transition directly from an equilibrium with outcome (I∗; y; x), (y = V [∗] − x) to one with outcome (I∗; y⁰ ; x⁰). Along this transition we will not allow the level of investment to change for any agent. Hence results in this section are essentially borrowed from Young's bargaining paper. Later we will worry about multi–step transitions in which one first changes the investment and then changes the demand following the efficient investment.

Lemma 7 From an outcome I^* ; y ; x_1 the easiest transition in which investment is at all times e-cient, but which ends with difierent demands, is to an outcome I^* ; y' ; x' where $x - x - y$; $x - y$ or V^* - -.

Proof: From Young (1993b) Lemma 1. ✷ The idea is that if one population changes their demand, then the increase in demand which will be least hard to get the other population to accept is an increase of –; while the decrease in demand that is easiest to get the other population to accept is a decrease all the way to –

Lemma 8

(i) Moving from x to $x - -$ takes $N(1 - \frac{-\delta}{\lambda})$ mutations to pop A. (ii) Moving from x to x – takes $N_i - \frac{V^* - \delta}{V^* - \delta}$ mutations to pop B. (iii) Moving from x to – takes $N \frac{V^*-\delta}{V^*-\delta}$ mutations to pop B. (iv) Moving from x to V^* – – takes $N \frac{V^* - \delta}{V^* - \delta}$ mutations to pop A.

Proof: This again follows immediately from Young's Lemma 1. All that needs be done is to note that both populations' 'sample size' is just N ✷

Lemma 9

(i) If – < x < V^* – –, then moving from x to x – – takes fewer mutations than moving from x to $-$, and moving from x to $x -$ takes fewer mutations than moving from x to V^* – –.

(ii) If x_{\perp} – then moving from x to

$$
\begin{array}{ccccccccc}\n\text{P} & \text{P} & \text{P} & \text{P} & \text{P} & \text{P} & \text{P} \\
\text{P} & \text{P} & \text{P} & \text{P} & \text{P} & \text{P} & \text{P} \\
\text{P} & \text{P} & \text{P} & \text{P} & \text{P} & \text{P} & \text{P} \\
\text{P} & \text{P} & \text{P} & \text{P} & \text{P} & \text{P} & \text{P} \\
\text{P} & \text{P} & \text{P} & \text{P} & \text{P} & \text{P} & \text{P} \\
\text{P} & \text{P} & \text{P} & \text{P} & \text{P} & \text{P} & \text{P} \\
\text{P} & \text{P} \\
\text{P} & \text{P} \\
\text{P} & \text{P} \\
\text{P} & \text{P} \\
\text{P} & \text{P} \\
\text{P} & \text{P} \\
\text{P} & \text{P} \\
\text{P} & \text{P} \\
\text{P} & \text{P} \\
\text{P} & \text{P} & \text{P} & \text{P} & \text{P} & \text{P} & \text{P} &
$$

Lemma 11 Let surplus be divided by the ultimatum game. The component with the subgame perfect outcome, $I^H: V^H - x^{\max} I^H \rightarrow x^{\max} I^H \rightarrow y$ is a subset of the unique locally stable set.

$$
\begin{array}{ccccccccc}\n\mathbf{a}^{r} & \mathbf{b} & \mathbf{c} & \mathbf{c
$$

Lemma 12 Let surplus be divided by the ultimatum game. Agents in population A receive a payofi of at least $V^H - I^H - x^{\max} I^H$, in every equilibrium.

 $\mathbf{P} \left(\begin{array}{ccc} \mathbf{P} & \$ \mathbb{P} (p gp p A of of p.g.] dipids \mathbb{P} . I dipids than the \mathbb{P} p p p l^H \Box

Lemma 13 Let surplus be divided by the 'ultimatum' game. If V I_N−I − x ≥ $V^H - I^H - x^{\text{max}} I^H$, then there exists an equilibrium μ such that $\mu \in \Theta^L$ and ‰ μ ₎ μ /; V I ₁ \rightarrow x; X ₁

 \textbf{P}^r is provided from \textbf{P}^r and \textbf{P}^r and \textbf{P}^r and \textbf{P}^r and \textbf{P}^r and \textbf{P}^r Note that of course if two outcomes give the same payoff to A, higher than that given by the hold–up equilibrium, then there are equilibria in which both outcomes are present. The above lemma is more a statement about the richness out external into the latter is the previous lemma. \mathfrak{g}^r ((From Proposition 5: From H) \mathfrak{g}^r and \mathfrak{g}^r and \mathfrak{g}^r $p_{\mathbf{q}}$, $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$, which $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, which $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, which $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ \mathbb{R} is \mathbb{R} stable set. \Box

6 References

 $\mathcal{F} = \mathcal{F}$ lutionary Stability in Alternations, \mathbf{S} and \mathbf{S} are $\$ Economic Theory $\mathbf s=\mathbf 1$ Binmore, Ken, Chris Proulx, Larry Samuelson and Joe Swierzbinski $(1-8)$ is a \mathbb{F} and \mathbb{F} and \mathbb{F} or \mathbb{F} and \mathbb{F} or \mathbb{F} exponential \mathbb{F} and \mathbb{F} are Economic Journal 108, $1\quad\downarrow$ 8 $\mathbf{y} = \mathbf{y}$ and $\mathbf{y} = \mathbf{y}$ (1999): Evolution (1999 $\mathbf{F} \cdot \mathbf{F} \cdot \mathbf{S}$ or $\mathbf{F} \cdot \mathbf{S}$. And $\mathbf{F} \cdot \mathbf{S}$ are $\mathbf{F} \cdot \mathbf{S}$ and $\mathbf{F} \cdot \mathbf{S}$ are $\mathbf{F} \cdot \mathbf{S}$. And $\mathbf{F} \cdot \mathbf{S}$ are $\mathbf{F} \cdot \mathbf{S}$ and $\mathbf{F} \cdot \mathbf{S}$ are $\mathbf{F} \cdot \mathbf{S}$ and van Damme, Erik, Reinhard Selten, and Eyal Winter (1990): Alternating $\mathbb{P}[\mathfrak{g}]$ is papel above $\mathbb{P}[\mathfrak{g}]$ of $\mathbb{P}[\mathfrak{g}]$ of $\mathbb{P}[\mathfrak{g}]$. Games and Economic Behavior \rightarrow 88 \rightarrow \overline{F} \overline{F} (1997): The Evolution of \overline{F} and \overline{F} are \overline{F} and \overline{F} \overline Journal of Economics i i \quad \cdot \cdot \cdot \overline{F} , Tore and Magnus Johannesson (2000): Is \overline{F} there are a Hold– \mathbf{u}^* . The problem of \mathbf{u}^* of \mathbf{u}^* of \mathbf{u}^* of \mathbf{u}^* of \mathbf{u}^* of \mathbf{u}^* P in P Fermal and $\begin{array}{ccc} F & \longrightarrow & \mathbb{R}^n \end{array}$ (1991): Striking for a Bargain for a Barga peter Two Completes Informed Agents, American Economic Review \mathbf{S} From and H. Peyton I. Peyton P Dynamics, Theoretical Population Biology 38,219–232. $f = f e$, John, Kenn Binmore, and Larry Samuelson (1995): Learning to \mathcal{F} \mathbb{P}^{\bullet} \mathbb{R}^{\prime} in \mathbb{P}^{\prime} \mathbb{P}^{\bullet} is \mathbb{P}^{\bullet} Games and Economic Behavior 8, 56–90. f_{α} F is an and α J. and β or β or D effects of A \mathbf{r} of \mathbf{r} and $\$ of Political Economy 1 441–1191 $\ell=\ell\cdot\mathcal{S}_\Delta$, P. p.p.p. $p=\max_{\mathbf{v}\in\mathcal{S}_\mathcal{S}}p_\mathbf{v}$ in the P. $p_\mathbf{g}$ $\mathbb{P} \bullet^r$ Econometrica \Box $H_{\text{eff}} = \frac{1}{2} \sum_{i=1}^{3} 1 - \frac{1}{2} \sum_{i=$ \mathbb{R} set of Reputations and Reputations and Repeated \mathbb{R} is \mathbb{R}^n Journal of Law, Economics and Organization 1 , 360

 \mathcal{V} is a Markets and Hierarchies: Analysis and Antitrust Implications, $\mathbf{r} \cdot \mathbf{r} = \mathbf{r}$ γ y is λ by Forting the Evonometrica $\frac{1}{2}$ $\frac{8}{2}$ Young, H. Peyton (1993b): An Evolutionary Model of Bargaining, Journal of Economic Theory \rightarrow 145–168. γ y is \aleph_{1} pppel p and P Review of Economic

Studies 50, 773–792.