Receiver Inattention and Persuading to Be Persuaded

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Abstract

Frictions in a Bayesian persuasion game, such as the receiver's rational inattention,

\The purpose of an elevator pitch is to describe a situation or solution so compelling that the person you're with wants to hear more even after the elevator ride is over."

| *-Seth Godin*[1]

1 Introduction

Bob's decision, based on his belief about the true state, can a ect Alice's payo . By designing the information structure|a rule that maps possible states to di erent signals|Alice can shape Bob's belief, thereby changing his decision to maximize her payo . In the canonical Bayesian persuasion model, Alice can commit to the signal revelation rule, and Bob processes the signal according to the Bayes' rule, making Bayes plausibility the only requirement for an information structure to be feasible.

This frictionless framework, despite its elegance, may oversimplify the real-world complexities. Alice and Bob may meet in an elevator. Since Alice's opportunity to present her idea is constrained to a few minutes during the elevator ride, she may not be able to deliver her best pitch. In a Bayesian persuasion game, frictions like the receiver's inattention may make a Bayes-plausible information structure infeasible. If Alice chooses to pitch as if she were in a frictionless scenario, she may be unable to nish her pitch, or Bob may only process the signal partially (Bloedel and Segal, 2018), making persuasion less e ective. How should a sender end for five months persuading former PepsiCo CEO John Sculley to join Apple. Given the informal nature of these meetings, nearly each attempt was an “elevator pitch,” with only limited attention from Sculley. Since persuasion rarely succeeds on the first attempt, it is important to make the receiver persuadable on the following attempts, just as Seth Godin explains with the elevator pitch. In contrast to most previous relevant studies where the game duration is exogenously determined, we assume that the sender can strategically choose the information structure to extend the persuasion game. In this framework, where the duration of the game is endogenous, we investigate what properties of the frictional constraint will motivate the sender to extend the game for higher effectiveness in persuasion.

The significance of whether sequential persuasion enhances persuasiveness hinges on another key question: how much does a sequential strategy—especially the first attempt—differ from the strategy that would optimize a one-shot persuasion game? If the optimal opening pitch in a sequential approach aligns with the best strategy for concluding persuasion in a single attempt, Alice need not worry about whether sequential persuasion improves her overall effectiveness. She can simply focus on persuading as hard as possible within the current constraint; if she fails, she waits for another opportunity from Bob to try again. However, this may not be the case. To secure a second chance in the case that her first attempt fails, Alice might need to adjust her opening pitch strategically to ensure that Bob's patience is not exhausted by her first failure. In such cases, overlooking the benefits of sequential persuasion could reduce Alice's overall effectiveness—or even cost her a second chance to persuade.

The intuition behind designing an optimal static versus sequential persuasion strategy generally differs, particularly in the sender's first attempt. In a sequential design, the first attempt serves as an opening, whereas in a static strategy, it is the sender's final opportunity. In some frictionless, static persuasion problems, maximizing the chance of success involves making the “bad signal” as bad as possible. As shown by Kamenica and Gentzkow (2011), if a good state does not produce bad signals, the sender can design the bad state to send a signal with maximum probability while the signal remains “good,” which recommends the receiver to “act.” This design maximizes the probability of success within the single attempt. However, if this attempt fails, the bad state is identi ed, leaving no room for further persuasion. This logic holds in a frictional setting where the receiver is rationally inattentive. If Alice aggressively persuades in her rst attempt, aiming for a one-shot success, the failure would lead Bob to believe that any future persuasion attempt is unlikely or impossible to change his mind. Consequently, if Alice anticipates the di culty of immediate success and values the chance to persuade again, she might adopt a more conservative approach, withholding some information in her initial pitch to preserve a backup opportunity.

To keep the receiver's patience and make him open to further persuasion, it is common in practice to keep an ace up the sender's sleeve. For example, a job candidate prepares multiple versions of her pitch. While each version provides a complete analysis of the same topic, the time required to nish the pitch|such as 1-minute, 5-minute, or 10-minute|determines the level of analytical detail. The candidate discloses more details one at a time, until she either convinces the search committee or is interrupted. This \piecemeal" information disclosure is proven to be e ective when the interviewer has limited time and attention. It endogenously egy is more effective than a static approach (Proposition 1). This benchmark shows that an endogenous sequential framework is necessary only when the frictional constraint renders some Bayes-plausible information structures infeasible. Otherwise, a sequential strategy never outperforms an optimal static one. Naturally, the receiver may become disappointed after a failed attempt, reducing the feasible set of information structures for future persuasion efforts (Proposition 2). If the receiver's motivation is sensitive to prior failures, the sender must adopt a conservative persuasion strategy in the initial attempt (Proposition 4); otherwise, she risks losing effectiveness or even the chance to persuade again (Proposition 3). Generalizing this finding, we examine conditions under which the sender benefits from extending the persuasion path beyond two stages (Proposition 5), where she chooses a "piecemeal" information disclosure strategy (Proposition 6). To explore to what extent a sequential strategy can beat frictional constraints due to the receiver's inattention, we allow for a sufficiently large attempt limit. While prolonging the persuasion path can enhance effectiveness, we find that the efficiency boundary remains determined by the frictional constraint (Proposition 7).

The endogenous dynamic nature of this study distinguishes it from related research, particularly those that also examine multiple signals for a single non-stochastic state. Here, the sender fully controls the length of the persuasion path,[2] with the option to make it either purely static or sequential with any duration. While signal realizations play a crucial role in prompting the sender to end the game, in this study's specific problem, only one of the two possible signals in each persuasion attempt triggers termination. Therefore, as long as the game is active, the persuasion process strictly follows the path set by the sender. It is the sender—not the game environment—who determines how the receiver's prior belief evolves beyond the initial attempt. When the prior belief is considered a "state variable" at each period, the sender's decision makes the problem intertemporal. Unlike typical intertemporal problems, here the sender also chooses the optimal stopping point for the persuasion process when the attempt limit is sufficiently large to be non-binding.

[2] It is important to distinguish the length of the persuasion path from the length of the persuasion. The former determines when the sender finishes the persuasion or gives up regardless of the outcome, which is purely the sender's decision. Besides the sender's decision, the latter is also determined by the experiment, where the good signal terminates the subsequent persuasion effort immediately.

dynamic. Among these studies, Che et al.(2022) and Su et al.(2021) are most relevant studies to our research. Both studies examined how the sender designs experiments to induce the ideal prior belief for the subsequent persuasion attempts. Information costs play an important role in Che et al. (2022) but the dynamism of their game is introduced by the uncertain arrival of the e ective signal. Su et al.(2021) discussed how sequential structure in the persuasion game may expand the Sender's constrained signal space but the sequential structure is pre-determined and the constrained is set by determine experiments. Since it is not the sender's choice to induce a subsequent persuasion attempt, the trade-o between immediate success and opportunity for subsequent attempt in persuasion was not fully characterized in both studies.

Rather than assuming a static state space with dynamic signal, some studies assume a resulting in 0 value. Although Alice understands her business idea, she is inexperienced and uncertain about its market viability. In this case, the information is symmetric between Bob and Alice at the beginning. They both believe the chance of success is 0.5. To launch her business, Alice requests a 0.6 (billion dollar) investment from Bob. Because the initial expected value of the business idea is less than the requested funds, Bob will decline the request if no further information is provided.

To change Bob's decision, Alice must introduce her idea in greater detail. Bob then evaluates the information provided and gives honest feedback based on his assessment. Alice can present her business idea in three ways: S, A, and B. Each of these ways is characterized by an information structure as follows, where both high-pro t (h) and low-pro t (l) business models can be recognized as a good (g) or bad (b) idea. Here, denotes the conditional probability of these events.

the idea. In this particular scenario, the information structure S is not feasible. Fortunately, the simpler information structures A and B meet the elevator pitch time constraint. If Alice successfully completes her initial pitch in the elevator with either information structure, she will gain a second opportunity to continue her pitching outside of the elevator if necessary, where the feasible information structures are still A and B.

In a single persuasion attempt, the information structure A outperforms B: it offers a 0.5 probability of generating the g signal, changing Bob's belief to 0.8, which leads to the approval of the investment. In contrast, the information structure B provides only a 0.4 chance of convincing Bob to invest with the signal g.

However, the information structure A loses its advantage when a subsequent persuasion chance is considered. If Alice starts with A and fails, Bob's belief drops to 0.2 upon receiving the signal b. This prior belief is too low for Alice to change Bob's decision with a second persuasion attempt, regardless of whether she uses A or B. Specifically, the signal g from A or B would only raise Bob's belief to 0.5 or 3/7, respectively—both below the 0.6 threshold needed to secure his investment. Thus, if Alice begins with A, her overall chance of success stays at 0.5, as the failure in the first attempt also eliminates any further opportunities to succeed.

By contrast, starting the persuasion with B preserves a second chance for Alice to succeed. If the signal b appears in Alice's first persuasion attempt, Bob's belief only falls to $1/3$. With a subsequent attempt using A, the signal g will appear with a probability of $2/5$, leading Bob's belief to surpass the 0.6 threshold. Overall, Alice will have a $0.4 + 0.6 \times 2/5 = 0.64$ chance of

[redacted]

be transmitted to change his belief. His payo ,$v =$ $[1(! = h) -]$, where

phase, where the sender will make a take-it-or-leave-it offer of the information structure and the receiver will accept it. When the frictional constraints at each period t are binding, the receiver is indifferent among the information structures proposed in different stages. Accepting the proposal as long as it satisfies the constraint becomes his best response. Therefore, we can use the backward induction to solve for the equilibrium.

4 Simplification

$$\min_{p_t \in [0;q_t) \times [\gamma;1]} \frac{p_t^g - q_t}{p_t^g - p_t^b} \tag{3}$$
$$s.t.\ M(p_t;q_t) \geq 0.$$

Lemma 1. *For any given $q_t \in (0;\ )$, if $M(p_t;q_t)$ is second-order differentiable in p_t for all $p_t \in [0;q_t] \times [\ ;1]$, there exists an optimal strategy $p_t \in [0;q_t] \times [p;1]$ for the problem (3).*

Depending on the level of inattention, the receiver may have different levels of motivation to engage in persuasion. This further restricts the feasible set of persuasion strategies beyond the conventional constraint, $p_t \in [0;q_t] \times [\ ;1]$. When the sender faces the constraints of both Bayesian plausibility and the receiver's inattention, Lemma 1 indicates the possibility of characterizing an optimal strategy in the sender's final persuasion attempt, given various prior beliefs. Specifically, when $t = 1$, this theorem predicts an optimal strategy in a static persuasion game. The existence of these optimal persuasion strategies helps characterizing the sender's optimal strategies preceding her final attempt with backward induction.

Lemma 2. *In an endogenous sequential Bayesian persuasion game, when $M(p_t;q_t)$ is quasi-concave in $p_1 \in [0;q_1] \times [q_1;1]$, choosing an objective posterior belief p_t^g within $[q_1;\ )$ for a persuasion attempt is strictly dominated.*

According to Lemma 2, even before the final persuasion attempt, the sender should adopt the experiment that recommends the receiver to either "act" and "not act". Just as in the final attempt, she aims to change the receiver's decision rather than solely changing his belief. Therefore, we can use p_t^θ, $\ = g;b$ to represent the sender's persuasion strategy in both attempts. The sender's problem can be reformulated as choosing the distribution of posterior beliefs in each period of the information transmission phase to minimize the chance that "bad" signal is realized in each period as follows. Based on Lemma 2, the sender's sequential persuasion problem is simplified to the following.

$$\min_{\tau \in \mathbb{Z}^+ \cap [1;T]} \min_{p_t \in [0;q_t) \times [\gamma;1]} \prod_{t=1}^{\tau} \frac{p_t^g - q_t}{p_t^g - p_t^b} \quad s.t.\ M(p_t;q_t) \geq 0; \tag{4}$$

where $p_t^g = q_{t+1}$ for all $t \in \mathbb{Z}^+ \cap [1; T-1]$.

Beyond the properties of $M(\cdot)$ supposed in Lemma 1 and Lemma 2 that establish the sender's simplified problem (3), we also need to make several axiomatic assumptions about the motivation function to make the problem of this research interesting. The failure persuasion comes at a cost. Whenever a persuasion attempt fails, the receiver lose patience and the sender's subsequent persuasion attempts lose effectiveness. We define this nature as disappointment-penalizing. Suppose that p_q^{θ} represents the optimal static information structure given the prior belief q, and that $y(p_t^b; q_t)$ represents the minimum p_t^g that satisfies M_t conditional on the prior belief q_t and the choice of p_t^b. Also denote $(p;q)$ as $\frac{p^g - q}{p^g - p^b}$, the chance of a failure in the a persuasion attempt given that the prior belief is q and the strategy is p. The following specifies two different levels of the receiver's motivation in terms of the disappointment-penalizing feature.

Definition 1. *The receiver's motivation is* ***weakly disappointment-penalizing (WDP)*** *if given that $q' > q''$, $y(p_t^b; q'') \geq y(p_t^b; q')$ for all $p_t^b \in [0; q'']$. If the motivation also satisfies $(p_{q''}; q'') \geq (p_{q'}; q'')$, it is* ***severely disappointment-penalizing (SDP)****.*

With a failing record of persuasion that causes the prior belief to decay, the sender is faced with a smaller successful rate for each information structure that contains a given p_t^b, making the receiver's motivation weakly disappointment-penalizing. On top of this property, the disappointment penalty can be more severe to make the motivation severely disappointment-penalizing. Not only some effective information structures available for previous persuasion attempt are no longer feasible, those information structures remains feasible now become less effective after the sender's failure in persuasion attempt. WDP and SDP are not fully related, but SDP generally implies WDP for relatively small prior belief. Therefore, we consider SDP a stricter punishment due to the receiver's disappointment. Based on this de nition, below gives the axiom that applies to the remaining analysis.

Axiom 1. *The receiver's motivation function satisfies Lemmas 1 and 2. Additionally, it is either WDP or SDP.*

This axiom rules out the counter-intuitive \sweet points" on the prior belief with which people become even more optimistic that future experiment will reveal a good state while they are just shown the evidence that this state is unlikely. With this punishment that re ects the real world more closely, we eliminate the sender's ad-hoc motivation to extend the game, with which she gets extra chance to persuade and a better prior belief if this additional attempt fails.

Kamenica and Gentzkow (2011) established a convenient way to characterize persuasion value of a static strategy, which is de ned as a distance between the value function and the a ne sphere determined by targeted posterior beliefs. The simpli cation allows the method to be applied on the sequential persuasion. According to the sender's simpli ed problem (4), she will make the subsequent attempt only when she fails the previous ones, which causes the receiver's prior belief updated to $q_{t+1} < q_t$. Then, the persuasion value of a sequential strategy is the weighted value of successful previous persuasions and the expected value of subsequent persuasions conditional on the failure of the previous persuasions. According to the conventional Kamenica-Gentzkow (K-G) framework, the preceding persuasion at period t should be represented as a line-segment connecting the line-segment that represents the following persuasion at q_{t+1}, and the value function. In panel (a1) and (b1) of Figure 2, which represents the game with $T = 2$, the line-segments representing the sender's rst and nal persuasion attempts are colored in blue and red, respectively. Their distances to the value functions at q_1 characterize the overall persuasion values of sequential persuasion strategies.

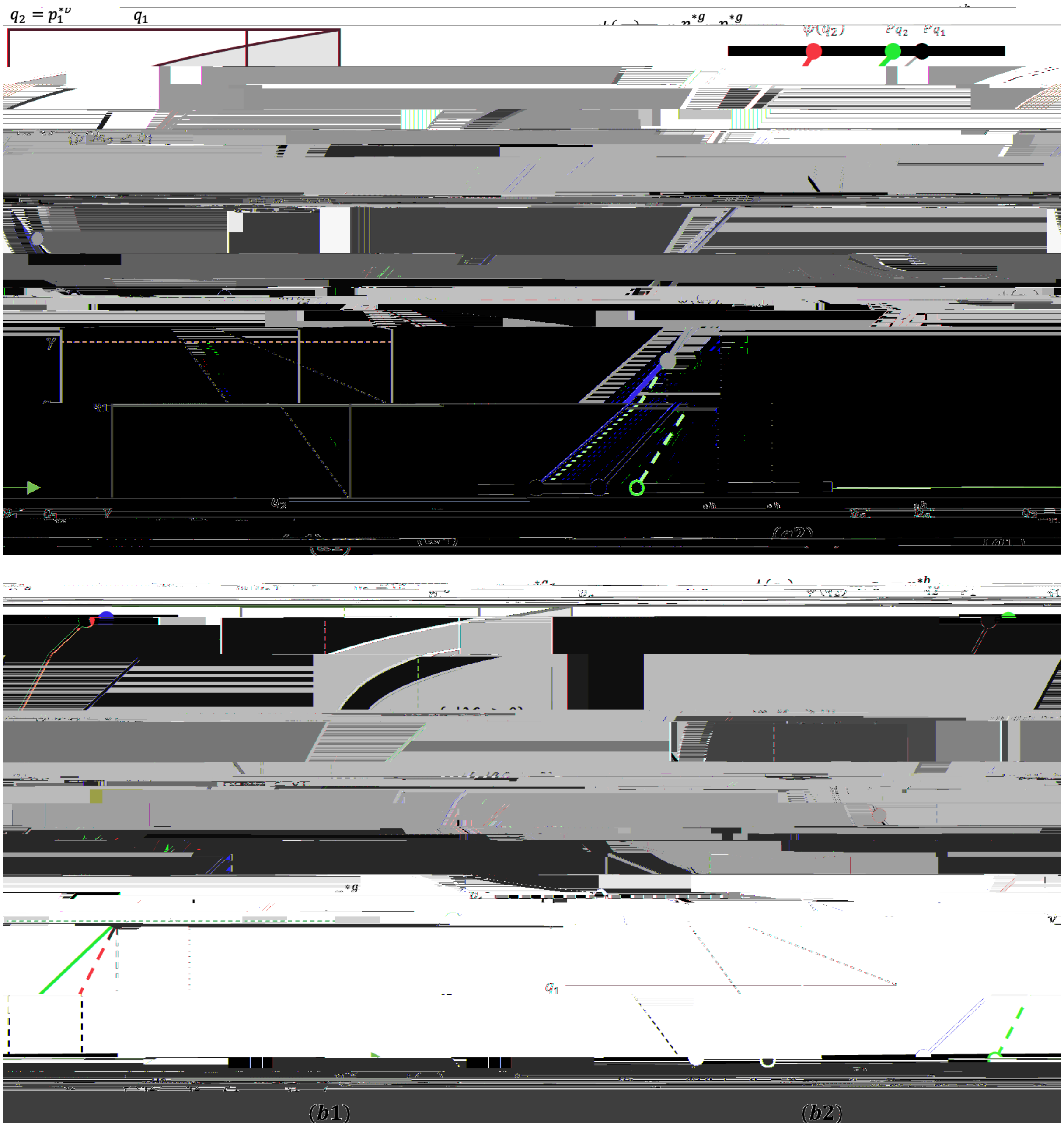

Figure 2: Equivalence Between Strategies with Feasible Sets

5 Endogenous Two-Stage Bayesian Persuasion

The model in this section assumes that $T = 2$. With this simplest setting in our problem, the sender is allowed at most one additional persuasion attempt beyond static persuasion game. She makes a binary choice of whether to remain in a static persuasion game, or extend the game to make it sequential. This binary choice allows us to investigate the underlying determinant that makes a sequential game more favorable than a static game for the sender.

5.1 An Additional Persuasion Attempt

With the receiver's varying motivation that may restrict the set of feasible persuasion strategies, the conventional K-G framework is not sufficient to visualize the sender's problem. To make the analysis more intuitive, we develop an alternative framework that is able to characterize the set of feasible persuasion strategies as well as their persuasion values. In a two-way coordinate system with horizontal and vertical axes respectively representing the receiver's posterior beliefs p given bad and good signals realizes, a feasible set of persuasion strategy at t is $[0; q_t] \times [\ ; 1] \bigcap \{p_t | M(p_t; q_t) \geq 0\}$. These sets are shaded regions in panel (a2) and (b2) of Figure 2. The persuasion value of a feasible strategy is $\frac{q_t - p_t^b}{p_t^g - p_t^b}$ in a static game. On the graph, this value is represented as the slope of the linear indifference curve connecting $p_t = (p_t^b; p_t^g)$ and $(q_t; q_t)$. For a given q_t, a flatter indifference curve is associated with a higher persuasion value.

In this framework, the overall persuasion value of a sequential persuasion is the composite of the persuasion values in different stages. Therefore, different persuasion strategies are not comparable directly regarding their overall persuasion values. In order to overcome this challenge, the comparison needs to come down to be between strategies within one persuasion attempt. This is achievable if it is possible to identify a sequential persuasion strategy that is equivalent to a static strategy or a static strategy that is equivalent to a sequential strategy.

Lemma 3. In a two-stage endogenous persuasion game, suppose that the receiver's motivation function is well-defined to satisfy Lemmas 1 and 2, and the players' common prior belief is q_1 at $t = 1$.

1) for each given q_2, a static persuasion strategy p_{q_1} is equivalent to the sequential persuasion

strategy where the sender chooses $p_1 = \; q_2; \; (q_2)$ and $p_2 = p_{q_2}$, if (q_2) , where

$$(q_2) = \frac{(p_{q_1}; q_1) q_2 \quad (p_{q_2}; q_2) q_1}{(p_{q_1}; q_1) \quad (p_{q_2}; q_2)};$$

2) for each given $q_2 = p_1^b$, the sender's optimal two-stage persuasion strategy $(p_1; p_2)$ is equivalent to a static persuasion strategy $p_{q_1} = \; (q_2); y(q_2; q_1)$, where

$$(q_2) = \frac{q_2 \quad [1 \quad (p_{q_2}; q_2)] y(q_2; q_1)}{(p_{q_2}; q_2)}:$$

Lemma 3 establishes mappings between a (set of) static persuasion strategies and a set of sequential persuasion strategies, indicating their equivalent persuasion values. The rst equivalence mapping identi es the rst-stage strategies, if they exist, that allow two-stage persuasions with arbitrary $q_2 = p_1^b$ to be equivalent to the optimal static persuasion strategy. An example of such rst-stage strategies is represented as a blue line-segment on panel (1a) of Figure 2. To ensure the equivalence between the two-stage and the static persuasions while leading to the designated q_2 that determines the nal-stage persuasion problem, this line-segment should attain the persuasion values of the static strategy and the second-stage strategy at q_1 and q_2, respectively. Accordingly, in panel (a2), (q_2) is determined by the intersection of the indi erence curves representing $1 \; (p_{q_1}; q_1)$ and $1 \; (p_{q_2}; q_2)$. This equivalence converts the optimal static persuasion strategies to a series of benchmark sequential persuasion strategies indexed by q_2. To determine whether making the strategy sequential is more bene cial than performing a static persuasion strategy, comparisons between feasible rst-stage strategies and these benchmarks is su cient.

From a di erent perspective, the second half of Lemma 3 converts optimal sequential persuasion strategies conditional on di erent choices of $p_1^b = q_2$ to equivalent static persuasion strategies. To nd such static strategies, the blue line-segment representing the rst-stage strategy is extended to make its both ends connect to the curve representing the value function on the graph (b1). Accordingly, this static strategy is identi ed on (b2) by projecting the point representing the rst-stage strategy left onto the indi erence curve for $1 - r(p_{q_2}; q_2)$. This equivalence allows for di erent optimal sequential persuasion strategies to be compared as static strategies.

When the receiver is willing to engage in persuasion for a second time as long as his motivation advises him to, it is the sender's decision that determines whether the persuasion is static or sequential. To make an optimal decision in this endogenous sequential persuasion problem, the sender needs to know whether sequential or static persuasion is optimal. The analytical framework and the equivalence mappings established in Lemma 3 make the comparisons between sequential strategies and the optimal static persuasion strategy straightforward.

Proposition 1. *Define a benchmark function as follows*

$$r(q_2) = \begin{cases} \frac{q_1 - \rho(p_{q_1};q_1)q_2}{1-\rho(p_{q_1};q_1)} & \text{if } \{p_2 \mid M(p_2;q_2) \geq 0\} = ; \\ \max\{ (q_2); \} & \text{if otherwise} \end{cases} :$$

Sequential persuasion is more advantageous than static persuasion for the sender if and only if there exists a $q_2 \in [0; q_1]$ *such that* $y(q_2; q_1) < r(q_2)$.

In the spirit of backward induction, the sender always chooses a best static persuasion strategy for her nal attempt, p_{q_2}, if the persuasion game proceeds to the second stage with prior belief being q_2. Therefore, the sender's rst-stage strategy is about choosing the subsequent prior belief q_2 $\rho(p_{q_2}; q_2$ a30 t3eeyst3e30ubse-

Proposition 1 reduces the comparison between the optimal static persuasion strategy and sequential persuasion strategies to the comparison between feasible rst-stage strategies and the benchmark. Accordingly, the investigation of whether static or sequential persuasion is better can largely be re ected by the properties of functions $r(q_2)$ and $y(q_2; q_1)$ under di erent conditions. When $p_{q_1} = (0; \ )$, the shape of $r(q_2)$ is pinned down, which produces Proposition 2 as the baseline case where the receiver has full attention to the persuasion.

Proposition 2. *If the strategy $p_{q_1} = (0; \ )$ is feasible in the static persuasion, static persuasion is never less advantageous than sequential persuasion; If $M(p_t; q_t)$ is always greater o11.9552 11.95h.668* make the optimal frictionless static persuasion strategy infeasible for the sender. This incentive incompatibility between the sender and the receiver is a necessary to initiate the intertemporal trade-o , with which the sender will consider persuade sequentially.

In contrast, when the information structure (0;) is infeasible due to the frictional constraint, it becomes possible a two-stage persuasion strategy can out-perform one that optimize the static persuasion problem. According to Proposition 1, this possibility relies on the comparison between $r(q_2)$ and $y(q_2; q_1)$, which are both determined by the properties of motivation function. According to Lemma 3 and Proposition 1, $(p_{q_t}; q_t)$ determines $r(q_2) = \max f \ (q_2); \ g$, where $r(q_2) > y(q_2; q_1)$ is possible. Since the sender is objective is given as xed, $(p_{q_t}; q_t)$ is a measure, although only partially, of how the receiver's motivation function and information cost vary in his prior belief q_t. $y(q_2; q_1)$, on the other hand, measures the lower boundary of the set of feasible information structure conditional on the given prior belief q_1. It re ects how the receiver's motivation and information cost varies in the information structure p_t.

How motivation function varies in p_t and q_t are independent for certain values of q_2, especially when it is distinct from q_1. As a result, $r(q_2)$ and $y(q_2; q_1)$ are generally independenteasure,) are g receiver's inattention, a static framework is not su cient to fully understand how the sender persuade a receiver. Introducing a framework that allows the sender to optimally choose sequential persuasion is necessary to capture insights in the persuasion behavior. Moreover, would be a waste of time, which makes him refuse to pay attention to any persuasion at the second stage. With a different philosophy, the sender may choose a p_1^b above $p_{q_1}^b$. This less aggressive strategy makes a bad signal "less bad", thereby expanding the available strategies for his second persuasion attempt, which can be a better strategy within the framework that allows for an endogenous sequential persuasion.

Since $q_2 < p_{q_1}^b$ makes subsequent persuasion attempt impossible, the sender needs to increase q_2 above $p_{q_1}^b$ to create a second chance to persuade had her first attempt fails. However, when the motivation function is WDP rather than SDP indicating a less severe punishment of failed persuasion attempts. Although $y(p^b_t; q_t)$ decreases as q_t decreases with SDP, if the decrease is not substantial for $p^b_t \leq p^{\,b}_{q_1}$, $q_2 = p_{q_1}$ can still produce a chance for subsequent persuasion. In this case, some sequential persuasion strategies de nitely out-performs any static one, as the sender owns additional persuasion value without losing any chance in her rst persuasion attempt.

However, the possibility persists that the optimal static persuasion strategy is not optimal as the rst-stage strategy in a two-stage sequential persuasion. If this is the case, not realizing that sequential persuasion strategies can out-perform the static ones at the start of the game to plan the strategy ahead accordingly still causes the sender to choose a sub-optimal information structure and miss the most e ective persuasion strategy.

Proposition 4. *Given that the motivation function is WDP that causes at least a sequential persuasion strategy to out-perform any static persuasion strategy, the optimal sequential* produces a higher persuasion value for the second persuasion attempt. Therefore, the left-hand-side of (5) is negative at $p^b_{q_1}$, leading to $p_1^b > p^b_{q_1}$.[3]

When the receiver is inattentive, his disappointment-penalizing motivation does not prevent the sender from planning for a subsequent persuasion in case her first attempt fail. But she becomes more conservative in her first attempt. In brief social interactions, immediately impressing the person one encounters is challenging. On the contrary, an aggressive self-introduction may have high chance to be deemed as bragging. However, when she chooses a more conservative approach, even if one does not buy what she says, he does not lose interest in building connection with her, making it a more effective approach in this scenario.

6 ESBP with Binding Attempt Limit

If the sender is allowed for more attempts in the persuasion game, she may find it beneficial to be the final one but have lower branches. To put it another way, the sender's evaluation now includes deciding whether to insert and identifying the best interim attempt, the failure of which leads to a subsequent interim stage of persuasion rather than the final attempt.

Proposition 5. *Suppose there is a sequential strategy with adjacent subgames Q_t and Q_{t+1} Q_t, in which q_t and $q_{t+1} < q_t$ are prior beliefs, respectively. For each belief q^0 2 $(q_{t+1}; q_t)$, adding an additional attempt into this structure in the persuasion path is more advantageous for the sender if and only if: when adding the attempt causes $y(q_{t+1}; q_t)$ increase to $y(q_{t+1}; q^0)$ by $\ _{t+1} > 0$, it creates a $y(q^0; q_t)$ that is smaller than $y(q_{t+1}; q_t)$ by more than*

$$_t = \frac{(q^0 - q_{t+1})\ _t\ _{t+1}}{(q_t - q^0)\ _{t+1} + (\ _{t+1} - \ _t)\ _{t+1}};$$

where $_t = \prod_{i=t}^{\tau}\ _i$ conditional on the original persuasion path with $\ < T$ attempts.

As discussed in the previous section, the optimal strategy for each persuasion attempt results in two direct recommendations for the receiver, \act" and \not act". Only under the latter recommendation the sender has incentive to keep persuading. Hence, given the receiver's motivation, we can characterize the sender's sequential persuasion strategy as a series of prior beliefs upon failed persuasion attempts $Q_\tau = (q_1; ::: q_\tau)$, $\ \leq T$. This series is essentially a Markov Chain where the probabilities of the recommendations and their related expected payoffs are determined by the prior belief of that stage.

Adding an interim attempt to a sequential strategy directly influences the prior belief of the persuasion attempt that immediately follows it. $\ _{t+1}$, defined as the change in $y(q_{t+1}; \cdot)$ due to the prior belief changing from q_t to q^0, indicates how a less favorable prior belief ($q^0 < q_t$) alters the highest chance of success in the next attempt to persuade. With this less favorable prior belief, it requires a more informative signal to make the receiver's belief cross the cutoff, , and change his decision to \act" accordingly. A positive $\ _{t+1}$ is consistent with Axiom 1, which defines the disappointment-penalizing nature of the receiver's motivation to engage in persuasion. This nature indicates that the deteriorated prior belief makes an original persuasion design prohibitively costly for the receiver to pay attention to persuasion.

transiting to another iso-value curve as we encounter their connecting points until we reach the bottom endpoint. The process is equivalent to pivoting the line segments from left to right around the connecting points in panel (a) to elevate the position of the line segment combination.

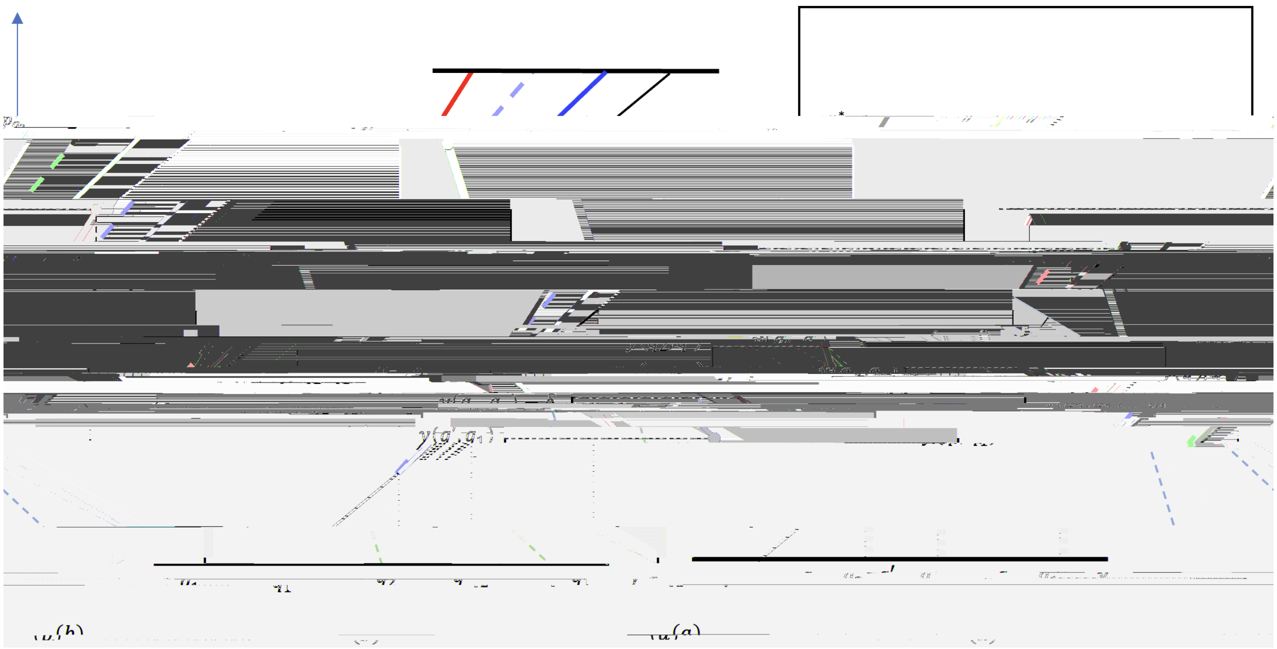

Figure 3: Value of an Additional Persuasion Attempt $(q^0; y(q^0; q_1))$

This extended framework provides an rudimentary yet intuitive approach to designing a sequential persuasion strategy. The sender must rst identify a starting point at the upper left of the graph in panel (b), which is attached to several line segments, which belong to indi erence curves of prior beliefs on the persuasion path, with decreasing slopes that connect to each other, forming a \wire" that transmits the initial point to the lower right and ultimately reaches the receiver's indi erence curve of his prior belief q_1 at the lowest position. Based on this intuition, Figure 2 generally implies how adding a persuasion attempt prior to an interim stage can improve the overall persuasiveness. In panel (b), the additional line segment causes the connecting point to retreat to a higher position. However, as long as the line segment attached to the connecting point crosses the benchmark level, the \wire" reaches a lower indi erence curve, and the additional persuasion attempt it represents leads to a higher overall e ectiveness. Panel (a) likewise re ects this improvement.

$1 - {}_t$ represents the sender's persuasion value of the given persuasion path, assuming that she is at stage t with prior belief q_t. The value of ${}_t$ and ${}_{t+1}$ are both unspeci ed in Proposition 5. In addition, the inserted stage can represent an attempt that is equivalent to the composition of multiple interim attempts. Therefore, Proposition 5 as a criterion of whether to include additional persuasion attempt is very general. When the sender is designing a persuasion path, this can be used as a rule of thumb to determine whether to extend the persuasion game for higher persuasiveness no matter what persuasion path is currently in her mind. For example, if q_t and q_{t+1} in Proposition 5 belong to the optimal persuasion path with attempts, the argument in the proposition becomes a su cient condition that the sender can improve her ex-ante persuasiveness by extending the persuasion path design beyond attempts.

According to Proposition 5, whether or not to include an additional attempt as an interim stage of a persuasion game depends on the comparison between the benchmark ${}_t$ and the actual variation of $y(\cdot; q_t)$. Given a q in general. On the other hand, while a two-stage sequential persuasion design may be less e ective than the optimal static persuasion attempt, incorporating additional stages could ultimately result in a sequential persuasion strategy that outperforms the static approach in overall persuasiveness. Therefore, although two-stage persuasion game reveals the underlying mechanism of endogenous sequential persuasion game intuitively, it may not precisely predict the sender's behavior of choosing an optimal persuasion path, which demonstrates the necessity of analysis under the general ESBP framework with T attempts. Additionally, this corollary also advises the sender to understand the limit on how many attempts are allowed to persuade the receiver, as well the form of receiver's inattention, so that she can optimally design the optimal persuasion approach, which may vary signi cantly with little changes to the circumstances.

6.2 Piecemeal Information Disclosure

Given that the sender is to design a persuasion strategy with $T < 1$ attempts, she is solving an intertemporal problem of choosing an optimal persuasion path, which can be formalized as a Bellman function as follow0s.

Euler condition below

$$\frac{1}{p^g_{t+1} - p^b_{t+1}} = \frac{1}{p_t^g - p_t^b} + y^0_{t;q_{t+1}} \frac{q_t - q_{t+1}}{(p_t^g - q_t)(p_t^g - q_{t+1})} + [y^0_{t+1;q_{t+1}} - 1] \frac{q_{t+1} - q_{t+2}}{(p^g_{t+1} - q_{t+1})(p^g_{t+1} - q_{t+2})}; \tag{7}$$

where $y^0_{t;q_{t+1}}$ *and* $y^0_{t+1;q_{t+1}}$ *respectively represent the derivatives of* $y(q_{t+1}; q_t)$ *and* $y(q_{t+2}; q_{t+1})$ *with respect to* q_{t+1}.

Similar to the two-stage persuasion game, the optimal design of a persuasion path in general ESBP game also solely rely on how the motivation function vary in the prior belief and the objective posterior belief in case of the current persuasion attempt failing. Based on Lemma 4, the speci cation of these properties can generate deterministic predictions on how the information structure evolves as the persuasion proceeds along the designed persuasion path.

Proposition 6. *Given that both* $y^0_{t;q_{t+1}}$ *and* $y^0_{t+1;q_{t+1}}$ *are negative for all possible* q_t *and* q_{t+1}, *the information structure employed at each stage is less informative than each structures employed at following stages, or that* $p^g_{t+1} - p^b_{t+1} > p_t^g - p_t^b$. *Specifically,* p_t^b *decreases and* p_t^g *increases as* t *increases from* 1 *to* T.

Since the persuasion process continues only when previous attempt only recommends \not act", Bayes plausibility indicates that p_t^b decreases as t increases. With the receiver's disappointment-penalizing motivation, if an information structure has both larger p^g and p^b than the other information structure, the former information structure should be dominated by the latter and should not be included in a persuasion design if possible. If there exists any information structure in the optimal path that does not satisfy $p^g_{t+1} - p^b_{t+1} > p_t^g - p_t^b$, the given attempt limit cannot be binding. As $y^0_{t;q_{t+1}} < 0$ ensures Proposition 6, it is also an important condition that motivates the sender to keep extending the game before the attempt limit is hit, which is consistent with Proposition 5.

When $y^0_{t;q_{t+1}} < 0$ and $y^0_{t+1;q_{t+1}} < 0$ are both satis ed, the sender is a \piecemeal" information disclosure strategy so that the sender to make a bad signal less bad and each possible failure only causes q_t to decay marginally. By doing so, she takes advantage of small ρ

7 Non-Binding Attempt Limit and Efficiency Boundary

Under certain conditions, making the persuasion game sequential can recover efficiency loss resulting from the receiver's inattention. This finding from the previous analysis is subject to the binding constraint of maximum persuasion attempt permitted. Allowing the sender to persuade as many times as she wishes can further relax the constraint to improve the persuasion effectiveness. With sufficiently large (but finite) T to make the attempt non-binding, the receiver can keep persuading as long as she is able to construct the necessary motivation with certain information structure to attract the receiver's attention. This relaxation may shape the sender's optimal persuasion design. It also allows us to characterize the efficient boundary in a sequential persuasion game with receiver's inattention, or how much can ESBP game beat the receiver's inattention.

The constraint of maximum persuasion attempts allowed in the game, if binding, has significant impact in determining the optimal persuasion path. As indicated in Proposition 5, it may be beneficial to include an attempt in the persuasion path design. However, if the path has reached the maximum limit of attempts, this inclusion implies a replacement of the attempt that has been included in a path and therefore turns out to be harmful. Formally, while there are many path satisfies the Euler condition (7) in the intertemporal persuasion problem, the backward induction indicates an unique terminal subgame and an unique path that omits many design in other candidate paths.

Nevertheless, if the attempt limit is no longer binding, this trade-off among different paths that satisfy Euler condition dissipates. If a persuasion attempt is beneficial to the overall effectiveness of the persuasion, the sender can include it into the persuasion path design. The following proposition indicates that as long as any persuasion attempt has sufficiently small impact on the subsequent persuasion design, the sender only need to consider whether to include a persuasion attempt into a given persuasion path rather than considering replacing any attempt that has been included in the path.

Lemma 5. *Given that Q_τ is the optimal persuasion path is the optimal persuasion path condi-*

tional on the length of the path being $< T$, there exist motivation functions with a sufficiently small disappointment-penalty for each $p^b_t = q_{t+1}$ such that: if q^0 that satisfies Proposition 5, given that q_t and q_{t+1} belong to Q, q^0 also belongs to Q.

When the disappointment-penalty, measured by $y^0_{t;q_{t+1}}$

this intuition, we can characterize the sender's optimal sequential persuasion design given that the disappointment-penalty is su ciently small. This persuasion design is uniquely determined only by the receiver's motivation function.

Proposition 7. *When the attempt limit is not binding, and that the disappointment-penalty is su* f *ciently small for each possible* q_t, *the optimal sequential persuasion strategy includes all information structures* $p_t^b; y(p_t^b; q_t)$ *that are not mutually dominated, where* p_t^b 2 $[p_\tau{}^b; p_1{}^b]$, *if there are only* f*nite of them. In this design, if* $y^0_{t;q_{t+1}}$! 0, *then* $p_1{}^g = y(p_1{}^b; q_1)$ *is the smallest possible* $p^g \geq$ *that satis*f*es the participation constraint* $M_t \geq 0$, *denoted as* p^g_{min}. $p_\tau{}^b$ *is the smallest possible* p^b *that satis*f*es participation constraint* $M_t \geq 0$, *denoted as* p^b_{min}.

In a Bayesian persuasion problem that our research focuses on, the sender should minimize both p^g and p^b

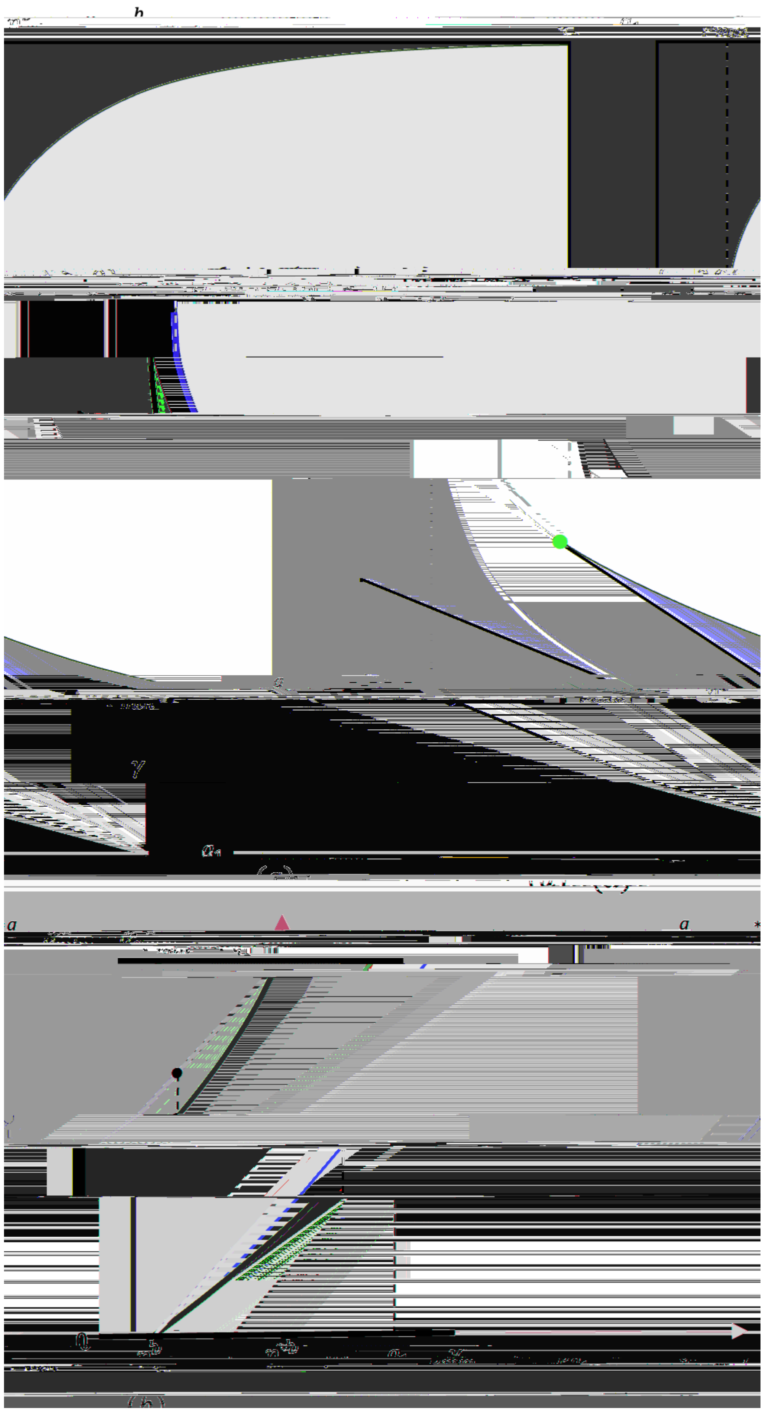

Figure 4: E cient Boundary of Sequential Persuasion Strategies

mutually-dominated information structures extend the bridge downward to connect with the lower endpoint de ned by p^g_{min}. Graphically, p^g_{min} ensures that the \bridge" extends as low as possible.p^b_{min} establishes the basis for the \bridge" to reach the furthest left positioning.

of her persuasion attempts. This improvement perhaps eventually exceeds the optimal e ec-tiveness in a static persuasion game. However, the e ciency boundary of this design is still determined by the constraint, where the initial and concluding strategies are determined by p^g_{min} and p^b_{min}, and the interim stages are determined by the Euler condition. More speci cally, if the frictionless optimal static persuasion strategy, (,0), is infeasible due to the frictional constraint, it is also not attainable by \piecemeal" information disclosure strategy. Extending the persuasion game may beat the static persuasion boundary under the receiver's inattention but not this inattention completely.

8 Conclusion

This paper discusses the Bayesian persuasion where the receiver is rationally inattentive. When it costs the receiver to process the signal sent by the Bayesian experiments, fewer Bayes plau-sible information structures are available to satisfy the receiver's participation constraint in paying attention to the persuasion. With this frictional constraint, sequential persuasion may be more e ective than static persuasion, causing the sender to emphasize a subsequent oppor-tunity to successfully persuade the receiver rather than concluding the persuasion immediately.

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