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Veto Bargaining with Incomplete Information and Risk Preference: An Analysis of Brinkmanship

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October 22, 2024

Give an inch and they'll take a mile.

American Proverb

This paper explains brinkmanship with in nitely repeated veto bargaining games

1 Introduction

Brinkmanship refers to the strategy of threatening the opponent with disaster to obtain a favorable outcome (Smith, Had eld, and Dunne 2008, p. 390 and Snyder 2001, p. 117). Schelling (1967, p. 91) explains brinkmanship the following way.

> The creation of risk—usually a shared risk—is the technique of compellence that probably best deserves the name of ”brinkmanship.” It is a competition in risk-taking. It involves setting afoot an activity that may get out of hand, initiating a process that carries some risk of unintended disaster. The risk is intended, but not the disaster.

The possible disaster is the leverage the party engaging in brinkmanship has to compel the other side to act. However, the disaster can strike both parties. So usually, the party engaging in brinkmanship also takes on the risk of disaster which means that the party's action can depend on its evaluation and preference regarding risk (p. 94).

Brinkmanship is a common negotiating strategy. It is found in labor contract, trade deal and peace treaty negotiations among others. A labor union that threatens the employer with a strike knows that a strike is costly for both sides but believes that the threat can get the union a better deal. In negotiating a trade deal, one side might threaten to walk away from the negotiations and start a trade war where both sides suffer high tariffs on exports. Similarly, for peace treaty negotiations, a nation may threaten prolonged war or total war.

Despite the common occurrence of brinkmanship in negotiating deals, brinkmanship often fails. The success of brinkmanship would mean that the disaster did not happen. In history, there are cases where the disaster happened. Also, for the party making the threat, often they backed off or the resulting deal was no better than the deal that they could have gotten without brinkmanship. For example, North Korea is a frequent user of brinkmanship.[1] Its relation with other countries is that it is one of the most isolated countries in the world.[2] In the United States, debt limit ghts and government shutdown ghts have been common. These ghts result in disaster if a deal is not reached. Most end with a deal that does little to change the status quo. Many shutdowns have happened.[3]

This paper is the rst paper to study how the use of brinkmanship changes depending on the risk preference of the party engaging in brinkmanship using game theory. Previous papers choose different explanations for brinkmanship. Some explained brinkmanship or strikes with irrational types.[4] It is also the rst to explain why brinkmanship is unlikely to succeed by analyzing how information revealed in a repeated game changes players' interaction. In the basic model of the paper, I solve for the Perfect Bayesian equilibrium (PBE) of a veto bargaining game with incomplete information — players' types are private information. The game is in- nitely repeated and asymmetric. In each period t of the game, the proposer proposes a new policy, action or a good a_t. The vetoer can veto this proposal. By making an proposal that has the risk of being vetoed, the proposer may engage in brinkmanship against the vetoer.

For the basic model, I nd Risk-Taking equilibria (RTE) and Risk-Avoiding Equilibria (RAE). Brinkmanship happens and happens only in the RTE. RTEs are caused by the proposer's risk love and RAEs are caused by the proposer's risk aversion. The key result of the basic

1. See Snyder (2001, p. 117–118), Ha and Chun (2010) and Shin (2020, p. 32–33).
2. See Rennolds (2024).
3. For information on debt limit ghts and government shutdown ghts, see Scholtes and Emma (2023), Prokop (2023), Hussein (2023), and Schaul and Uhrmacher (2024).
4. See Fanning (2016), Acharya and Grillo (2015) and Calabuig and Olcina (2000).

model that differs from models with irrational types is that brinkmanship is unlikely to succeed

Figure 1: Game Tree for the Basic Model

On the other hand, in Hayes (1984), the firm has information about profitability that the union does not. In Hayes (1984), the union may use strikes as a tool for gaining information. Cheung and Davidson (1991) assumes that a union can represent workers at more than one firm and has private information about its utility. A union representing multiple firms is more likely to strike because it does not want to signal weakness.

Two recent papers that are closely related to this one analyze repeated games where in each period, the parties bargain over a new contract for the period. First, Robinson (1999) defines a game where the firm has private information about its profitability. In the model where this information is subsequently fully revealed, the union does not go on a strike. In the model where this information is not subsequently fully revealed, the union may go on a strike. In Robinson (1999) strikes are a tool to punish the firm not to extract concessions from the firm. Second, in Calabuig and Olcina (2000), both parties have private information. Both try to build reputations of being "stubborn" and irrational which results in strikes.

3 Basic Model

The veto bargaining game has two players, a proposer (he) and a vetoer (she). Only the proposer makes offers. This allows me to demonstrate brinkmanship as a unilateral strategy by the proposer.[5] This is an infinitely repeated game. At the start of the game, the proposer observes S_p and the vetoer observes S_v. Both are independent private random variables and specify players' types. In each period $t \in \{1, 2, \ldots\}$, proposer offers a proposal or an action a_t to the vetoer. Then, vetoer decides whether to accept or reject

5. This unilateralism is consistent with Snyder (2001, p. 117).

3.1 Payoffs

Players' payoffs are constructed from period utilities. When the offer is rejected, both players

$f_H \quad P(p_H) = 1 \quad f_L > 0$

decision is made by her cut-off function. Once the inputs of the function are known, the period does not matter at all for the player's strategy for the period. This helps me drastically simplify the players' strategies in an equilibrium.

An equilibrium satisfying assumption 1 has Markov properties in that for any period, the player's strategies and utilities are given by the state indicators S_V, S_P and b. In other words, players' types and the proposer's beliefs are what matters. The history of the game does not affect the players types and once the proposer's belief is known, the history has no further useful information.

The following de nition lists the three main beliefs that the proposer can have. b_0 is the proposer's initial belief in the game.

De nition 1.

- b_L means that proposer believes that the vetoer is low type.
- $b_{\ L}$ means that proposer believes that the vetoer is medium type with probability $\frac{g_M}{g_M + g_H}$ and high type with probability $\frac{g_H}{g_M + g_H}$.
- b_0 means that proposer believes that the vetoer is low type with probability g_L, medium type with probability g_M and high type with probability g_H.

Any proof missing in this section is found in appendix 1.

4.1 Risk-Taking Equilibria

I de ne a risk-taking equilibrium (RTE) using players' strategies.

De nition 2.

Figure 2: Offers on the equilibrium path in an RTE

From the vetoer's strategy, (iv), (v) and (viii) are the important parts. By (iv) and (viii), when the vetoer's type has not been revealed, the low type vetoer accepts greater and the medium etoer

the effect on current utility but because of the effect on future utilities as well. Therefore, the medium and high type vetoers reject the initial offer $\underline{a}$ and build a reputation to be not the low type. Also, $\underline{a}$ must be suf ciently high that the low type vetoer considers it an adequate compensation for giving up the possibility of higher offers in the future.

In the RTEs, players sources of surplus are different. The proposer's surplus comes from rst-mover advantage and information about the vetoer. The rst-mover advantage allows the proposer to propose a low policy or action that the vetoer would not propose. The proposer's information about the vetoer lets him offer a low policy and action that will still be accepted.

On the other hand, the vetoer's surplus comes from information rent. The vetoer's information rent comes from the fact that the vetoer knows her own type but there are circumstances where the proposer thinks that the vetoer's type can be higher. In period 1, the low type vetoer derives surplus from the fact that the proposer is unable to distinguish her from a medium or high type vetoer. In period 2, the medium type vetoer derives surplus from the fact that the proposer is unable to distinguish her from a high type vetoer. Vetoer's expected payoff in an RTE is

$$g_L(\underline{a} \quad v_L) + g_M f_H \sum_{i=1}^{\infty} d_V^i (v_H \quad v_M) = g_L(\underline{a} \quad v_L) + g_M f_H \frac{d_V}{1 \quad d_V}(v_H \quad v_M): \tag{4}$$

Proposition 1.
The following is suf cient for an RTE to exist. (i) and (ii) are also necessary for an RTE to exist.

(i) $(1 \quad g_L)u(p_H \quad v_H) \quad g_L[u(p_H \quad \underline{a}) \quad u(p_H \quad v_H) + \frac{d_P}{1 \quad d_P}(u(p_H \quad v_L) \quad u(p_H \quad v_H))]$

(ii) $0 \quad u(p_L \quad \underline{a}) + \frac{d_P}{1 \quad d_P} u(p_L \quad v_L)$

(iii) $d_V \quad 0:5$

(iv) $\underline{a} = v_L + f_H \frac{d_V}{}$

$\underline{a}$. Since the proposer did not gain any information, the vetoer can offer $\underline{a}$ again in the next period. This is not optimal for the low type proposer. A low f_H means that the proposer has a low probability of being the high type. A high d_V means that the vetoer cares a lot about the future utility gain from rejecting the brinkmanship offer. (iv) and lemma 1 shows that a low f_H and low d_V make brinkmanship feasible.

Proposition 2. Suppose d_V 0:5, $\underline{a} = v_L + f_H \frac{d_V}{1 \quad d_V}(v_H \quad v_L) < v_H$ and the parameters of the basic model are given. Then, there exists some $\underline{h}$ for which an RTE exists if and only if h $\underline{h}$.

The above proposition establishes the relationship between the proposer's risk preference and the existence of RTEs. When the other parameters of the model allow for an existence of an RTE, an RTE exists when the proposer is suf ciently risk-seeking. $\underline{h}$ is the highest amount of risk aversion that allows an RTE to exist. Engaging in brinkmanship entails the risk of brinkmanship failure for the proposer. As a proposer becomes in nitely risk-seeking, he will come to value the potential gain from successful brinkmanship succeeds to be more valuable than the potential loss from failed brinkmanship. As a proposer becomes in nitely risk-adverse, he will come to feel the opposite way. Also, an in nitely risk-seeking proposer is willing to take a loss in period 1, for future gains.[11] Therefore, under the proposition's conditions, $\underline{h}$ always exists and is somewhere in the middle of those extremes. For any h, lower risk aversion, an RTE exists.

4.2 Risk-Avoiding Equilibria

The following de nes the Risk-Avoiding Equilibrium (RAE).

De nition 3.
A Risk-Avoiding Equilibrium (RAE) is a PBE satisfying assumption 1 where the following holds.
Proposer's strategy:

(i) $a(p_H; b_0) = a(p_H; b\ _L) = v_H$

(ii) $a(p_H; b_L) = a(p_L; b_0) = a(p_L; b_L) = a(p_L; b\ _L) = v_L$

Vetoer's strategy:

(iii) $g(v_L; b_0) = g(v_L; b_L) = v_L$

(iv) $g(v_L; b\ _L) = g(v_H; b_L) = v_H$

(v) $g(v_M; b_L) = v_M$

(vi) $8b\ 2\ f\ b\ _L; b_0 g : g(v_M; b) = g(v_H; b) = v_H$

The above de nition de nes the players' strategies using the three beliefs de ned in de nition 1, b_L, $b\ _L$ and b_0. For these beliefs, the proposer only offers v_H when he is high type and he believes $b\ _L$ or b_0 and in all other cases, he offers v_L. In other words, when the proposer thinks that the vetoer has a positive probability of not being the low type and gets positive utility from v_H being accepted, he offers v_H. When this is not true, the offer is v_L.

From the vetoer's strategy, (iii) and (vi) are the important parts. In (iii), when the proposer thinks that the vetoer may be the low type, the low type vetoer's cut-off point is v_L. In (vi), when

Figure 3: Offers on the equilibrium path in an RAE

the proposer believes that the vetoer may be the medium or high type, the medium or high type vetoer's cut-off point is v_H.

Now that I have spelt out the players' strategies, I will explain the progress of the game on the equilibrium path for an RAE depicted in gure 3. In the gure, the ovals contain the offer for the period. In all periods, the low type proposer offers v_L and the high type proposer offers v_H. The v_L offer is accepted by the low type vetoer and rejected by the medium and high type vetoers. The v_H offer is accepted by all types of vetoers.

This means that if the vetoer is low type, the high type proposer can make low offers (v_L in this case) and get it accepted in all periods. However, offering v_L entails the risk of rejection by the medium and high type vetoers. Thus, for the RAE, brinkmanship refers to the following strategy by the high type proposer. In period 1, he offers v_L. If it is accepted, he makes the same offer in subsequent periods. If it is rejected, he offers v_H in subsequent periods. This brinkmanship does not happen in the RAE because the proposer weakly prefers to not take the risk of rejection from the brinkmanship. Instead, the high proposer plays it safe by making offers that any type of vetoer will accept.

When the vetoer knows that the proposer is not engaging in brinkmanship, she knows that the proposer is “honest”. In other words, for a proposer who never engages in brinkmanship, his type is fully revealed to the vetoer from his period 1 offer. The low type vetoer accepts an offer of v_L because she knows that the low type proposer only offers v_L. The medium and high type vetoers accept v_H because this is what the high type proposer will offer them and the the best offer they can get.

In an RAE, the proposer's surplus comes from his rst-mover advantage. On the equilibrium path, the proposer makes the lowest offer that the types of vetoer that he will make a deal with will accept for sure. Like in an RTE, vetoer's surplus in an RAE comes from information rent. The vetoer's surplus is only positive when the vetoer is low or medium type and the proposer is high type. In this case, the vetoer gets surplus because the proposer is unable to distinguish the low or medium type vetoer from the high type vetoer and does not risk rejection by making an offer lower than v_H. Vetoer's expected payoff in an RAE is

$$f_H \sum_{i=0}^{\infty} d_V^i (v_H \quad E(S_V)) = \frac{f_H}{}$$

(i) $g_L \frac{1}{1 \quad d_P}(u(p_H \quad v_L) \quad u(p_H \quad v_H)) \quad (1 \quad g_L)u(p_H \quad v_H)$

(ii) $d_V \quad 0{:}5$

The above proposition states a suf cient condition and a necessary condition for the existence of an RAE. The proofs of this proposition and lemma 11 in appendix 1 explain the progress of the game and the beliefs in the game in more detail.

the proposer to be low type. Also, the vetoer believes that v_L is the best offer she can get from the low type proposer. Thus, the vetoer does not demand compensation for revealing her own type. So the vetoer's trust in the proposer's honesty in a RAE creates a greater incentive for the proposer to deceive the vetoer and engage in brinkmanship.

In an RAE, the high type proposer is unwilling to take the risk of brinkmanship despite the fact that he only has to offer v_L to engage in brinkmanship. Then, he will prefer not to engage in brinkmanship in a RTE where he has to offer a greater amount $\bar{a} > v_L$ to engage in brinkmanship. Brinkmanship in RTE creates loss of trust by the vetoer. This leads the low type vetoer to set a higher cutoff point of $\bar{a} > v_L$ which is unfavorable to brinkmanship.

For certain parameters of the model, gure 4 answers when an RTE or an RAE exists depending on h, the proposer's coef cient of absolute risk aversion. In the gure, the lines draw the proposer's expected bene t from strategies. The “RAE strategy” line draws the right side of proposition 3's (i), the expected bene t from the RAE strategy in the RTE or the RAE. The “Brinkmanship in RAE” line draws the left side of proposition 3's (i), the expected bene t from brinkmanship in the RAE. The ”RTE strategy” line draws the right side of proposition 1's (i), the expected bene t from the RTE strategy in the RTE. Note that the “Brinkmanship in RAE” line is above the RTE strategy. The conditions for a brinkmanship are more favorable for the proposer in a RAE compared to an RTE. This is con rmed by the fact that the expected bene t from brinkmanship is higher in a RAE.

Figure 4 demonstrate how the expected payoffs, optimal strategies and PBEs change depending on h, the coef cient of absolute risk aversion for the proposer. In the gure, the parameters are such that an RAE exists if and only if proposition 3's (i) is satis ed. Also, an RTE exists if and only if proposition 1's (i) is satis ed. When h & 0:09, the expected bene t is weakly greater for the RAE strategy compared to the brinkmanship in RAE. This means that the proposer weakly prefers to not deviate and that an RAE exists. When 0:32, the expected bene t is greater for the RTE strategy compared to the RAE strategy. Therefore, the proposer weakly prefers to not deviate to the RAE strategy and an RTE exists.

Thus, following proposition 4, when h & 0:09, the proposer is risk averse enough that a RAE exists. On the other hand, following proposition 2, when 0:32, the proposer is risk-seeking enough that a RTE exists. Since RAEs only exists for

4.4 Lack of Pooling Equilibria

A pooling equilibrium of the basic model is a PBE where $g(v_L; b_0) = g(v_M; b_0) = g(v_H; b_0)$. This means that in a pooling equilibrium, the vetoer sets the same initial cut-off point for all of

Figure 5: Game Tree for the Extended Model

In specifying the threshold y, I am modeling a situation where the veteor can decide to impose conditions what proposals she will consider and announce it but has dif culty adjusting these conditions once they are announced. For instance, in negotiating a labor contract, the labor union may announce that it will not accept any wage lower than $10 per hour. The actual negotiation would be handled by a union leader who repeats in every meeting that she does not have the authority to accept any offer below $10 and that any proposal given to her below $10 will be discarded without being heard or read. If the rm attempts to bargain with the members of the union directly, the members could say that they have entrusted bargaining process to the union leader and they will not bargain directly. Such a situation would correspond to setting $y = 10$ initially and sticking to it.

To rescind the announcement, the union can remove the leader and communicate with the proposing rm directly or retract the announcement. By doing so, the union will hear any proposal the proposing rm makes. However, setting a new consequential threshold different from the previous one make be problematical. Suppose the union leader claims that she has received a new direction from the union members lowering the threshold for negotiation to $9. Then, the union loses credibility for the claim that because it has entrusted its leader with negotiations and because its members will not listen to any proposals, its leader is the only one the rm can talk to.

Now, rm reasons that the union members are in communication with the leader and do change the terms of the negotiation based on new information. In this case, the members should be able to listen to information from the rm and readjust their demands. Furthermore, if the previous statement that no offer will be considered below $10 was in fact wrong, the new statement

Figure 6: Offers on the equilibrium path in a TPE

The vetoer's strategy means that at the start of the game, vetoer sets the threshold, y_2 to equal the high type proposer's type θ_H. Once set, this threshold never changes. From the offers that make through the threshold and are seen by the vetoer, the vetoer accepts any offer equal to or greater than her type S_V.

In the proposer's strategy $S_P < y$ is a case where the current threshold is greater than the proposer's type S_P. In this case, since all offers that make it through the threshold and are accepted give the proposer negative utility, proposer deliberately makes any offer y_1 that will be rejected without being viewed. The other case S_P,

Vetoer's expected payoff in an TPE is

$$f_H \sum_{i=0}^{\infty} d_V^i (p_H \quad E(S_V)) = \frac{f_H}{1 \quad d_V}(p_H \quad E(S_V)) = \frac{f_H}{1 \quad d_V}(p_H \quad g_L v_L \quad g_M v_M \quad g_H v_H): \quad (7)$$

The vetoer uses her unchanging high threshold to take all the surplus. Therefore, the proposer's expected payoff is 0.

I will use the above equation to compare the vetoer's expected payoffs in an TPE, an RAE and a RTE by discussing how the game plays on the equilibrium paths. It is trivial to see that the expected payoff in the equation is always greater than the expected payoff for an RAE in equation 5. The expected bene t of a TPE compared to an RAE is that the threshold of $y = p_H$ negates the proposer's rst-mover advantage. Unlike in an RAE, the high type proposer can

6 Discussion

6.1 Brinkmanship Failure

Schelling (1981, p. 91, 105) states that limited war can also be a form of brinkmanship when it increases the risk of a major war. A prominent case where brinkmanship failed is President Nixon's application of the “Madman Theory” to the Vietnam War. The “Madman Theory” was a game theory based approach. According to his chief of staff, H. R. Haldeman, Nixon said the following.

> I call it the Madman Theory, Bob. I want the North Vietnamese to believe I've reached the point where I might do anything to stop the war. We'll just slip the word to them that, `for God's sake, you know Nixon is obsessed about Communism. We can't restrain him when he's angry–and he has his hand on the nuclear button'–and Ho Chi Minh himself will be in Paris in two days begging for peace.[13]

On Nixon, Defense Secretary Melvin Laird said “...he wanted adversaries to have the feeling that you could never put your nger on what he might do next.”

During the Vietnam War, the theory was applied by warning the North Vietnamese that if no major progress was made in the peace talks by November 1st of 1969, U.S. will be compelled to “take measures of the greatest consequence.” In October, the Soviet ambassador, Anatoly Dobrynin met with Nixon. Dobrynin reported to the Kremlin that Nixon said “he will never

caps on how much the government can spend. It also created a “supercommittee” which was supposed to come up with a de cit reduction deal. As an incentive for the committee to come up with a de cit reduction deal, the debt ceiling deal stated that unless the de cit reduction deal is passed by congress, sequestration will happen where across-the-board spending cuts take place. These cuts were split evenly between domestic and defense programs and were disastrous for both parties.[18] The supercommittee failed to reach a deal. Sequestration was delayed and then it took place in March 2023. However, in the same year, a congressional deal succeeded in providing “sequester relief” which reduced the cuts. Also, the budget caps were repeatedly raised allowing the government to spend more money.[19]

Assuming that both parties are rational, the reason that brinkmanship fails may be explained by the basic model of this paper. In the RTE, once the vetoer succumbs to brinkmanship, the proposer realizes that the vetoer is “weak” (low type) and will succumb to brinkmanship again in the future. This leads to worse offers from the proposer in the future. Because of this, the vetoer who is not “weak” (medium or high type) rejects the brinkmanship offer.

If the party engaging in brinkmanship is irrational, brinkmanship will likely fail in reality. An irrational proposer may demand unrealistically too much from the vetoer. Also, an irrational proposer may not honor deals. In this case, the vetoer should not make concessions to get an unreliable deal. Finally, if a proposer does not rationally consider deals, the vetoer should make nominal concessions instead of material concessions.

The reasons that I have listed for why brinkmanship may fail are not purely theoretical. For some of the examples of brinkmanship I have listed, people who were subject to brinkmanship stated or hinted less rigorously similar reasons for not succumbing to brinkmanship.[20] For the Russian invasion of Ukraine, Ukrainian President Zelenskyy insinuated that making concessions for someone who repeated engages in brinkmanship or whose promise is unreliable will lead to bad outcomes.[21] During the 2018-2019 government shutdown, House Speaker Nancy Pelosi said that if Trump got what he wanted though the shutdown, he will continue to apply the strategy for future demands as well.[22]

6.2 Brinkmanship Success

Lemma 1 shows some necessary conditions for the existence of an RTE. From it, I deduce necessary conditions for the success of brinkmanship. First, early brinkmanship cannot be too unfavorable for the vetoer. In lemma 1, the brinkmanship offer, $\underline{f}$, is high enough that the low type vetoer deems it worth the cost of succumbing to brinkmanship. Second, similarly, the vetoer needs to be okay with with the long term consequences of succumbing to brinkmanship. This may mean that the vetoer is myopic (low δ) or that the future harm from succumbing to brinkmanship is small (low $v_H - v_L$). Lastly, the vetoer needs to believe that because of the proposer's situation that the proposer likely nds a favorable offer for the vetoer unacceptable (low f_H).

Russia was involved in the First and Second Chechen Wars, the Russo-Georgian War, the Syrian civil war and the Russian invasion of Crimea. From those wars, Russia and Putin may

18. See Drawbaugh et al. (2011), Barrett et al. (2011), Matthews (2013), Kosnar and Rafferty (2013),

preconditions is not expected to happen. Since this vetoer will not try something different when the preconditions don't work, she tries to set preconditions that will lead to a deal.

In the basic model, high cut-off points fail because if the vetoer examines all offers, the proposer is going to try the make the worst offer the vetoer will accept. Therefore, to have

Appendix 1. Lemma and Proofs in Section 4

Only lemma 1 is here. All other lemmas are in the supplement. Lemma 1 is used to prove lemma 3. Lemma 2 is used to prove lemma 3 and proposition 1. Lemma 3 is used to prove propositions 1 and 5. Lemma 4 is used to prove propositions 1 and 3.

Proof of Lemma 1.

Using de nition 2, I can gure out how the game plays on the equilibrium path. The proposer offers $\underline{a}$ in period 1. Suppose the proposer is low type and v_H. Then, proposer's belief as he makes the period 1 offer is b_0. This offer is always accepted. After it is accepted, proposer's belief is still b_0. So for every period, the offer is $\underline{a}$ and it is always accepted. Proposer's payoff is negative. Proposer prefers to offer f_L every period which will give a non-negative expected utility every period. This means $\underline{a} < v_H$.

Suppose that the vetoer is low type and $\underline{a} < v_L + f_H \frac{d_V}{1 \quad d_V}(v_H \quad v_L)$.

$$\underline{a} \quad v_L < f_H(v_H \quad v_L)\frac{d_V}{1 \quad d_V}$$

$$\underline{a} \quad v_L < f_H \sum_{i=1}^{\infty} d_V^i (v_H \quad v_L) \tag{8}$$

If the vetoer rejects the period 1 offer, proposer believes v_L. If the proposer is high type, starting from period 2, the vetoer can get an utility of $v_H \quad v_L$ by accepting v_H which will not change the proposer's belief. If the vetoer accepts the period 1 offer, proposer believes b_L. Then, on the equilibrium path, vetoer's utility for every period starting from period 2 is 0. Inequility 8 means that the vetoer prefers to deviate.

Proposition 1 is used to prove proposition 2. Lemma 5 is used to prove lemma 6. Lemma 6 and 7 are used to prove lemmas 8 and 9. Lemma 8 is used to prove propositions 2 and 4. Lemma 9 is used to prove proposition 2.

Proof of Proposition 1.

I will prove suf ciency rst. In the RTE of the suf ciency proof, the proposer's belief is formed the following way. Proposer can have 3 beliefs, b_0, b_L and $b_{\ L}$. His belief is initially b_0. From b_0, proposer's belief can change to b_L or $b_{\ L}$. Proposer's belief can also change from $b_{\ L}$ to b_L. However, once the proposer's believes b_L, his belief is xed. A proposer who believes b_0 at the start of period t has the same belief at the start of period $t+1$ if the vetoer accepts an offer $a_t \quad v_H$ or rejects an offer $a_t < \underline{a}$. The same proposer will believe b_L in period $t+1$ if $a_t < v_H$ is accepted. If $a_t \quad \underline{a}$ is rejected, he will believe $b_{\ L}$ in the next period. A proposer who believes $b_{\ L}$ at the start of period t changes his belief to b_L if the vetoer accepts $a_t < v_H$. Otherwise, his belief is the same at the start of period $t+1$.

I will check whether the above speci cation is consistent with a PBE. For a proposer who believes b_0, $a_t \quad v_H$ will always be accepted and $a_t < a \quad v$

b_L, medium and high type vetoers will accept $a_t \geq v_H$ and reject $a_t < v_H$. So upon acceptance or rejection of $a_t \geq v_H$, he can believe b_L at the start of period $t+1$. Upon rejection of $a_t < v_H$, he can believe b_L at the start of period $t+1$. Also, upon acceptance of $a_t < v_H$, he can believe b_L at the start of period $t+1$.

I will prove that the proposer's strategy is optimal. Once the proposer believes b_L or

period t^0, proposer switches to $a_{t^0} = \underline{a}$.

$$u^0 \quad g_L[u(p_H \quad \underline{a}) + \sum_{i=1}^{\infty} d_P^i u(p_H \quad v_L)] + (1 \quad g_L) \sum_{i=1}^{\infty} d_P^i u(p_H \quad v_H)$$

Weak inequality 9 means $\sum_{i=0}^{\infty} d_P^i u(p_H \quad v_H) = \frac{1}{1 \quad d_P} u(p_H \quad v_H) \quad u^0$.

$$u(p_H \quad v_H) + d_P u^0 \quad u^0$$

This means proposer weakly prefers to switch at period t^0 1 instead. $a_t = \underline{a}$ is optimal.

Now, I will prove that the vetoer's strategy is optimal. By de nition 2, proposer never offers more than v_H. High type vetoer's utility is always 0 or less. A cut-off point of v_H for the high type vetoer is optimal for all future periods. Given this, the cut-off point is also optimal for the current period.

If the proposer's belief as he makes the offer is b_L, proposer's belief and offer will not change in the future. Therefore, low and medium type vetoer's strategy is optimal when proposer's belief as he made the offer was b_L.

Suppose that in period t, the proposer's belief as he makes the offer is b_S. I will solve for the low or medium type vetoer's optimal strategy. If v_H, proposer's belief will be the same at the start of period t+ 1, therefore, the vetoer's strategy is optimal.

On the equilibrium path, low type proposer offers $a_t = v_L$ and high type proposer offers $a_t = v_H$. Off the equilibrium path, if $a_t < v_H$ and $a_t \in v_L$, vetoer can believe that the proposer is high type. Off the equilibrium path, if $a_t = v_L$, vetoer can believe that the proposer is low type.

Consider the above cases of vetoer's beliefs about the proposer. According his strategy in de nition 2, low type proposer who believes b_S will always offer b_L in period t and the future. Low or medium type vetoer's strategy is optimal when she believes the proposer to be low type. Consider the case where she believes the proposer to be high type. If she accepts $a_t < v_H$, proposer changes his belief to b_L and offers v_L in all subsequent period. If she rejects $a_t < v_H$, proposer offers v_H in all subsequent periods. When I apply lemma 4 to (iii), I get the following.

$$v_H \quad S_V \quad \sum_{i=1}^{\infty} d_V^i (v_H \quad S_V)$$

In this case, the lower or medium type vetoer weakly prefers to reject $a_t < v_H$.

Suppose that in period t, the proposer's belief as he makes the offer is b_0. Consider the low or medium type vetoer's optimal strategy. If the vetoer rejects $\underline{a}$ and the proposer is high type, she will be offered v_H for all subsequent periods. If the vetoer rejects $\underline{a}$ and the proposer is low type, she will be offered v_L for all subsequent periods. If she accepts, she will be offered v_L for all subsequent periods from both proposer types. Take (iv).

$$\underline{a} \quad v_L = f_H(v_H \quad v_L) \frac{d_V}{1 \quad d_V}$$

$$\underline{a} \quad v_L = f_H \sum_{i=1}^{\infty} d_V^i (v_H \quad v_L)$$

Therefore, the low type vetoer's strategy is optimal. I will transform the above equation.

$$\frac{v_H \quad v_M}{v_H \quad v_L} (\underline{a} \quad v_L) = \frac{v_H \quad v_M}{v_H \quad v_L} f_H \sum_{i=1}^{\infty} d_V^i (v_H \quad v_L) = f_H \sum_{i=1}^{\infty} d_V^i (v_H \quad v_M)$$

If $\underline{a} \quad v_M > 0$, $\frac{v_H \quad v_L}{\underline{a} \quad v_L} < \frac{v_H \quad v_M}{\underline{a} \quad v_M}$. Thus, if $\underline{a} \quad v_M > 0$,

$$\underline{a} \quad v_M < f_H \sum_{i=1}^{\infty} d_V^i (v_H \quad v_M):$$

Note that the above inequality also holds when $\underline{a} \quad v_M \quad 0$. The inequality means that the medium type vetoer prefers to reject in period 1. The high type vetoer's optimal strategy is trivial.

Necessity is proven by lemma 3.

Proof of Proposition 2.

Since proposition 1's (iii) and (iv) are already satis ed, an RTE exists, if and only if proposition 1's (i) and (ii) hold. I will start with proposition 1's (i).

$$\frac{1 \quad g_L}{g_L} \quad \frac{u(p_H \quad \underline{a})}{u(p_H \quad v_H)} \quad 1 + \frac{d_P}{1 \quad d_P} (\quad u(p_H \quad v_L$$

I will prove suf ciency rst. In the RAE for the suf ciency proof, the proposer's belief is formed the following way. The proposer can have 3 beliefs b_0, b_L and b_{-L}. Proposer initially believes b_0. A proposer who believes b_0 can switch his belief to b_L or b_{-L}. A proposer who believes b_{-L} can change his belief to b_L. However, once a proposer believes b_L, his belief will never change. Consider a proposer who believes b_0 at the beginning of period t. If the vetoer accepts or rejects $a_t \geq v_H$, proposer does not change his belief. Also, if the vetoer rejects $a_t < v_L$, proposer does not change his belief. If the vetoer accepts $a_t > v_H$, proposer changes his belief to b_L. If the vetoer rejects $a_t \in [v_L; v_H)$, proposer changes his belief to b_L. A proposer who believes b_{-L} at the start of period t changes his belief to b_L if $a_t < v_H$ is accepted. Otherwise, his belief is the same at the beginning of period $t+1$.

I will check whether the above speci cation is consistent with a RAE. For a proposer who believes b_0 at the start of period t, $a_t \geq v_H$

is always 0 or less. A cut-off point of v_H for all future periods gives him 0 utility for all future periods. Given this, a cut-off point of v_H is also optimal for the current period.

Moving on, I will prove the low or medium type vetoer's optimality. If the proposer believes b_L, his belief will not change. Therefore, the vetoer's strategy for this case is optimal.

Suppose the proposer believes b_L or b_0. If the proposer offers $a_t = v_L$, the vetoer can believe that the proposer is low type. In this case, the vetoer's strategy is optimal because a low type proposer will always offer v_L in future periods.

If the proposer offers $a_t \in R^1 \setminus \{v_L\}$, the vetoer can believe that the proposer is high type. Vetoer finds it optimal to accept $a_t = v_H$ in this case. Applying lemma 4 to (ii) gives the following.

$$v_H - S_V = \sum_{i=1}^{\infty} d_V^i (v_H - S_V)$$

If the veteor accepts $a_t < v_H$, she will only be offered v_L after period t. If the vetoer rejects $a_t < v_H$, she can get $v_H - S_V$ in all future periods. Vetoer's strategy is optimal.

Necessity is proven by lemma 11.

Proof of Proposition 4.

Apply proposition 3. An RAE exists if and only if the following holds.

$$g_L \frac{1}{1-d_P} u(p_H - v_L) \geq [(1-g_L) + \frac{g_L}{1-d_P}] u(p_H - v_H)$$

$$g_L \frac{1}{1-d_P} \geq [(1-g_L) + \frac{g_L}{1-d_P}] \frac{u(p_H - v_H)}{u(p_H - v_L)} \tag{10}$$

By lemma 8's (1), $\frac{u(p_H - v_H)}{u(p_H - v_L)}$ is increasing in h.

$$\lim_{h \to \infty} \frac{u(p_H - v_H)}{u(p_H - v_L)} = 1$$

Therefore, there exists some h for which formula 10 holds. By lemma 8's (2),

$$\lim_{h \to \infty} \frac{u(p_H - v_H)}{u(p_H - v_L)} = 0$$

Therefore, there exists some h for which formula 10 doesn't hold. The intermediate value theorem completes the proof.

Proof of Proposition 5.

Suppose an RTE exists. I start by combining lemma 3's (1) and (2).

$$(1-g_L) u(p_H - v_H) < g_L [u(p_H - v_L) - u(p_H - v_H) + \frac{d_P}{1-d_P}(u(p_H - v_L) - u(p_H - v_H))]$$
$$= g_L [\frac{1}{1-d_P}(u(p_H - v_L) - u(p_H - v_H))]$$

However, this means that the necessary condition for lemma 11 does not hold.

Proof of Proposition 6.

I will use proof by contradiction. Suppose that I am at the PBE.

$$\bar{g} \quad g(v_L; b_0)$$

In any period where the proposer's belief is b_0 as he acts, vetoer rejects any offer below $\bar{g}$ and accepts any offer equal to or greater than $\bar{g}$. If the vetoer plays the PBE strategy, proposer's belief at the start of any period is b_0. Then, the cut-off point is always $\bar{g}$.

If $\bar{g} > p_H$, the vetoer knows that the probability of a deal is 0 for all periods. The high type vetoer prefers to deviate to $b : g(v_H; b) = \frac{v_H + p_H}{2}$.

Let $a_{H;t}$

offer, proposer's belief then and afterwards is p_L. This is the only case where the proposer's belief changes.

I will check that the proposer's belief is consistent with a PBE. If the proposer believes p_L since he believes that the vetoer's type is low type with probability 1. His belief can always remain the same in the future.

Suppose $y_t = p_H$

Finally, in period t, for a given y_t, de nition 4's (iv) is optimal. This is because by de nition 4's (i), vetoer's acceptance or rejection does not affect how the proposer will play in the future.

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