DISCUSSION PAPERS IN ECONOMICS

Picking Winners: technology-specific policies can be welfare improving

By Sean Ericson

I show that the commonly held belief that policy should not pick winners is not always valid. Picking winners can increase social welfare above the decentralized equilibrium even when the policymaker has no exclusive knowledge of which technologies are most viable, and even when the market has private information unavailable to the policymaker. Innovation requires the use of scarce resources to bring a new product to market and improve the quality of existing products. Product redundancy, where the improvement of a product's value comes partly at the expense of substitute products, reduces the incremental value of additional products. When the number of products in the market is endogenous, there exists a tension between the benefits of developing a larger suite of technologies and the benefits of allocating more innovative resources towards each technology developed. Product redundancy in conjunction with product innovation can lead to the market developing more products than is socially optimal. A policy which selects a subset of technology options to support{picking winners{can increase social welfare. The results of this paper contribute to the ongoing discussion of industrial policy and are of particular importance for policies aimed at mitigating climate change.

A commonly held belief is that policy should not \pick winners" but should instead allow market forces to decide how resources are allocated (Aghion et al., 2011, 2015; Nathan and Overman, 2013; Rosenberg, 1998; Schultze, 1983). The aversion to picking winners is used as an argument against targeted research

(Rosenberg, 1998), enforced standardization, or industrial policy (Aghion et al., 2011). It is assumed that \targeted subsidies to speci c types of research" leads to wastage \because of the ine ciency of picking winners" (Acemoglu et al., 2016). Even when a class of technologies is socially preferred, such as with clean versus dirty technologies, the assumed optimal policy supports a broad sector and is \not biased towards individual rms within the sector" (Aghion et al., 2011).

I show that picking winners can increase social welfare above the decentralized equilibrium even when the policymaker has no exclusive knowledge of which technologies are most viable, and even when the market has private information unavailable to the policymaker. A social planner balances the bene ts of developing a larger suite of technologies with the bene ts of allocating more scarce resources towards each technology developed. New technologies produce new products, and the value of introducing a new product partially comes at the expense of lower utilization of substitute products. Because substitute products are partially redundant, the social value of a new product is less than the private value generated by that product. Product redundancy combined with innovation in new technologies results in the decentralized equilibrium developing more technologies, with less resources devoted to innovating each technology developed, than is socially optimal. A policy which supports a subset of technology options{picking winners{can increase sprb(iable,)-347(a4d6rs{pic)28u).

the two can result in technology-speci c policies being welfare improving.

The negative view of picking winners largely stems from the fact that policy-makers often do not have the information needed to determine which potential innovations are most viable (Nathan and Overman, 2013;

Choosing the \right" number of options to pursue is especially important when allocating resources towards research and development. Dasgupta and Maskin (1987) list how many and what kinds of options to pursue as among the major questions in the economics of science and technology.[1] For many important problems there is no technology which currently o ers an adequate solution, but instead there are an array of technologies which, with additional research and development, could contribute to the solution. Early stages of development are marked by a plethora of competing options, large uncertainty over the relative merits of each option, and additional resources required to develop each option.

The modeling framework of this paper has several connections to the economic growth literature, such as Schumpeterian models with product diversity (Akcigit and Kerr, 2018; Howitt, 1999; Young, 1998), and models of semi-endogenous growth (Jones, 1995; Kortum, 1997; Segerstrom, 1998). The results of this paper connect to the broader literature of innovation and the role of government, and contributes to the ongoing debate on the merits of industrial policy.

Selective government intervention, often referred to as industrial policy, is one of the most contested topics in economics. Critics argue that the government cannot pick winners and may end up picking losers (Klimenko, 2004; Krueger, 2011; Pack and Saggi, 2006; Schultze, 1983). Proponents of industrial policy cite a variety of market imperfections which call for government intervention such as imperfect competition (Aghion et al., 2015), coordination failures between sectors (Greenwald and Stiglitz, 2013; Rodrik, 2004), knowledge spillovers (Lin, 2012; Rodrik, 2014; Stiglitz et al., 2013), information asymmetries (Cohen, 2006; Rodrik,

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[1] The major questions in the economics of science and technology according to Dasgupta and Maskin (1987) are:

> (1) What problems ought to be on the agenda? (2) How many and what kinds of research projects (or research strategies) ought to be pursued in tackling them? (3) How ought resources to be allocated among the chosen research projects? (4) Who ought to be conducting the research? and (5) How ought research personnel to be compensated?

2009), and environmental externalities (Aghion et al., 2011; Rodrik, 2014). But even proponents readily admit the picking winners counter-argument (Greenwald and Stiglitz, 2013; Rodrik, 2004). Those in favor of industrial policy recommend government policies that support sectors with the most positive spillovers and shift support away from sectors with negative spillovers; with papers such as Liu (2019) and Hausmann et al. (2008) suggesting methods for how sectors could be ranked.

This paper contributes to the literature by showing the targeting of speci c rms, technologies, or sectors can increase welfare when there is both innovation to develop new products and product redundancy from substitute products. This suggests a policy of picking winners in markets where products are close substitutes and signi cant innovation in new products is present. Additionally, I discuss how policy intervention in such markets can be welfare improving even when decentralized actors have private information unavailable to the policymaker.

The results of this paper are of particular importance for policies aimed at mitigating climate change. Innovation in clean technologies has a large impact on the total damages caused by climate change (Acemoglu et al., 2012, 2016; Barrett, 2006; Goulder and Mathai, 2010). A large suite of potential technologies can lower emissions, but there are limited resources which can be devoted towards the research and development of these clean technologies (Pless et al., 2020). Finding the corre7r6ovatwhenofis an technol537miool537mt supp andFinable th1(a)2o324((P)1(t334(the3(s3e

the market has private information unavailable to the planner. I show that it still may be optimal for the planner to pick winners, and that a policy instrument such as a tax or quota on the number of technologies developed can be welfare improving. Section

the cost of production or increase the quality of the associated product. More scientists lead to more innovation, so $v(i; n; s_n(i))$ is increasing in $s_n(i)$. However, there are diminishing returns in the productivity of scientists. This is due to duplicate discoveries (Hill and Stein, 2019; Merton, 1961, 1968; Stephan, 1996), approaches to innovation being substitutes for each other (Bloom et al., 2013), and the fact that innovation expands the technological frontier making new discoveries harder to nd (Bloom et al., 2017; Kortum, 1997; Porter and Stern, 2000), which results in diminishing returns to research e ort at any given time and across time (Fischer and Newell, 2008; Popp, 2004).

The value of each product is decreasing in the number of competing products, $v(i; n; s_i)$ is decreasing in n. A new product increases welfare more when there are few competing products than when there are a plethora of competing products. A new medicine that treats a previously untreatable disease bene ts society more

bered technologies before higher numbered technologies. Mathematically, I assume $v(n; i; s) \geq v(n; j; s)$ for all $i < j$. This ordering captures ex-ante knowledge

are due to differences in the number of technologies developed.

Due to free entry, firms will enter until the next entrant makes zero profits. Hence, as shown in Appendix **??**, in the decentralized equilibrium $\hat{n}$ technologies are developed, where $\hat{n}$ is such that the following zero profit condition holds:

$$(6) \qquad \underbrace{v(\hat{n};\hat{n};s_{\hat{n}}(\hat{n}))}_{\text{Product Value}} = \underbrace{F + \hat{n}s_{\hat{n}}(\hat{n})}_{\text{Development Costs}}$$

C. Comparison of Social Planner Solution to Decentralized Equilibrium

The motivation to innovate in novel projects is well known to any researcher who strains to find "gaps in the literature", and the reality of product redundancy is even better understood by all who have gone to graduate school and inevitably have nightmares about discovering their research is redundant. Comparing equations (5) and (6), the social planner balances the benefits of increased product variety against both the cost of developing the new technology and the opportunity cost of making other innovations redundant. Individual firms do not internalize the opportunity cost of making other products redundant in their decision process. Therefore, the decentralized equilibrium innovates in at least as many technologies as the social planner.

If product redundancy is non-zero and the optimal developed is less than N, then the number of technologies developed in the decentralized equilibrium is greater than in the planner's solution. Additionally, Since $s_n(i)$ is decreasing in n, $s_n(i) \geq s_{\hat{n}}(i) \ \forall i \in [0;n]$.

Proposition 1. *The decentralized equilibrium develops more technologies than the social planner and the number of scientists per technology in the decentralized equilibrium is less than the social planner solution.*

Proposition 1 is a direct result of the opportunity cost of innovation lost due

to product redundancy appearing in the planner solution (5) but not in the decentralized equilibrium (6). The result that more technologies are developed in the decentralized equilibrium than in the planner's solution leads directly to our next proposition, which is a central result of this paper.

Proposition 2.   Picking winners{developing less technologies than would be developed in the decentralized equilibrium{can increase social welfare.

From Proposition 1 the number of technologies developed in the social planner solution is less than the number developed in the decentralized equilibrium. Because welfare is maximized by the social planner and the decentralized equilibrium is di erent from the social planner solution, welfare in the decentralized equilibrium is less than the social planner solution. Furthermore, the planner's solution can be met by supporting a subset of available technologies.

Proposition 1 holds even though the policymaker does not have exclusive knowledge of which technologies are best to support. This result highlights an additional externality caused by product redundancy when in conjunction with innovation, which results in the decentralized equilibrium di ering from the planner's solution.

It is important to reiterate that these results occur when substitute products

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II. Private Information

Limited knowledge from the policymaker's point of view is often brought up as to why policymakers should not pick winners. Knowledge of the viability of various technologies may be private knowledge held by individual rms or scientists. As Hayek (1945) states:

> practically every individual has some advantage over all others because he possesses unique information of which bene cial use might be made, but of which use can be made only if the decisions depending on it are left to him or are made with his active cooperation.

If decentralized actors have better information than centralized planners then the market allocation of resources may be more e cient than the centralized

Let technologies be are ordered such that $(i) > (j)$ whenever i is less than j. Thus technology i is expected to be more viable than technology j whenever $i < j$. Because the planner chooses technologies and allocates scientists before uncertainty is revealed, the planner's problems is equivalent to that of Section I.A with the indicator i replaced by (i).
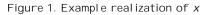
The planner develops n technologies, with n being determined implicitly by:

$$(8)\qquad v((n); n; s_n(n)) = F + s_n(n)\,_n \int_0^n v_n((i); n; s_n(i))\,di$$

$$v_s((i); n; s_n(i)) = \,_n \quad \forall i \in [0; n]$$

Decentralized rms have better information on which technologies are most promising and therefore observe each $x(i)$ before entering. While technology i is expected to be more promising than technology j whenever $i < j$, the order may be reversed once $x(i)$ and $x(j)$ are realized. Therefore, even if the planner and the market develop the same number of technologies, the technologies chosen will be di erent under the two solutions.

Let be a function which orders the realizations of x so that $(i; x)$ is the i'th highest x draw. Let $(i; x)$ converge to (i) and let $_m(i)$ denote the number of scientists allocated to the i^0th most viable technology given m technologies are developed.

Figure 1 displays an example with $x(i)$ normally distributed around (i) and (i) decreasing linearly with n. The gray points denote x draws of. The line is the distribution given the x draws are reordered, which is what the decentralized market makes decisions based on.

Figure 1. Example realization of *x*

The number of technologies developed in the decentralized equilibrium is implicitly given by:

(9)

$$v(\ (i); m;\ _m(m)) = F +\ _m(m)\ _m(x)$$

$$v_s(\ (i); m;\ _m(i)) =\ _m(x)\ \ 8i\ 2\ [0; m]$$

The derivation follows the same steps as in Appendix B. We cannot say whether the decentralized equilibrium develops more or less technologies than the planner. If one technology is far superior to the others then the market will only develop the superior technology while the ignorant planner has to develop several technologies to guarantee the superior technology is among them. Thus, the market can

III. Discussion and Applications

It may be difficult to envision what a tax or quota on innovative activities would look like, or why such a policy would be useful. Those familiar with the innovation literature may find it strange to even consider a tax on innovation. Innovation has clear positive spillovers which rightfully should be subsidized.

The results of Propositions 2 through 4 can best be understood in the context of a decision maker endowed with a limited budget to devote to innovative activities. Examples include the National Science Foundation allocating grants to a portfolio of research projects, the Department of Energy choosing which energy technologies to support, or the Department of Health and Human Services allocating funds to speed up the development of a vaccine. The decision maker can direct how funds are allocated or can allow market forces to decide, such as by offering subsidized loans. Private information held by decentralized actors may mean the market can more efficiently allocate resources than the planner could. Because of product redundancy however, the optimal allocation likely consists of supporting a smaller number of projects with more funding than would result from relying on market forces alone. In this case, policy instruments may be effective at leveraging market information while also accounting for project redundancy. In the context of a decision maker choosing how to allocate funds, a quota can be seen as the decision maker offering a limited number of grants, and a tax can be seen as any policy which raises the barriers to apply to or receive a grant.

The result that the market may develop more technologies than is socially optimal suggests policies which focus on a few technology options may be preferable to policies which provide an even playing field to all technologies. The NSF may be most effective by giving a smaller number of larger grants instead of a larger number of smaller grants. Instead of the \all of the above" policy the DOE

currently promotes, which funds a wide array of potential energy innovations, a policy which targets research funding to a smaller number of technologies could bring about more impactful innovations; a result which is especially important given innovation in the energy sector is essential for mitigating climate change. Directing more funding, research, and clinical trial participants to a smaller number of vaccine candidates could reduce the time for an effective vaccine to reach the market.

The optimal portfolio size depends on the amount of product redundancy and the concavity of returns to scientific effort. High product redundancy and low concavity (slowly low decreasing returns) of scientific effort recommends a small portfolio, while low product redundancy and high concavity (quickly decreasing returns) of scientific effort recommends a large portfolio. Therefore, picking winners is most likely to increase welfare in the case of markets with high levels product redundancy. This insight leads to different policy recommendations for seemingly similar problems.

One example of where the results of this paper imply different policy recommendations to seemingly similar problems is with regards to policy options to reduce car emissions. Emissions from transportation is the leading contributor of greenhouse gas emissions,[3] so reducing the carbon intensity of cars is a critical step in mitigating climate change. The two means of reducing emissions from cars are to increase the fuel efficiency of gas cars and to switch to an alternative fuel source{namely electric batteries or hydrogen fuel cells.

Battery electric cars require a significant build-out of charging stations along

network, and require innovation to reduce costs and improve performance. A car will only use a single power source, which means that there is significant product-level redundancy between these options. A fleet of fuel cell cars would largely come at the expense of a smaller fleet of battery-electric cars.

Innovations to increase the gas mileage of cars, such as better designed engines or the use of lighter composite materials, can be incorporated into the same car and therefore largely do not conflict, and can in fact be complementary innovations. Innovations to increase fuel efficiency therefore likely have low product-level redundancy.

The low level of product redundancy between efficiency improvements suggests a policy which supports a broad range of options. Such policies may include a gas tax or fuel efficiency standards, which benefit all means of improving efficiency equally. Meanwhile, high level of redundancy between battery-electric, and hydrogen powered cars implies a policy of picking a single winner to devote research funds towards may actually be preferred to a policy of supporting each option equally.

A. Application to Winner-takes-all Market

Innovation is often modeled as a winner-takes-all market due to rights of first discovery, such as patents or recognition from publication, which lead the bulk of compensation for innovation to often go to the first to innovate (Dasgupta and Maskin, 1987; Hill and Stein, 2019; Merton, 1961; Stephan, 1996). Additionally,

market share. This framework also incorporates markets where the ﬁrst to in-
novate captures the market by translating the level of innovation to speed of
innovation. With perfect substitutes the best product is used, the ﬁrst to make
a scientiﬁc discovery get the bulk of credit, the ﬁrst vaccine to complete clini-
cal trials is likely the one distributed. Ex-ante uncertainty regarding the ex-post
level of innovation can lead to multiple technologies being developed however.
Because the best choice is not known ahead of time, developing several technolo-
gies increases the pool of technologies to draw from. Competing technologies are
therefore not fully redundant even though only one will be used in the market.
At the same time, product redundancy is clearly baked into a winner-takes-all
market because the chance of a new product being the one developed decreases
as competing products are added.

Let the level of innovation in technology i be given by the random variable
$X(i; s_n(i)) \sim F_i(x; s)$ where $s_n(i)$ is the number of scientists devoted towards
technology i. Again, the level of innovation may stand for the speed of discovery
with a larger innovation denoting a quicker discovery. I assume $F(x; s)$ ﬁrst-
order stochastically dominates $F(x; t)$ whenever $s > t$ and $F_i(x; s)$ dominates
$F_j(x; s)$ whenever $i < j$. This ensures research increases expected innovation
and establishes earlier numbered technologies to be at least as viable as later
numbered technologies. I assume stel-457(ad [(9Jorst-)].849 0dom)-4 < j

the expected level of innovation given that technology is the most innovative:

(13)

$$v(i; n; s_n(i)) =$$

$$E[X(i; s_n(i)) j X(i; s_n(i)) \quad X(j; s_n(j)) 8j \quad n] P[X(i; s_n(i)) \quad X(j; s_n(j)) 8j \quad n]$$

We have mapped the framework for a winner-takes-all market to a discrete version

Table 1| Social and private value for winner-takes-all market

n	= 1=2		= 2=5		= 1=3	
	Social Value	Private Value	Social Value	Private Value	Social Value	Private Value
1	0.960	0.960	0.960	0.960	0.960	0.960
2	<u>0.981</u>	0.490	1.057	0.528	1.111	0.555
3	0.938	0.313	<u>1.061</u>	0.354	1.151	0.384
4	0.882	0.220	1.037	0.259	<u>1.152</u>	0.288
5	0.821	0.164	0.999	0.200	1.135	0.227
6	0.760	0.127	0.956	0.159	1.108	0.185
7	0.700	0.100	0.911	0.130	1.075	0.154
8	0.641	0.080	0.863	0.108	1.039	0.130
9	0.583	0.065	0.815	0.091	1.000	0.111
10	0.526	**0.053**	0.766	**0.077**	0.960	**0.096**

to develop a subset of technologies even though it is unknown which technologies will be ex-post preferred. Using the words of Rosenberg (1998) quoted in the introduction, the \virtue of the marketplace" that \in the face of huge ex-ante uncertainties concerning the uses of new technological capabilities, it encourages exploration along a wide variety of alternative paths" in some cases may turn out to be a vice of the marketplace by encouraging exploration along too many alternative paths with insu cient support to any path explored.

IV. Conclusions

Common wisdom goes that government does not have the knowledge required to pick winners, and therefore should leave such decisions up to the market. I have shown that picking winners can increase social welfare above the decentralized equilibrium. The optimal policy balances the bene ts of developing a larger suite of technologies with the bene ts of allocating more scarce resources towards each technology developed. The decentralized equilibrium can result in more technologies being developed with less resources devoted to supporting each technology developed than is socially optimal. Technology-speci c policies{picking winners{

are most likely to be welfare improving in markets with both innovation in new products and where products are close substitutes so that product redundancy is high.

When decentralized agents have knowledge unavailable to the policymaker, such knowledge should be leveraged to determing which technologies to develop. However, the tendency of the market to develop more products than is optimal is still present. Policy instruments which leverage market information while incentivizing decentralized agents to account for product redundancy may be e ective. Additional research may be warranted to determine preferred set of policies to implement.

I have not argued that that technology-speci c policies are always welfare improving. Beyond the costs and challenges to implementing such policies, concerns

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Therefore $n \int_0^n \frac{\partial s_n(i)}{\partial n} di = s_n n$, which gives us:

$$(17) \qquad v(n\,;n\,;s_n(n)) = F + n\, s_n(n) \int_0^n v_n(i;n\,;s_n(i))\,di$$

Which is equation (5).

APPENDIX B   Derivation of Competitive Equilibrium

Let w denote the wage rate for scientists. given n technologies are in the market, rm i maximizes pro ts as:

$$(18) \qquad (i;n;s_n(i)) = v(i;n;s_n(i)) \quad F \quad w s_n(i)$$
$$=) \quad v_s(i;n;s_n(i)) = w$$

The market for scientists then determines the wage rate such that:

$$(19) \qquad \int_0^n s_n(i)\,di = 1$$

From equation (18), we have that $v_s(i;n;s_n(i)) = v_s(0;n;s_n(0))$ for i

the wage rate into the pro t function gives:

$$(i; n; s_n(i)) = v(i; n; s_n(i))($$

The difference between welfare in the competitive equilibrium and in the planner's solution can be written as follows:

$$\int_0^{Z_m} v(\theta(i); m; \sigma_m(i))\, di - mF - \int_0^{Z_n} v(\theta(i); n; s_n(i))\, di - nF$$

$$= [A - mF] - [B - nF]$$

$$= [A - mF] + [C - C] + [D - D] - [B - nF]$$

$$= [D - B] + [C - D - F(m - n)] - [C - A]$$

(25)

$$= \int_0^{Z_n} v(\theta(i); n; \sigma_n(i)) - v(\theta(i); n; s_n(i))\, di$$

$$+ \int_0^{Z_m} v(\theta(i); n; \sigma_m(i))\, di - \int_0^{Z_n} v(\theta(i); n; \sigma_n(i)) - F(m - n)$$

$$- \int_0^{Z_m} v(\theta(i); n; \sigma_m(i)) - v(\theta(i); m; \sigma_m(i))\, di$$

Which is equation (10)