RESEARCH ARTICLE | AUGUST 25 2017

Uncovering low dimensional macroscopic chaotic dynamics of large finite size complex systems

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Chaos 27, 083121 (2017)
https://doi.org/10.1063/1.4986957

out noise in the original dataset. The reconstructed low dimensional dynamics can then be used to accurately calculate important dynamical invariants such as fractal dimensions and Lyapunov exponents. As an illustrative example, we apply this procedure to a system of globally coupled Landau-Stuart oscillators that appears to exhibit macroscopic chaos with substantial superposed noise-like finite-size fluctuations.

The remainder of this paper is organized as follows. In Sec. II, we describe the system of Landau-Stuart oscillators and summarize its dynamics and observed finite-size fluctuations. In Sec. III, we describe our method for reconstructing the low dimensional dynamics. In Sec. IV, we demonstrate our method's utility in calculating invariants of the macroscopic dynamics of the Landau-Stuart system. In Sec. V, we conclude with a discussion of our results.

II. SYSTEM DYNAMICS

In this paper, we use as our primary example a system of globally-coupled Landau-Stuart oscillators whose dynamics are governed by

$$\dot{} = 1 - |\quad|^2 +$$

section resulting in the ($\ -1$)-dimensional state vector $\tilde{\boldsymbol{x}}$, whose components we denote as $\tilde{\boldsymbol{x}} = [\tilde{}^{(1)}; \ldots; \tilde{}^{(\ \ -1)}]$. We will from here onwards denote the original (noisy) and reconstructed (denoised) state variables as $\tilde{\boldsymbol{x}}$ and $\boldsymbol{x}$, respectively. The surface of section for the noisy data defines a noisy mapping of the form

$$\tilde{\boldsymbol{x}}_{\ +1} \approx \boldsymbol{F}(\tilde{\boldsymbol{x}}\ \ ); \tag{2}$$

where $\tilde{\boldsymbol{x}}$ represents the th piercing of the surface of sections by the noisy data and $\boldsymbol{F} : \mathsf{R}^{\ \ -1} \rightarrow \mathsf{R}^{\ \ -1}$ is an unknown mapping function which we assume is continuously differentiable and encodes the hypothesized low

$$\lambda = \lim_{\to\infty} \frac{1}{} \ln \frac{|\boldsymbol{u}\ \ |}{|\boldsymbol{u}_0|}; \tag{5}$$

where $\boldsymbol{u}$

as ours is, this reduces to $= 1 + \lambda_1 = |\lambda_2|$. Given in Table I, this is our first fractal dimension for the strange attractor.

We next consider two other fractal dimensions, the information and correlation dimensions.[36,37] Formally, these dimensions are measured by partitioning the domain of the attractor into $(\square)$ cubes each of unit size $\square$ and calculating the fraction of time μ spent by typical orbits in each box . The information and correlation dimensions are defined, respectively, by

$$_1 = \lim_{\square \to 0^+} \frac{\sum_{=1}^{(\square)} \mu \ln \mu}{\ln \square} \quad \text{and} \quad _2 = \lim_{\square \to 0^+} \frac{\sum_{=1}^{(\square)} \mu^2}{\ln \square} : \qquad (7)$$

We proceed by calculating these quantities using the methods outlined in Refs. 38 and 39 with 10^5 points generated by the reconstructed dynamics and report them in Table I.

Finally, we comment briefly on the Kaplan-Yorke conjecture.[35] The Kaplan-Yorke conjecture states that for typical systems (i.e., systems that are not pathologically engineered), the information dimension is equal to the Lyapunov dimension, $_1 =$

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