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Transport through chaos

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Abstract. Certain orbits of area preserving maps of the plane appear to wander [illegible]

determine how ensembles of points are transported. The action principle of MacKay, Meiss and Percival [4] can be used to compute areas of pieces of the grid. Thus knowledge of trellis geometry together with area computations will form the

[redacted]

Rearranging the sum gives

$$\int_D dp \wedge dq = \sum_{j=-\infty}^{\infty} \alpha^j [F(b_j) - F(a_j)].$$

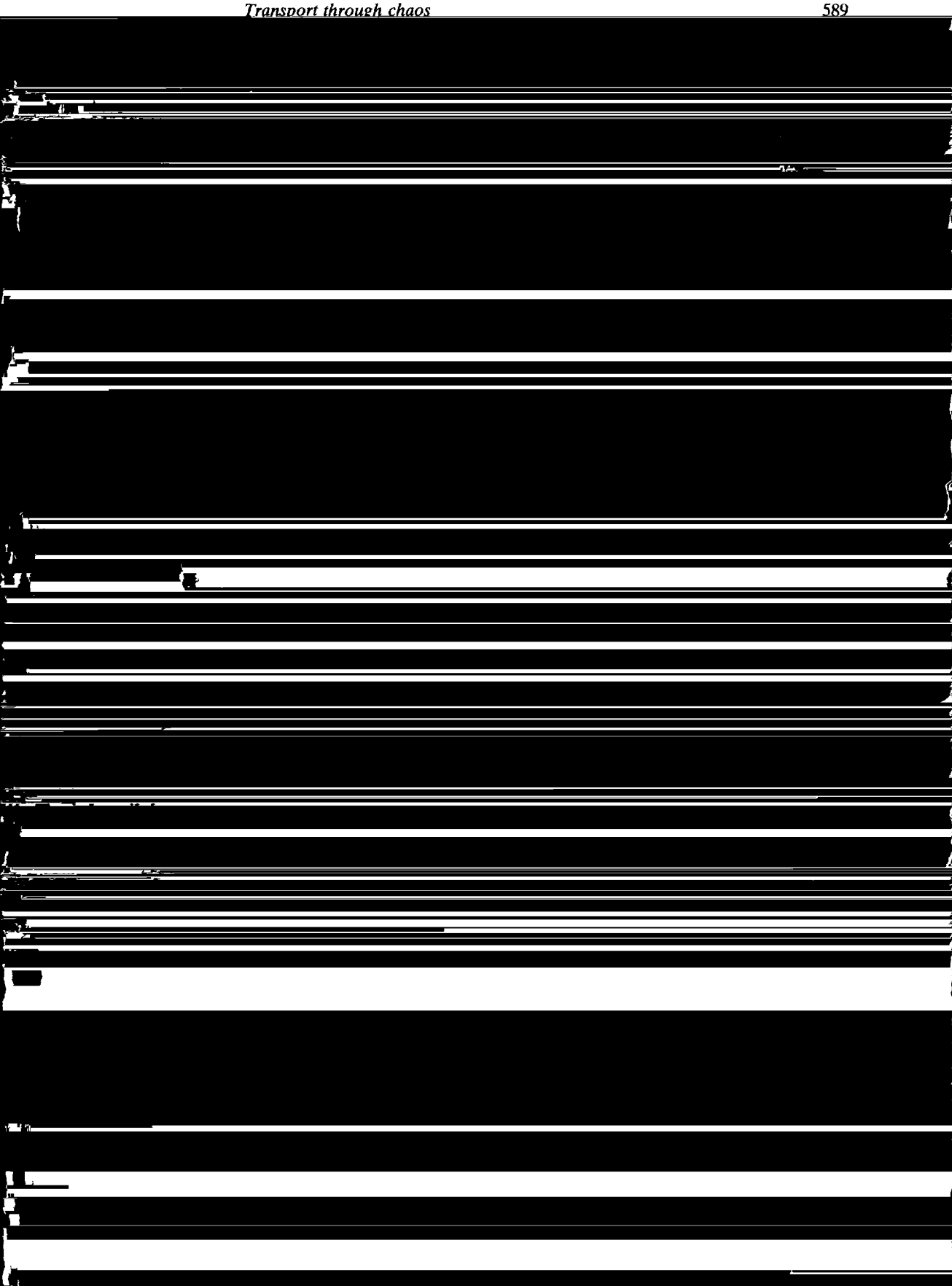
In general suppose that D is a disc bounded by alternating segments of stable and unstable manifold. Suppose that the endpoints of these segments are indexed $a^0, a^1, \ldots, a^{2m}$ (with $a^{2m} = a_0$) in a counterclockwise order around the boundary of D. Suppose that the segment joining a^0 and a^1 is contained in a stable manifold. Then by the preceding argument

$$\int_D dp \wedge dq = \sum_{j=-\infty}^{\infty} \alpha^j \sum_{k=0}^{m-1} [F(a_j^{2k+1}) - F(a_j^{2k})]. \tag{2.4}$$

This formula expresses the MacKay–Meiss–Percival action principle

Proposition. Discontinuity points of t^+ occur on R-stable manifolds. Similarly, discontinuity points of t^- occur on R-unstable manifolds. Hence the internal trellis of the resonance zone partitions the zone into its exit time decomposition.

Proof. For simplicity the proof will be given for the resonance zone pictured in



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