





The variance of the nondimensional field is  $\langle \delta x^2(t) \rangle = \frac{2}{T_0}$  with

$$\frac{2}{T_0} = \frac{T}{T_0}, \quad T_0 = \frac{\mu_0 M_s^2 V}{2 k_B}, \quad (5)$$

where  $k_B$  is the Boltzmann constant,  $V = \frac{4}{3} \pi r^3$  is the characteristic micromagnetic volume, and  $T_0$  is the nondimensional scaling of the absolute temperature. Table I includes

$$\dot{u} = v + \dots, \tag{11b}$$

$$\dot{u} = \left( \frac{\partial}{\partial t} \right), \tag{11c}$$

$$\dot{u} = v, \tag{11d}$$

where

$$= \frac{1}{2} \operatorname{sech}^2 \left( \frac{1}{2} \right), \tag{12a}$$

$$= h_0 + \frac{1}{2} \tanh \left( \frac{1}{2} + 1 \right), \tag{12b}$$

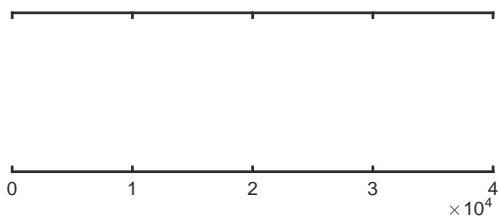
$$v = 2 + \dots \tag{12c}$$

It is necessary to carefully choose parameters so that this fixed point is stable, i.e., so that all eigenvalues in Eq. 4 are negative. The condition  $h_0 > \dots$  is sufficient for  $\dots < 0$ , but in order to ensure that  $\dots < 0$ , we require additionally that

$$(2 + h_0) > \frac{1}{2} \tanh \left( \frac{1}{2} + 1 \right). \tag{13}$$

Note that the inequality requirement for stability in Eq. 13 was not identified previously [1], and is essential to understanding the dynamics of the droplet. It is possible to visualize the region of linear stability in the  $(h_0, \dots)$  plane as in Fig. 2. The left







plitude dynamics, respectively, of spatially uniform STOs.  
For the linearized system, the resulting generation linewidth is linearly dependent on temperature, whereas the nonlinear system exhibits a linewidth enhancement when approaching room temperature, reflecting the coupling between the droplet's constituent variables. Full-scale micromagnetic simulation, including the fully nonlinear spatial variation of the system, qualitatively agree with the numerical results. However, we do not observe convergence toward the linear theory at low temperatures using a standard micromagnetic package [19]. This suggests the study of droplet generation linewidth as a test problem for stochastic micromagnetic codes [20].

The analytical and numerical linewidths obtained are two orders of magnitude below the typical linewidths observed in experiments. This disagreement may be caused by the small NC radii used experimentally, the existence of nonlocal dipolar and current-induced Oersted fields, and the aforementioned drift instabilities for data-acquisition timescales. In fact, micromagnetic simulations performed with a radius similar to those experimentally fabricated to date return linewidths in the same order of magnitude when both nonlocal and current-induced Oersted fields are included. The relevance of such fields in the generation linewidth motivates their inclusion in the analytical theory. For thin PLMs, the effect of nonlocal dipole fields on deterministic droplet dynamics has been shown



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