Program in Applied Mathematics
PROBABILITY AND STATISTICS PRELIMINARY EXAMINATION
January 11, 2019

Instructions: Do four of the following six problems. Place an X on the lines next to the problem numbers that you are NOT submitting for grading. Please do not write your name anywhere on this exam. You will be identi ed only by your student number. Write this number on each page submitted for grading. Show all relevant work.	1. ____ 2. ____ 3. ____ 4. ____ 5. ____ 6. ____ Total ____

Student Number ________________________

1. Let $N(t); t$ 0, be a Poisson process with rate that is independent of the sequence of iid random variables $X_1; X_2; :::$ with density $f(x)$.

   (a) Find the moment generating function of the compound Poisson process

   $$Y(t) = \sum_{i=1}^{N(t)} X_i :$$

   (b) Find the mean and variance of $Y(t)$.

2. Let $N(t); t$ 0, be a Poisson process with rate.

   (a) Find the covariance function for N. That is, nd Cov($N(t_1); N(t_2)$) for arbitrary 0 t_1 $t_2 <$ 1 .

   (b) Let A_1 be the rst arrival time of $N(t)$. Show that the conditional distribution of A_1 given $N(t_0) = 1$ is $U(0; t_0)$.

   (c) Show that $N(t)$ obeys the Markov property in that

   E f (N(t)) N(u) for all 0 u s = E f (N(t)J/F17 7.9701 Tf 4.227 -1.793 Td [

4. Suppose that $X_1; X_2; :::; X_n$ is a random sample from the uniform(0) distribution where 1 2.

   (a) Find the maximum likelihood estimator (MLE) for based on $X_1; X_2; :::; X_n$.

   (b) Is this estimator for unbiased? Justify your answer.

   (c) Now suppose that we do not get to observe $X_1; X_2; :::; X_n$ but instead we observe $Y_1; Y_2; :::; Y_n$ where

   $$Y_i = \begin{matrix} 1 \; ; & \text{if } 0 \quad X_i \quad 1 \\ 0 \; ; & \text{otherwise} \end{matrix}$$

   Find the MLE for based on $Y_1; Y_2; :::; Y_n$.

   (d) Compute the relative e ciency of these two estimators of .

5. An ecologist studying a particular population of grasshoppers measures the height (in cm) of grasshoppers' initial jump under a replicable set of conditions in the eld. A combination of empirical study and theoretical considerations has caused the ecologist to assume that each of these heights $H_1; :::; H_n$ are independent random samples from a Normal distribution with mean and standard deviation 1 cm. You should assume this too.

   (a) Show under what conditions on $f a_1; :::; a_n g$; where a_i 2 R, that $W = \sum_{i=1}^{n} a_i H_i$ will be an unbiased estimator of .

   (b) Find the unbiased estimator of this form that has minimum variance. What is the variance of this estimator?

   (c) Is this estimator the UMVUE? Show your work for full credit.

   (d) Now consider that the ecologist wants to estimate the probability that a grasshopper from the same population (under the same eld conditions) will jump higher than 10 cm on its initial jump. Find the MLE of this probability. You can express your answer in terms of the c.d.f. of the standard Normal distribution, ( ).

   (e) For her research, the ecologist wants to know if the jumping behavior of the same population of grasshoppers changes during a total solar eclipse. She measured 6 new grasshoppers' initial jumps during the latest eclipse. Recognizing that the eld conditions had changed (the eclipse being gras9gras9gras9gras9gra1o-9r6(ein333(that)-334(the)-332(MLE27(stimaat)-natioin333(