

Piston Dispersive Shock Wave Problem

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two
$$i\varepsilon \partial_t = -\frac{\varepsilon^2}{2} \partial_x^2 + V_0(x, t) + | \cdot |^2, \quad 0 < \varepsilon \ll 1.$$

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$\varepsilon = 0.015$

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$H(\cdot)$

$V_0(x, t) = V_{\max} H(\cdot - \cdot)$

$(\cdot, 0) \rightarrow \sqrt{\rho_R} \cdot p \rightarrow \infty (\cdot, 0) \rightarrow$

$0 \rightarrow -\infty$
 $V_{\max} \gg \rho_R$

$t \leq 0$

$V_{\max} H(\cdot - \cdot)$
 $\rho_R = 0.133$

$$E(\rho) = \sqrt{\rho} \exp\left[\frac{i}{\varepsilon} \int_0^t \rho(t') dt'\right]$$

$$\rho_t + (\rho^2)_t = 0, \quad (1)$$

$$(\rho^2)_t + \left(\rho^2 + \frac{1}{2}\rho^2\right) = \frac{\varepsilon^2}{4} [\rho(\log \rho)]_t - \rho V_0,$$

$$\rho_t + \rho^2 = \frac{\varepsilon^2}{4} \rho(\log \rho)_t - \rho V_0,$$

$$\rho_s^- = \frac{1}{2} p + \sqrt{\rho_R}, \quad \rho_s^+ = \frac{2 \frac{p^2}{2} + 4 p \sqrt{\rho_R} + \rho_R}{p + \sqrt{\rho_R}}. \quad (1)$$

$$\rho_{\min} = \left(\sqrt{\rho_R} - \frac{1}{2} p \right)^2, \quad \rho_{\min} = - p \left(\frac{\sqrt{\rho_R} + \frac{1}{2} p}{\sqrt{\rho_R} - \frac{1}{2} p} \right). \quad (2)$$

$$\rho_{\max} = \rho_L = \left(p/2 + \sqrt{\rho_R} \right)^2 \quad \rho_{\max} = L = p$$

$$E(\sigma) = V = \frac{2\epsilon K(4\rho_R/\frac{2}{p})}{\rho_p} \left(\frac{\sigma}{\rho_p + (\rho_p + 3\sqrt{\rho_R}) \left[\frac{E(4\rho_R/\frac{2}{p})}{(\rho_p - 2\sqrt{\rho_R})K(4\rho_R/\frac{2}{p})} - 1 \right]^{-1}} \right)$$

$$N_{\text{vac}}(t) \approx \left[\frac{\sigma}{l} t \right] = \left[\frac{(\rho_p - 2\sqrt{\rho_R}) \rho_p}{2\epsilon K(4\rho_R/\frac{2}{p})} t \right]$$

$$\left(\rho_p = \sqrt{\rho_R} \right) \quad \left(\rho_p = 2.5\sqrt{\rho_R} \right)$$

$$= \rho_p \quad E(\sigma)$$