

On the finiteness in the deformed Hamiltonian mean-field model

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Abstract: We study the finiteness of the number of islands in the deformed Hamiltonian mean-field model. We show that the number of islands is finite for a large class of deformations. The proof is based on the study of the topology of the phase space and the properties of the Hamiltonian. We also discuss the implications of our results for the stability of the system.

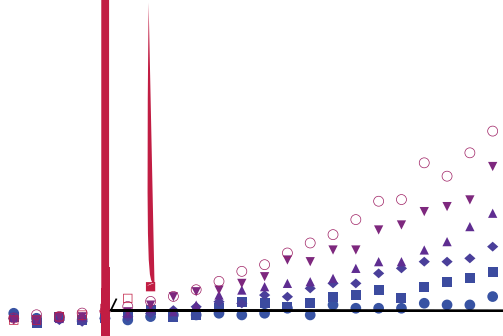
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I. INTRODUCTION

The Hamiltonian mean-field model (HMF) is a paradigmatic model of a many-body system. It consists of a collection of particles interacting with each other through a long-range potential. The model has been extensively studied in the context of statistical mechanics and chaos theory. In this paper, we study the finiteness of the number of islands in the deformed HMF model. We show that the number of islands is finite for a large class of deformations. The proof is based on the study of the topology of the phase space and the properties of the Hamiltonian. We also discuss the implications of our results for the stability of the system.

The evolution of the density is given by the continuity equation



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