







with  $x_j(t) = x_j$  and will show  $\lim_{t \rightarrow \infty} |x_1(t) - x_2(t)| = 0$ . At  $O(\epsilon)$ , we find Eq. (2) with a corresponding linear operator  $Lu = \epsilon u + w[f(U) \cdot u]$ . Solvability is enforced by ensuring the right-hand side of Eq. (2) is orthogonal to the null space  $V$  of the adjoint  $L^*p = \epsilon p + f(U) \cdot w(\epsilon p)$ , yielding the Langevin equation, Eq. (4). The Lyapunov exponent associated with the stability of the absorbing state  $x_1(t) = x_2(t)$  is then approximated by Eq. (14).

To compare our analytical results for traveling waves with numerical simulations, we compute from Eq. (14) when  $f(u) = H(u - x)$ ,  $w(x) = \cos(x - x)$ , and  $C(x) = \cos(x)$ . Stable traveling waves have a profile  $\phi(x) = \cos[\sin(x - x) + a]$ , width  $a = \sin^{-1}[\text{sec}]$  defined by thresholds  $\phi(x_1) = U(x_2) = \text{where } x_1 = x - a \text{ and } x_2 = x$ , and speed  $c = \tan[\text{28}]$ . The null vector can also be computed explicitly:

$$V(x) = \sum_{k=1}^2 (\sin)^k H(x - x_k) + \frac{\coth(x/c) - 1}{2} e^{(kx - x)/c}.$$

Fourier coefficients of  $W(x, t)$  in Eq. (6) are thus given  $b_{\pm 1} = (1 - c)/$

so  $\langle W_\ell(\mathbf{x}) \rangle = 0$ ;  $\langle W_\ell(\mathbf{x}) W_\ell(\mathbf{x}') \rangle = 2C_\ell(\|\mathbf{x} - \mathbf{x}'\|)$  ( $\ell = 1, 2, c$ ) with  $C_\ell(r) = \int_0^\infty a \cos(r\omega)$ . The degree of correlation between layers is controlled by the parameter  $\mu$ .

Our analysis proceeds by considering stationary bumps in a network with even symmetric connectivity [ $\tilde{c}(\mathbf{x}) = \tilde{c}(-\mathbf{x})$ ]. As in the main text, we characterize stochastic bump motion by applying the ansatz  $\varphi_\ell(\mathbf{x}) = U(\|\mathbf{x} - \Delta_\ell(\mathbf{x})\|) + \varepsilon \Phi_\ell(\|\mathbf{x} - \Delta_\ell(\mathbf{x})\|) + O(\varepsilon^2)$ , and  $\Delta_\ell(0) = \mathbf{0}$ . Plugging this ansatz into Eq. (A1), expanding to  $O(\varepsilon)$ , and applying a solvability condition, we find that each  $\Delta_\ell$  ( $\ell = 1, 2$ ) obeys the Langevin equation

$$\Delta_\ell = \varepsilon, \frac{-\nabla V(\Delta_\ell)}{V''(\Delta_\ell)}$$

