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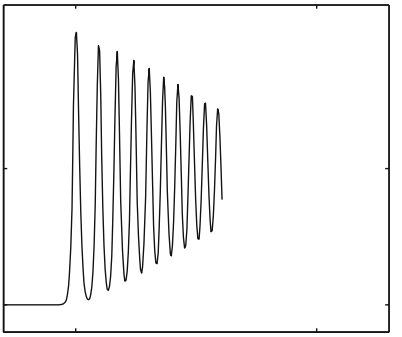


3 **(dispersive operator)** $D(\rho, u)$ D (2.2)

$$\omega \quad u_0 k \pm \omega_0(k, \rho_0), \tag{2.3}$$

ρ_0, u u_0 ρ $(kx \quad \omega t)$. $\pm$ (2.3) $\omega_0(k, \rho_0) \geq 0, \quad k \geq 0, \rho_0 \geq 0.$

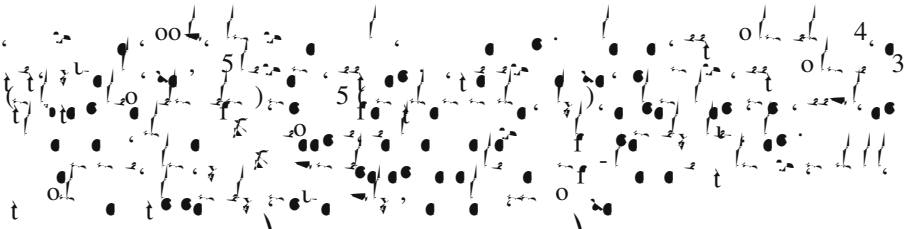
$$\omega_0(k, \rho_0) \quad c_0 k \quad \mu k^3 \quad o(k^3), \qquad k \to 0, \qquad \mu \quad 0. \tag{2.4}$$

$\omega_0''(k \ \rho_0), \quad k > 0,$ (2.4) ω_0 $k,$

$$\frac{\omega_0(k, \rho_0)}{k} \qquad \frac{\partial^2 \omega_0}{\partial k^2}(k, \rho_0).$$

$k,$

4 **(Whitham averaging)** (2.1)



$$u(x,0) = \begin{cases} u_1 & x < 0 \\ u_2 & x > 0, \end{cases} \qquad \rho(x,0) = \begin{cases} \rho_1 & x < 0 \\ \rho_2 & x > 0, \end{cases} \tag{2.5}$$

u_j R, ρ

f

$$u \quad V \quad \frac{A}{\rho}, \tag{3}$$

$$\rho^{2} \quad \int_{\rho_1}^{\rho} f(\rho)\,\rho \quad B\rho^{2} \quad C\rho \quad \frac{A^{2}}{2} \equiv G(\rho). \tag{3}$$

G $\rho_1 \le \rho_2 \le \rho_3$ $A, B,$ C ρ_2 $\rho_1,$ $V.$ (1

. 1, ,)

$$\mathcal{E} \equiv \frac{\rho u^{2}}{2} \quad \frac{\rho_x^{2}}{\rho} \quad \int_{0}^{\rho} f(\rho)\,\rho,$$

$$\mathcal{E}_t \quad u\,\mathcal{E} \quad P(\rho)_x \quad \frac{1}{4}\,u\rho_{xx} \quad \frac{(\rho u)_x \rho_x}{\rho}_x,$$

4, 6.

$$u_0 \equiv \lim_{\xi\to\infty} u(\xi), \qquad \rho_0 \equiv \lim_{x\to\infty} \rho(\xi), \qquad \rho \equiv \lim_{\xi \mathrm{R}} \rho(\xi).$$

(3) G ρ_1 ρ , ρ_2 ρ_3 $\rho_0,$ V

$$(s \quad u_0)^{2} \quad \frac{2\rho}{(\rho_0 \quad \rho)^{2}} \left((\rho_0 \quad \rho)\, f(\rho_0) \quad \int_{\rho}^{\rho_0} f(\rho)\,\rho \right). \tag{3.10}$$

(3).

(. 1, , 0 , 1, , 5), 1 (. 200); (2011 . 2012).

3.2

Two-temperature collisionless plasma:

$\sigma > 0$ ... 1 4 (2010). ... σ 0 ... (2006).

... (3.11) ... σ 0 ... (1, 2) ... (2001). ... (1, 4). ... 10.

4 Background: Dispersionless Limit

... (2.1) ...

$$\rho_t \quad (\rho u)_x \quad 0,$$

$$(\rho u)_t \quad \rho u^2 \quad P(\rho) \Big]_x \quad 0, \tag{4.1}$$

... $D \equiv 0$. ... (4.1) ... $P(\rho)$ (... 1, 5). ... (1, 4)

$$r_1 \quad u \quad \int^{\rho} \frac{c(\rho')}{\rho'} \rho', \qquad r_2 \quad u \quad \int^{\rho} \frac{c(\rho')}{\rho'} \rho', \tag{4.2}$$

$$\lambda_1 \quad u \quad c(\rho), \qquad \lambda_2 \quad u \quad c(\rho), \tag{4.3}$$

$$\frac{\partial r_j}{\partial t} \quad \lambda_j \frac{\partial r_j}{\partial x} \quad 0, \qquad j \quad 1, 2. \tag{4.4}$$

$$g(\rho) \quad \int^{\rho} \frac{c(\rho')}{\rho'} \rho',$$

... (4.3) ...

$$u \quad \frac{1}{2} \quad 1 \quad r_2), \qquad \rho \quad g^{1} \quad \frac{1}{2} \quad r_1) \, . \tag{4.5}$$

(4.1)

4.1

$\rho > 0$ (4.) o $a > 0$ z

$a(t)$

$$A \quad a, \qquad B \quad a, \tag{4, }$$

R_A R_B m t o

x_0 $z(0)$ $\frac{\partial u}{\partial x} \quad \frac{c(\rho)}{\rho}\frac{\partial \rho}{\partial x} \quad \frac{c(\rho)}{\rho}^{1/2}$

$\partial a/\partial r_1 < 0$.

(4.), o x_0 m

(4.10) (4.10). $m > 0$,

$\partial a/\partial r_1 < 0$, R_A R_B (4.12) (4.12), A

B (4). (4.11).

2

(4.10) (4.10),

o R_A R_B $r_2(x_0, 0$

4.3

(4.1)

1 u_1 u_2 $\left[\frac{\rho_2}{\rho_1}\frac{c(\rho)}{\rho}\rho,\right.$ $\rho_1 > \rho_2,$ (4.16)

2 u_1 u_2 $\left[\frac{\rho_2}{\rho_1}\frac{c(\rho)}{\rho}\rho,\right.$ $\rho_2 > \rho_1,$ (4.16)

oo λ_j *increase* (4.1) (1, 4 200) $\mathcal{O}(t^{1/2})$ $\mathcal{O}(t^{2/3})$. $\mathcal{O}(t^{1/3})$ (1, 4), $\mathcal{O}(t)$.

4.4

(1, 3 1, , 4). (1, 5). u_1 u_2 0. (ρ, u). $\rho_1 < \rho_2$, 1- 2- (4.13) ( ) (4.16), $P(\rho)$ $\kappa\rho^\gamma$

1 u $\left[\frac{(\kappa\rho^\gamma \kappa\rho_1^\gamma)(\rho \rho_1)}{\rho \rho_1}\right]^{1/2}$,

2 u $\frac{2(\kappa\gamma)^{1/2}}{\gamma 3}\rho_2^{(\gamma 1)/2} \rho^{(\gamma 1)/2}\Big]$. (4.1)

ρ , u

5 Background: Simple DSWs

(2.1) (2005).

1-

λ_1 (4.1)

2-

λ_2

o (. 200) / (. 2012), (2011).

o o (4.2) (2005). o

$$1 \quad u_2 \quad u_1 \quad \int_{\rho_1}^{\rho_2} \frac{c(\rho)}{\rho}\,\rho, \qquad \rho_2 > \rho_1, \tag{5.1}$$

$$2 \quad u_2 \quad u_1 \quad \int_{\rho_1}^{\rho_2} \frac{c(\rho)}{\rho}\,\rho, \qquad \rho_1 > \rho_2. \tag{5.2}$$

1- (2-) r_2 (r_1) λ_1 (λ_2) (5.1), (5.2) *DSW loci* (4.13), (4.16) (4.16). *inadmissible* o *Temple systems* o (1, 3)

. 4.2 o (5.1), (5.2) (4.13)

5.1

o k $k(\overline{\rho})$, a 0 ( ) o $\overline{u}$ $\overline{u}(\overline{\rho})$

$$\frac{k}{\overline{\rho}} \quad \frac{\omega_{\overline{\rho}}}{\overline{u}(\overline{\rho}) \quad c(\overline{\rho}) \quad \omega_k}, \tag{5.3}$$

(5.1)

$$\overline{u}(\overline{\rho}) \quad u_1 \quad \int_{\rho_1}^{\overline{\rho}} \frac{c(\rho')}{\rho'}\,\rho'. \tag{5.4}$$

$$\omega(k,\bar{\rho}) \quad \omega(k,\bar{\rho}) \quad \bar{u}(\bar{\rho})k \quad \omega_0(k,\bar{\rho})$$
$$\bar{u}(\bar{\rho})k \quad \omega_0(k,\bar{\rho}). \tag{5. }$$

$k \to 0$ $a \to 0.$

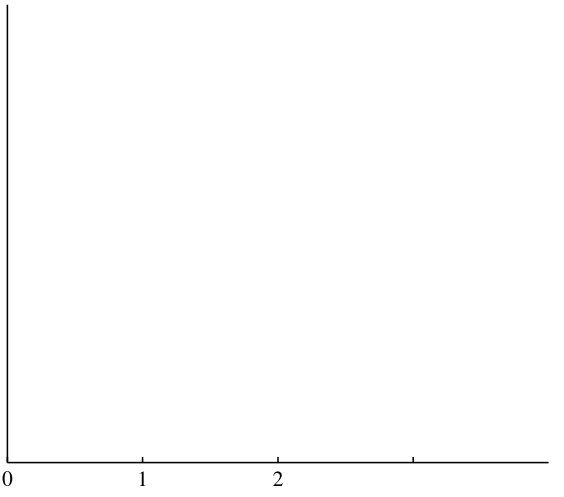
k 0

$$\frac{k}{\bar{\rho}} \quad \frac{c(\bar{\rho})k/\bar{\rho} \quad \omega_{0_{\bar{\rho}}}}{c(\bar{\rho}) \quad \omega_{0_k}}, \tag{5, }$$

(5.3) $k \quad 0.$ (5.6),

$\bar{\rho} \quad \rho_j \quad k(\bar{\rho} \quad \rho_j)$

$k(\rho_j \quad \rho$


0
1
2

1- t (5.1).
o o $(\rho_1, \rho_2, u_1, u_2)$ 1-
t o 1 (5.11) (5.)
t 1
t

$\omega(k, \overline{\rho}, \overline{u})$ [illegible] (2.3).

$\nu_{1,2,3}$ [illegible] $r_{1,2,3}$ [illegible]

$(\nu_1, r$

x/t. o , $k' \to \infty$, $\partial v / \partial \rho_2$ 0 (6.16) (6.16) ρ_2.

f o t oo - f o t (1, 6 2002 t 2006 200 200,). o- o t f t o o - f o (2013). o t o f (1, 4, .15.4) o f t t o o $k \to 0$ o u ω_0/k (5.11),

$$\lim_{k\to 0} \frac{\lambda_2}{\overline{\rho}}_{k\ k_2,\overline{\rho}\ \rho_2,\overline{u}\ u_2} \left(\frac{\partial}{\partial k}\frac{\omega_0}{k}\right) k' \left(\frac{\partial}{\partial \overline{\rho}}\frac{\omega_0}{k}\right) \overline{u}'_{k\ k_2,\overline{\rho}\ \rho_2,\overline{u}\ u_2} 0.$$

1- f t (5.1) (5,) o -

$$kc \left(\omega_0 \quad \omega_{0\overline{\rho}} \quad \frac{ck}{\overline{\rho}}\right)_{k\ k_2,\overline{\rho}\ \rho_2} 0.$$

o t , (6.3),

$$\omega_{0\overline{\rho}} \quad \frac{ck}{\overline{\rho}}_{k\ k_2,\overline{\rho}\ \rho_2} 0, \qquad (6.1\ )$$

t (2.3), f oo o (6.1), o f tf ω_0 0. o t $\partial s / \partial \rho_2$ 0. (6.1) $s(\rho_1, \rho_2)$ o ρ_2. t f (6.1) (6.4). I t o t o f t o t

$$\frac{\omega_0}{k} \qquad k \; k \quad k_2, \overline{\rho} \quad \rho_2 \qquad 0.$$

$$v_j(\rho_1, \rho_1 \quad) \sim \lim_{\rho_2 \to \rho_1} v_j(\rho_1, \rho_2) \quad \frac{\partial}{\partial \rho_2} v_j(\rho_1, \rho_2) \quad . \qquad (\ .1)$$

f o (2.4) o
t (5.6),

$$\lim_{\rho_2 \to \rho_1} v_j(\rho_1, \rho_2) \quad u_1 \quad \lim_{k \to 0} \omega_{0_k} \quad u_1 \quad c_1.$$

(.1) j 1 t t f (5.5) (5.)

$$\lim_{\rho_2 \to \rho_1} \frac{\partial}{\partial \rho_2} v_1(\rho_1, \rho_2) \quad \lim_{\rho_2 \to \rho_1} \omega_{0_{kk}} \frac{\partial k}{\partial \rho_2} \quad \lim_{\rho_2 \to \rho_1} \omega_{0_{kk}} \frac{k}{\rho_1}$$
$$\lim_{k \to 0} \omega_{0_{kk}} \frac{c_1 k/\rho_1 \quad \omega_{0_{\overline{p}}}}{c_1 \quad \omega_{0_k}}$$
$$2 \quad \frac{c_1}{\rho_1} \quad c_1' \quad . \qquad (\ .2)$$

(.2). o
ρ_2, t t t t t o

$$\frac{\partial k}{\partial \rho_2}(\rho_1 \ \rho_2) \quad \frac{k}{\rho_1}(\rho_1 \ \rho_2). \qquad (\ .3)$$

(.2) o o (2.4).
o t j 2

$$\lim_{\rho_2 \to \rho_1}$$

.2

$$s_j(\rho_1, \rho_1) = \lim_{\rho_2 \to \rho_1} s_j(\rho_1, \rho_2) = \frac{\partial}{\partial \rho_2} s_j(\rho_1, \rho_2) = \cdots .$$

(2.4), (5.), (5.11), (5.10)

$$\lim_{\rho_2 \to \rho_1} s_j(\rho_1, \rho_2) = u_1 = \lim_{k \to 0} \frac{\omega_0(k, \rho_1)}{k} = u_1 = c_1.$$

$\frac{\partial}{\partial \rho_2} s_j(\rho_1, \rho_1)$ j. j 1, (5.11)

$$\lim_{\rho_2 \to \rho_1} \frac{\partial}{\partial \rho_2} s_1(\rho_1, \rho_2) = \lim_{\rho_2 \to \rho_1} \frac{\omega_{0_k} k - \omega_0}{k^2} \frac{\partial k}{\partial \rho_2}$$
$$\lim_{\rho_2 \to \rho_1} \frac{\omega_{0_k} k - \omega_0}{k^2} \frac{k}{\rho_1}$$
$$\lim_{k \to 0} \frac{(\omega_{0_k} k - \omega_0)(c_1 k/\rho_1 - \omega_{0_{\bar{\rho}}})}{k^2 (c_1 - \omega_{0_k})}$$
$$\frac{2}{3} \frac{c_1}{\rho_1} - c_1' .$$

j 2,

$$\lim_{\rho_2 \to \rho_1} \frac{\partial}{\partial \rho_2} s_2(\rho_1, \rho_2) = \lim_{\rho_2 \to \rho_1} u'(\rho_1) - \frac{\omega_{0_{\bar{\rho}}}}{k} - \frac{\omega_{0_k} k - \omega_0}{k^2} \frac{dk}{d\bar{\rho}}$$
$$\lim_{k \to 0} \frac{c_1}{\rho_1} - \frac{\omega_{0_{\bar{\rho}}}}{k} - \frac{(\omega_{0_k} k - \omega_0)(c_1 k/\rho_1 - \omega_{0_{\bar{\rho}}})}{k^2 (c_1 - \omega_{0_k})}$$
$$\frac{1}{3} \frac{c_1}{\rho_1} - c_1' .$$

$$s_j(\rho_1, \rho_1) \sim u_1 - c_1 - \frac{3 - j}{3} \frac{c_1}{\rho_1} - c_1' , \qquad \ll 1. \qquad (.5)$$

.3

(.4) (.5)

1 - (6.1) (6.1) > 0

$\omega_{kk} > 0$. 1 - (6.2) (6.2) > 0

$\omega_{kk} < 0$. $1^{\pm}$-

t (6.6), $1^{\pm}$-

$$s_{\pm}^{(1)}(\rho_1, \rho_1 \quad) \sim u_1 \quad c_1 \quad \frac{3 \mp 1}{6} \quad \frac{c_1}{\rho_1} \quad c_1' \quad , \qquad (.6)$$

$$v_{\pm}^{(1)}(\rho_1, \rho_1 \quad) \sim u_1 \quad c_1 \quad \frac{1 \pm 3}{2} \quad \frac{c_1}{\rho_1} \quad c_1' \quad , \quad 0 < \quad \ll 1, \quad (.6)$$

(.6) (.6) 1-

(4.14)

2- t (5.2) $(c_1/\rho_1 \quad c_1')$ *negative*

$$s_{\pm}^{(2)}(\rho_2 \quad , \rho_2) \sim u_2 \quad c_2 \quad \frac{3 \mp 1}{6} \quad \frac{c_2}{\rho_2} \quad c_2' \quad ,$$

$$v_{\pm}^{(2)}(\rho_2 \quad , \rho_2) \sim u_2 \quad c_2 \quad \frac{1 \pm 3}{2} \quad \frac{c_2}{\rho_2} \quad c_2' \quad , \quad 0 < \quad \ll 1.$$

.4

$1^{\pm}$- $s_{\pm}^{(1)}$ 0

t

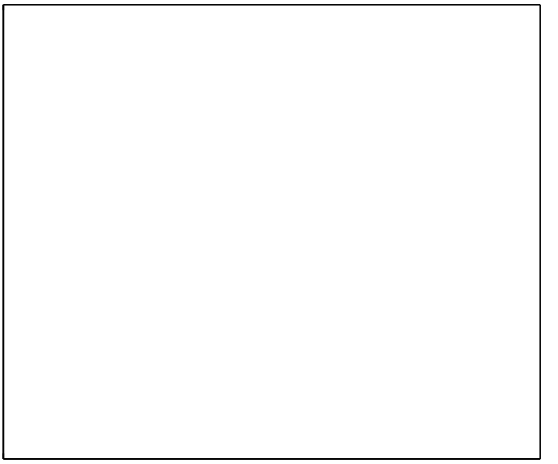
$$\pm \sim \frac{2\rho_1 c_1}{(1 \mp 1/3)(c_1 \quad \rho_1 c_1')}(M_1 \quad 1),$$

$$M_{2,\pm} \sim 1 \quad 2^{\pm 1}(M_1 \quad 1), \quad 0 < M_1 \quad 1 \ll 1,$$

(4.15)

.5

$$\rho \quad \rho_1 \quad \rho^{(1)}(\xi, T) \quad {}^2\rho^{(2)}(\xi, T) \quad \cdots,$$
$$u \quad u_1 \quad u^{(1)}(\xi, T) \quad {}^2u^{(2)}(\xi, T) \quad \cdots,$$
$$\xi$$



$$\rho_1 \quad 1, \qquad \rho_2 \qquad > 1,$$

1- t 1- t (5.1)

$$u_2 \quad u_1 \quad \int_{1}^{} \frac{f'(\rho)}{\rho}^{1/2} \rho.$$

f t tf 1/2

$$\alpha(\overline{\rho}) \quad \frac{\omega_0(k,\overline{\rho})}{c(\overline{\rho})k} \quad 1 \quad \frac{k^2}{4c(\overline{\rho})^2} \qquad (, .5)$$

(, .2) t (, .3) $\alpha \to \alpha$

$$\alpha(1) \quad 1. \qquad (, .6)$$

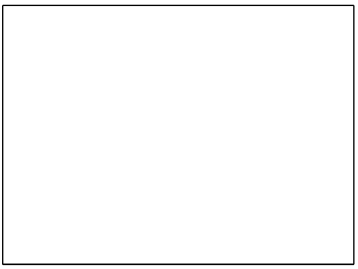
o f o t (5.) (5.11), α α,

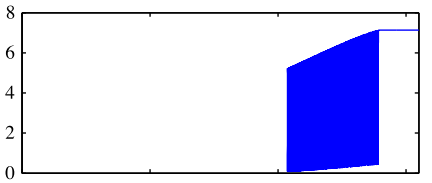
v

f t - t (, .15) o f o α( ) f t

$$\alpha(\ ) \searrow \frac{p\ \ 2}{2p} > \frac{1}{2}, \qquad \to \infty, \qquad p > 1, \qquad (\ , .1,)$$

$$0 < p < 1, \alpha(\quad)$$

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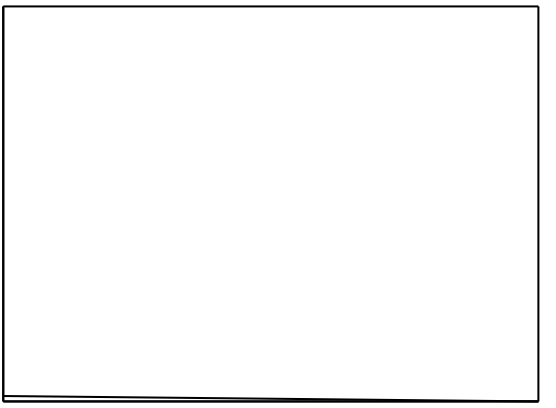
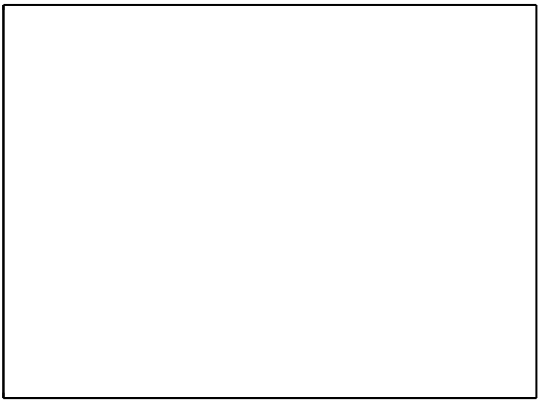
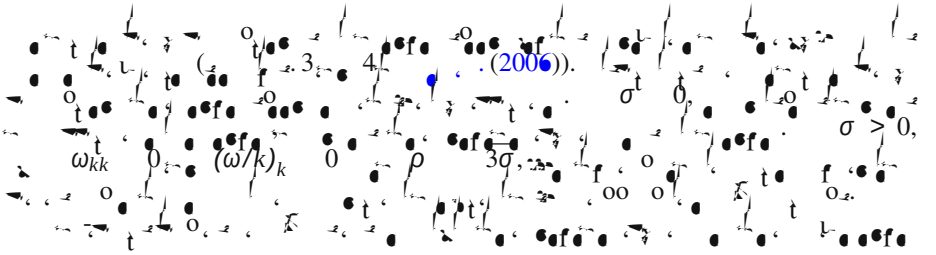
6
4
2
0







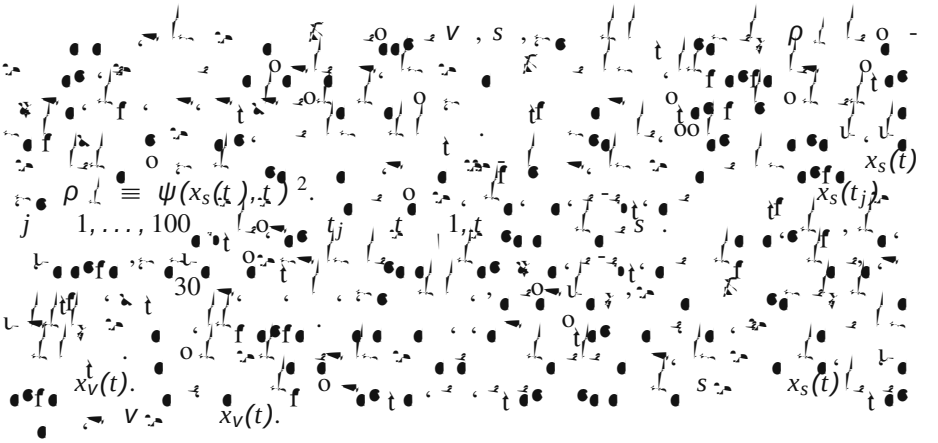
$\omega_{kk} \quad 0 \quad (\omega/k)_k \quad 0 \quad \rho \quad 3\sigma,$ $\sigma > 0,$

Acknowledgments

Appendix: Numerical Methods

(3.1)

$$\psi(x,$$



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32(20), 2, 30 2, 32 (200)

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31(5), 24 422 (200)
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45, oo. 1615 5653. o. (200).
78(1), 013 2, (200)
41(1), 26 5 (200)
5(5), 611 613 (1, 64)
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1.
oo . **36**(3), 253 2, 0 (1, 3)
2.
oo . **36**(5), 5 1 5, 3 (1, 3)
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oo . **36**(6), 0, 30 (1, 3)
.I. o o. **21**(10), 23, 1 240 (200)
(1, 5) . I. . **718**, 524 55 (2013)
40(3 4), 322 326 (1, 2)
73(5), 05 602 (2006)
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10(3), 536 53, (1, 6,)

(1, , ,)

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17(143), 151 (1, 3)

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oo . **38**, 125 155 (1, 5)

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3(1), 46 51 (200)

. **104**( ), 0 3, 03 (2010)

283, 23 261 (1, 65)

(1, 4)

4, 36, 3, 6 (1, 2)