



Dynamics in hybrid complex systems of switches and oscillators

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Dynamics in hybrid complex systems of switches and oscillators

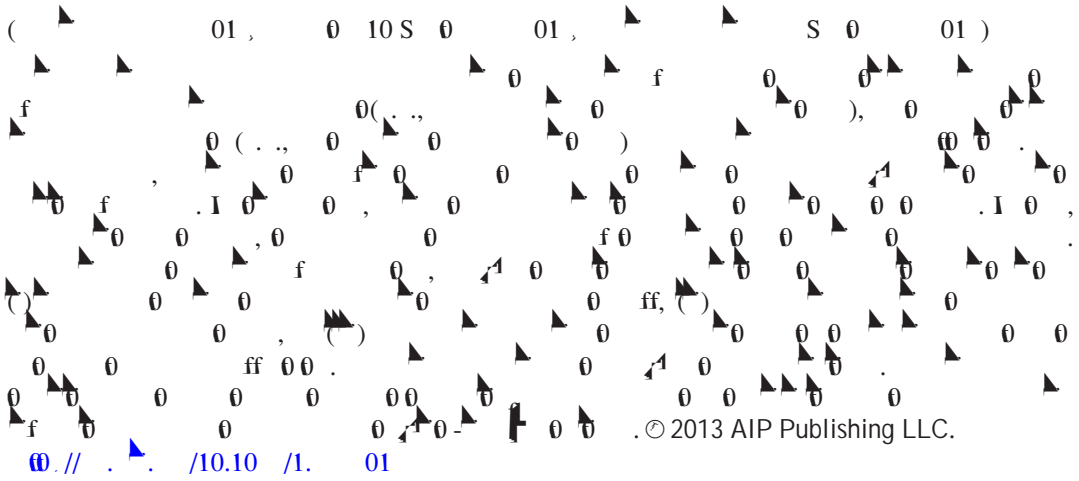
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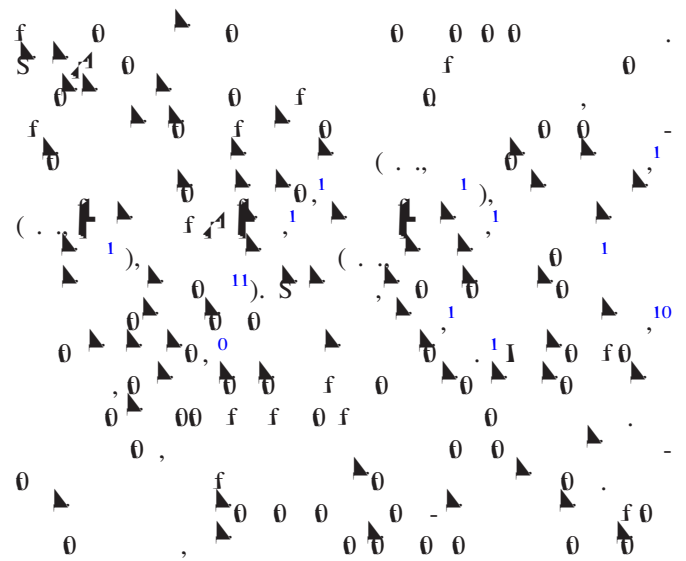
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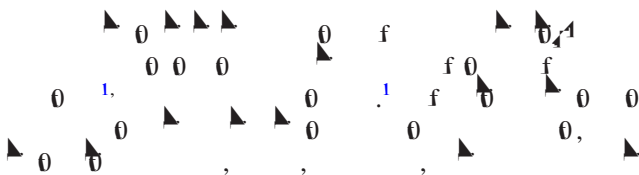
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Although extensive theoretical progress has been made in understanding collective behavior in large systems containing a single type of component (such as a switch¹ or oscillator²), there has been less development for diverse systems containing more than one type of component. However, many complex systems are composed of various types of units.³⁻⁹ For example, the system-wide dynamics of the yeast cell cycle may be modeled as a system of coupled switches and oscillators.^{8,9} Extending the numerical work of Ref. 9, we study interconnected Hopfield switches¹⁰ and Kuramoto oscillators¹¹ with positive feedback. We find three steady state solutions that may coexist: (i) the Incoherent-Off (I-Off) state in which the oscillators are incoherent and all switches are permanently off, (ii) the Synchronized-On (S-On) state in which the oscillators synchronize and all switches remain on, and (iii) the Synchronized-Periodic (S-P) state in which the oscillators synchronize and the switches periodically turn on and off. Numerical experiments confirm our predictions for these steady state solutions and the transitions between them. Our model demonstrates how the interplay between different units can result in rich dynamics.



I. INTRODUCTION



$\theta_1, \dots, \theta_N$ are the parameters to be estimated. The model is defined by the following equations:

 (S. I) $\dot{\theta}_n = \omega_n + \frac{k}{N} \sum_{l=1}^N (\theta_l - \theta_n)$

 (S. II) $\theta_n(t) = \int_0^t \Omega(\omega) k(t) dt + \theta_n(0)$

 (S. III) $\theta_n = \theta_n(t)$

II. MODEL

The model is described by the following equations:

 (1) $\dot{\theta}_n = \omega_n + \frac{k}{N} \sum_{l=1}^N (\theta_l - \theta_n)$

 (2) $\theta_n(t) = \int_0^t \Omega(\omega) k(t) dt + \theta_n(0)$

 (3) $\theta_n = \theta_n(t)$

$$\dot{\theta}_n = \omega_n + \frac{k}{N} \sum_{l=1}^N (\theta_l - \theta_n), \quad (1)$$

The model is defined by the following equations:

 (1) $\theta_n(t) = \int_0^t \Omega(\omega) k(t) dt + \theta_n(0)$

 (2) $\theta_n = \theta_n(t)$

x_m for $m = 1, \dots, M$. The model is defined by the following equation:

$$\dot{x}_m = -x_m - \eta + \frac{K^x}{M} \sum_{l=1}^M \tilde{x}_l, \quad ()$$

The model is defined by the following equations:

$$\dot{x}_m = -x_m - \eta + \frac{K^x}{M} \sum_{l=1}^M \tilde{x}_l$$

$$\tilde{x}_m = 1(x_m = 0) \text{ f } x_m > 0 (x_m \leq 0)$$

$$\tau \dot{k} = -k + \frac{K}{M}$$

$\eta > 0 \quad N \rightarrow \infty.$ I θ , θ

$\theta_n(t) = \omega_n t + \theta_n(0)$

I θ , θ $N = M = 1000$

θ $k(0) =$,

$\{\theta_n\}$ θ $\theta r_\theta(0) \approx 0.$,

$\{x_m\}$ θ $\theta r_x(0) \approx 0$ θ $\{x_m(0)\}$ θ $\theta - 1.$

θ $-\eta,$ θ r_θ k θ $0.$ $(.)$,

θ θ θ f k

C. The synchronized-periodic state

$\bar{x}_m = 1$ $\text{ff}(\bar{x}_m = 0)$ $\{1, \omega_0^{-1}\}$ $(\cdot)(\cdot)$
 $\tau \gg \{1, \omega_0^{-1}\}$ $S \cdot I$
 $B(\beta)$ (1) $\beta_m = \beta$ $m \cdot I \cdot S \cdot I \cdot 1$

1. Uniformly distributed phase lag

$\beta \in [-\pi, \pi]$ $B(\beta) = (\pi)^{-1} f$ $\tau \gg \{1, \omega_0^{-1}\}$ $N, M \rightarrow \infty$
 r_x r_θ $k = Kr_x$ $k = Kr_x 0$ 1 0 (\cdot)

$\{A, B, C, D\}$

I

(K^x, η)

f

K^x

η

$r_x^{(s)}$

K^θ

$F = 0, dF/dr_x = 0, \quad d^2 F/dr_x^2 < 0$

$\sigma_\beta \rightarrow 0$. $\langle r_x \rangle$ $B(\beta)$ $r_x(t)$
 S ∞ $\langle r_x \rangle$ $\sigma_\beta = \infty$ (S . III 1) $r_x(t)$
 $\sigma_\beta = 0$ (S . III 1). $\sigma_\beta \in \{0, 1, 10\}$
 $\langle r_x \rangle$ $r_x(t)$ K^x $\sigma_\beta \in \{0, 1, 10\}$
 $K^\theta = 10, K = , K^x$

$\tau = 0$.
 $K^x f$
 $f \langle r_x \rangle$
 $f \tau$
 $S \cdot III \rightarrow$
 $f \tau = 10,$
 f
 f
 f

APPENDIX: THE S-P STATE FOR IDENTICAL PHASE LAGS

$$\begin{aligned}
 & \text{III}^f, f \\
 & \beta_m = \beta^f \\
 & N, M \rightarrow \infty
 \end{aligned}$$

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 (00).
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 ... 81, 0 1 (010).
 ... S.S ... 0 (01).
 ... S ... 21, 0 1 (011), S. 0 ...
 S0f ... 86, 0 1 (01).