

## Wavefront sets of solutions to linearised inverse scattering problems

Weita Chang<sup>†</sup>, Philip Carrion<sup>‡</sup> and Gregory Beylkin<sup>§</sup>

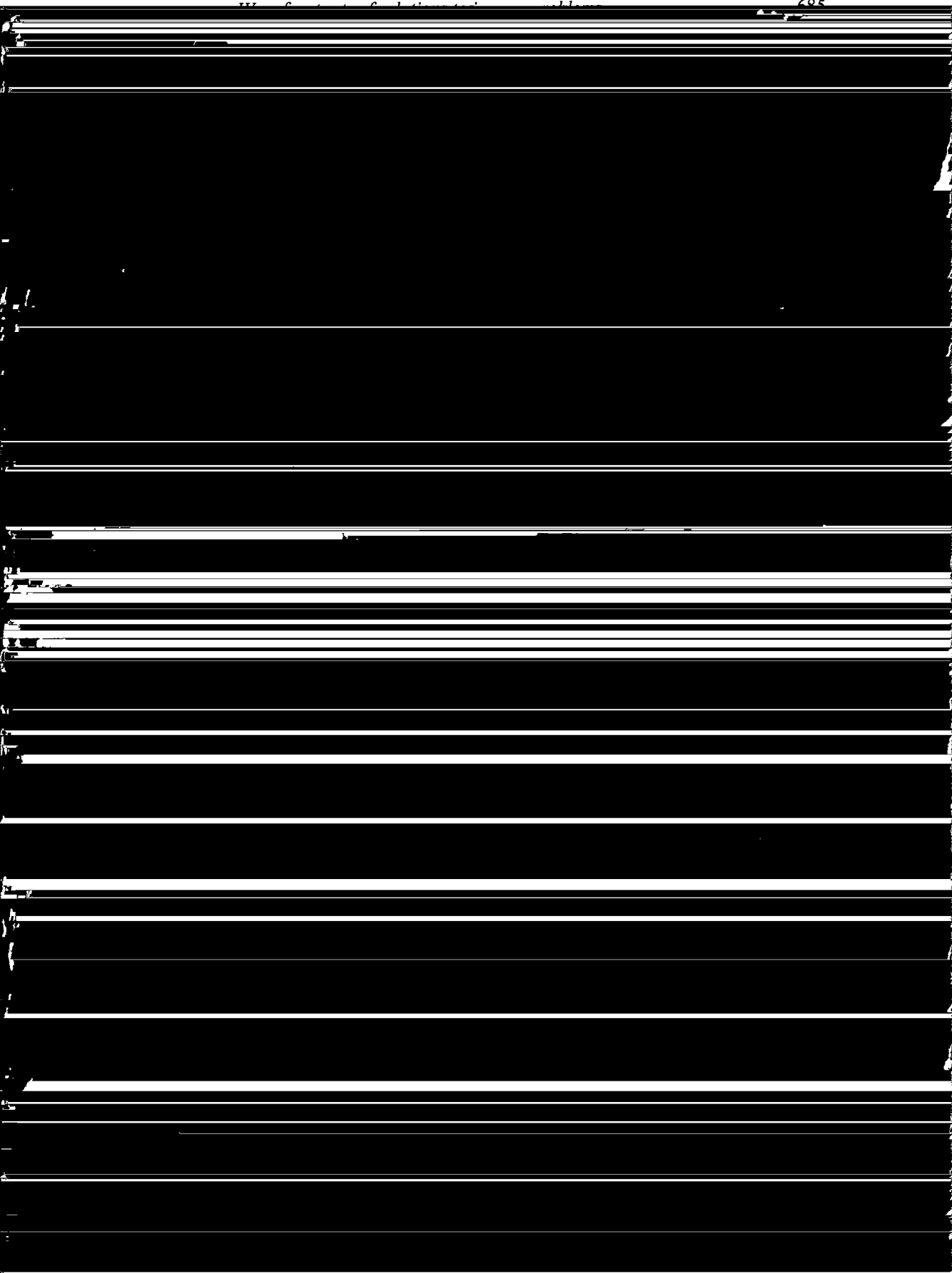
<sup>†</sup> Naval Underwater Systems Center, Code 3315, New London, CT 06320, USA

<sup>‡</sup> Georgia Institute of Technology, School of Geophysical Sciences, Atlanta, GA 30332

In [10] precise answers to these questions are given using the notion of a generalised Radon transform [23]. In particular the answer is affirmative to the first question.

The purpose of this short paper is to reformulate the results of [10] in terms of wavefront sets. This provides a natural framework for formulating the inverse problem in general as one of determination of wavefront sets of unknown parameters of PDEs, which is a meaningful question to ask from the point of view of applications.

Such a formulation is also meaningful from the pure mathematical point of view. In other words, the wavefront set of a function seems to be a natural notion to use in problems of reconstructing discontinuities. Recall that the concept of the wavefront set is



satisfying the eikonal equation

$$|\nabla_x \varphi(x, \xi)|^2 = n_0^2(x) \quad (3.2)$$

for  $x \in X$ , and  $\xi \in \partial X$ , such that

$$\varphi(x, \xi) \rightarrow 0 \quad \text{as } x \rightarrow \xi. \quad (3.3)$$

We also assume that  $\varphi(x, \xi)$  satisfies conditions (2.1), (2.2). For example if  $n_0(x)$  is constant, say  $n_0(x) = 1$  — the constant background model — we have

states that the locations of discontinuities together with their infinitesimal directions can be (at least partially) recovered. In fact the migration schemes described in [13–21] all have various choices for  $B$ , motivated in simple cases (e.g. a constant background) by some kind

**Appendix**

Here we state the basic facts about pseudodifferential operators. Let  $X$  be an open set of  $R^n$ .

*Definition (cf [7] vol. 1, p 13).* The space of symbols of degree  $m$ , denoted by

$$S^m(X \times X \times R^n \setminus \{0\})$$

consists of all complex-valued functions

$$C(x, y, \theta) \in C^\infty(X \times X \times R^n \setminus \{0\})$$

where

$$p = \nabla_x \Phi(x, x, \theta).$$

Since the determinant factor is non-zero and positive homogeneous degree 0 in  $p$ , we conclude that  $T$  is elliptic at  $(x_0, p_0)$  if

$$|C(x, x, \theta)| \geq d|\theta|^m$$

for all  $(x, \theta)$  in some conic neighbourhood of  $(x_0, \theta_0)$  where

$$p_0 = \nabla_x \Phi(x_0, x_0, \theta_0)$$

and  $d$  is some positive constant. From this we get the following corollary.

*Corollary A.2* Let  $Tf = g$  be as in proposition A.1, then

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