Stationary Expansion Shocks for a Regularized Boussinesq System

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1. Introduction

, 1 1 ,
1 1 1
1
1 ,
,
1 3, 4
1
5 B -B (BB) , 1
(1).
B (1)

(3) 1 $u_\pm$ $h_\pm$
1 *RH locus*

$$u_\pm \quad h_\mp \left(\frac{2}{h \quad h}\right)^{1<2} \tag{4}$$

(2) (3) 1
1 1 (1),

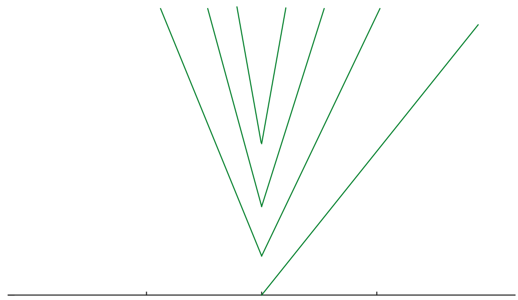
2. Expansion shock Riemann data

h_t (

1 1 , ,
1 , ,
1 1 1
$h_\pm$ $\frac{1}{16}(r_\pm$ $s_\pm)^2$ (10),
1 (14) (15). 1
$s_\pm$, $r_\pm$ 1 H 1, 1 1
A H 1 1 (11
, (10)
1 1 ,
5.

3. BBM approximation and the structure of the expansion shock

1
(11) 1 (11 1
1 ,



1 , , δ-

4. Expansion shock for the Boussinesq equations

(17) 5 , 1 1 . 5 1 D $x<\delta$, T δt , B (17) 1 x . t 1 1 1 . 1 , , 1 . 1 1 1 $\sim H$ 1 ((13)) 1 1 δ, , 1 1 H 1.

4.1. Inner solution: first-order approximation

A 1 δ , (17) $x<\delta$, t, 1 0 $\ll 1$ 1 1 , , (17)

$$r \quad \frac{1}{4\delta}(3r \quad s)r \quad \frac{}{6\delta^2}\left(r \quad s\right)$$

$$s$$

$r^{(0)}, r^{(1)}, r^{(2)}, s^{(0)},$ $s^{(2)}$ (1) $\to 0,\ \delta \to 0,$ $\to 0.$
1 1 1 1 r (15),
1 s $s^{(1)}$ 0,
1 ,
.
1 (21) (20),

$$\left(r^{(1)} \quad \cdots\right)$$

$$\frac{1}{4\delta}\left(3r^{(0)} \quad s^{(0)} \quad 3r^{(1)} \quad {}^{2}(3r^{(2)} \quad s^{(2)}) \quad \cdots\right)\left(r^{(1)} \quad {}^{2}r^{(2)} \quad \cdots\right)$$

$$\overline{6\delta^2}\left(r^{(1)} \quad {}^{2}\left(r^{(2)} \quad s^{(2)}\right) \quad \cdots\right) \tag{22}$$

$$\left({}^{2}s^{(2)} \quad \cdots\right) \quad \frac{1}{4\delta}\left(r^{(0)} \quad 3s^{(0)} \quad \cdots\right)\left({}^{2}s^{(2)} \quad \cdots\right)$$
$$\overline{6\delta^2}\left(r^{(1)} \quad \cdots\right). \tag{23}$$

B $\ll 1/\delta,$ (22)

$$\left(\frac{}{\delta}\right): \quad \frac{1}{}$$

$$\frac{2a}{9a^2} \qquad \frac{ff'}{f''} \qquad K. \tag{30}$$

$K\ /\ 0$ 1 . a f BB
5

$$a(\ ) \qquad \frac{A}{\frac{9}{2}AK \qquad 1}. \qquad f(\ ) \qquad B \left(\frac{B}{2K}\right. \qquad . \tag{31}$$

$A\ /\ 0$ $B\ /\ 0$ 1
1

$$B \qquad 1. \qquad K \qquad \frac{1}{2}. \tag{32}$$

1 , 1 A,
, (31)

$$a(\ ) \qquad \frac{A}{\frac{9}{4}A \qquad 1}. \qquad f(\ ) \qquad (\ ) \tag{33}$$

, 1 1

$$r^{(\ )}(\ .\ ) \qquad \frac{s^{(0)}}{3} \qquad \frac{A}{\frac{9}{4}A \qquad 1} (\ ) \qquad (^2). \tag{34}$$

$$s^{(\ )}(\ .\ ) \qquad s^{(0)} \qquad (^2) \tag{35}$$

1 $s^{(0)}$ A
, (34), (35) 0, → , 1
, 1 1 (12)
1 :

$$r_\pm \qquad \frac{s^{(0)}}{3} \pm A \qquad (^2). \tag{36}$$

$s_\pm$

$> _t >> _x$ 0, $> \frac{1}{4}(s^{(0)}$ $3r)$ ((19)).
1 BB (18). ,
>
BB 1 5 .

E . 1 1

$$r^{( )}(\; . \;) \sim 1 \quad \epsilon \left(\frac{1}{4} \quad \frac{\Delta( )}{1 \quad \frac{9}{4}} \right)$$

$$\frac{\epsilon^2}{3} \left(C \quad \frac{2 \quad 17 \quad {}^2( ) \quad D \, \Delta( ) \quad E \quad ( )}{16\left(1 \quad \frac{9}{4} \quad \right)^2} \right).$$

$$s^{( )}(\; . \;) \sim 3 \quad \frac{3}{4}\epsilon \quad \epsilon^2 \left(C \quad \frac{3 \quad {}^2( )}{16\left(1 \quad 9\right.}\right.$$

$$\epsilon r^{( )} \quad \frac{1}{4}(3r^{( )} \quad s^{( )})r_X^{( )} \quad 0.$$
$$\epsilon s^{( )} \quad \frac{1}{4}(r^{( )} \quad 3s^{( )})s_X^{( )} \quad 0 \qquad (51)$$

1 1 , 1

$$s^{( )}(X. \ ) \quad 3 \quad \frac{3}{4}\epsilon \quad \frac{1}{32}\epsilon^2 \quad \cdots .$$
$$r^{( )}(X. \ ) \quad 1 \quad \epsilon\left(\frac{1}{4} \quad r_1(X. \ )\right) \quad \epsilon^2\left(\frac{1}{96} \quad r_2(X. \ )\right) \quad \cdots \qquad (52)$$

1 , (5) $s^{( )}$ r .
$r^{( )}$, 1 1 ϵ

(58). (50),

$$\lim_{X\to 0^{\pm}} r_2(X, \ ) \quad F_2^{\pm}\frac{1}{\left(1 \quad \frac{9}{4} \ \right)^2} \qquad \to\pm \quad r^{(2)}(\ . \ ) \quad \frac{1}{24\left(1 \quad \frac{9}{4} \ \right)^2}. \tag{60}$$

F_2 F_2 $1<24$,

$$r_2(X, \ ) \quad \frac{1 \quad 3 \ X}{24\left(1 \quad \frac{9}{4} \ \right)^2}. \tag{61}$$

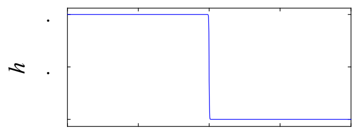
11

$$r^{(\ )}(X, \ ) \quad 1 \quad \left(\frac{1}{4} \quad \frac{X \quad 3X}{1 \quad \frac{9}{4}} \right)$$

$$\frac{^2}{24}\left(\frac{1}{4} \quad 1 \quad 3 \right.$$

, 1

$r^{( )}(X.) \quad 1$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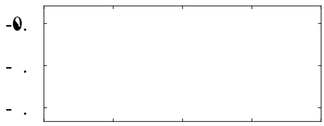

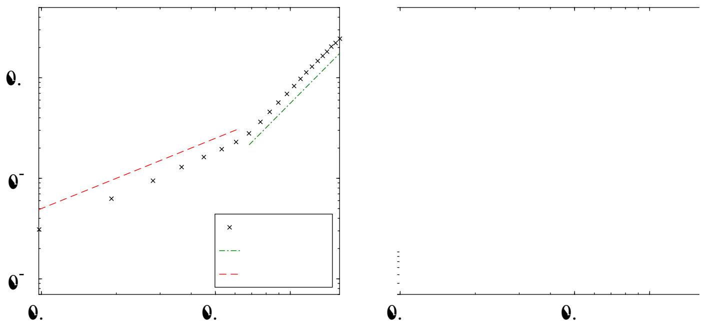

6. Discussion

[illegible]

Acknowledgments

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Appendix

A 1 1 1 11 . $L.\ L$ x_n L $2Ln/N$, n $0, \dots, N$ 1. 1 h u, 1 g h_x, u_x

$$g_t \quad (ug)_x \quad (h\ )_x \quad 0.$$

$$\ _t \quad (u\ )_x \quad g_x \quad \frac{1}{3}\ _{xxt} \quad 0 \tag{A1}$$

B L $h(\pm L, t)$ $h_\pm$ $u(\pm L, t)$ $u_\pm$ 10^{9} g 1 (A1) 1 1 1 1 1 ,

$$(ug)_x\ _n \quad ik_n\ ug\ _n, \quad n \quad N/2, \dots, N/2 \quad 1. \tag{A2}$$

1 1 k_n $n\ L$ h 11 g

$$h(x_n, t) \quad h \quad ^{1}\ g(t)\ _n \quad \frac{1}{2L}(h \quad h\ )(x_n \quad L). \tag{A3}$$

$$g_n(t) \quad \begin{cases} \dfrac{2L}{N} \displaystyle\sum_{m \quad N/2}^{N/2 \quad 1} x_m g(x_m, t) & n \quad 0 \\ \dfrac{g_n(t)}{ik_n} & n \quad 0 \end{cases} \tag{A4}$$

n 0 (A4) 1 11 $\int_{L}^{L} xg(x, t)dx$ h g . A 1 1 u.

1 1 , .

1 $> {}_{xxt} < 3$, (A1)

1 0.002 t 45. L 120 (.

2 3 1) ,

N 2^{14}. , 1,