Discrete Radon Transform

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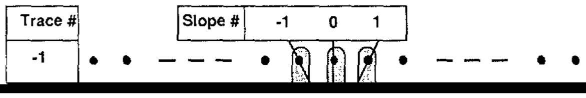
***Abstract*—This paper describes the discrete Radon transform (DRT)**

various discretizations of Radon's inversion formula. We show that DRT can be used to compute various generali-

where

$$x(n) = \begin{bmatrix} x_{-L}(n) \\ \cdots \\ x_0(n) \\ \cdots \end{bmatrix}.$$

sets of points of the lattice with a weight coefficient assigned to each point. The family of objects is constructed by invariant shift of such objects. Given a function defined on the lattice, its transform is a new function defined on such family. Its value on a given subset is the sum over this subset of values of the function weighted by cor-



This is the key observation which follows from the periodicity condition (i). (Discussion of properties of the conjugation, it is sufficient to consider (3.4) for $k = 0, 1, \cdots, N/2$. (Here and elsewhere in the paper, $N/2$ should be replaced by $(N - 1)/2$ if N is odd.)

Definition 2: We say that the DRT in (2.1) is uniquely invertible within the normalized frequency band $[k_{min}/N,$

and matrices $\hat{R}(k)$ are as follows[1]

$$\hat{R}(k) = \sum_{m=-M}^{m=M} R_m e^{-2\pi i(mk/N)} \tag{4.4}$$

Since $x(n)$ is a real vector-sequence, it is sufficient to

It follows from (4.6) that if $\sigma = 1$, matrices R_m are given by

$$(R_m)_{jl} = \delta_{m,jl}.$$

These are the matrices considered in the example in Section II. In the definition (4.7), the description of straight

[illegible]

where $j = 0, \pm 1, \cdots, \pm J$. This transform reduces to the ordinary DFT for $\alpha = 1$ and $L = J$. We consider now the following problem: given α and $\hat{w}_\alpha(j)$ for $j = 0, \pm 1, \cdots, \pm J$, find $w(l)$. To solve this problem, we apply the normalized adjoint transform (if $\alpha = 1$ and $L = J$ this is the inverse DFT)

[illegible]

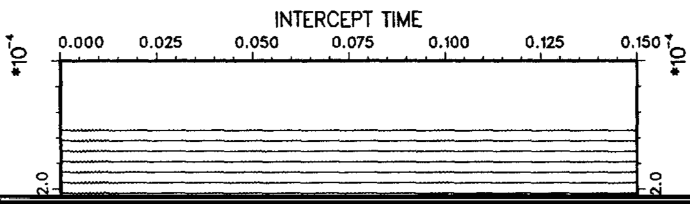
$= N/k_0(2J + 1)$, where $k_{min} \leq k_0 \leq k_{max}$), estimates of the eigenvalues of the matrix $\hat{H}_m(k)$ can be obtained using

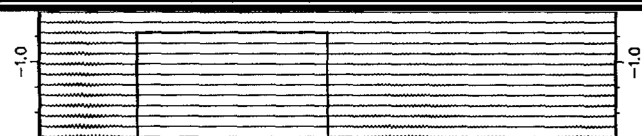
[illegible]

Inversion formula (6.1) also implies the discrete Parseval's identity. In the continuous case, Parseval's iden-

[illegible]

One can see now that the expression in (4.8) is a discrete analog of the kernel in the inner integral in (7.2). If we





mask and the approximate inversion was a problem in using the tau-P representation for the velocity filtering.

APPENDIX

Lemma 1 and Lemma 2 are essentially similar. Their proof is elementary. We use the notation of Lemma 1.

$$\hat{z}(k) = \sum_{m=-2M} H_m \sum_{n=0} x(n+m)\, e^{2\pi i(nk/N)},$$

or