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> 0, $\delta > 0$, $r^{-\beta}$, $\beta > 0$,

$$\left| r^{-\beta} - \sum_{m=1}^{M} w \right.$$

1. *Let us assume that* (4) *holds. For any* > 0 *and* $t_0 \in \mathbb{R}$, *we have*

$$\left|\int_{\mathbb{R}} f(t)\,dt - h\sum_{n\in\mathbb{Z}} f(t_0+nh)\right| \leqslant \tag{6}$$

provided that the Fourier transform of f *satisfies*

$$\left|\hat{f}(\xi)\right| \leqslant c_1 e^{-q|\xi|}, \tag{7}$$

for some positive constants c_1, q *and step size* $h \leqslant q/\ (2c_1\ ^{-1}+1)$ *or, alternatively,*

$$\left|\hat{f}(\xi)\right| \leqslant \frac{c_2}{|\xi|^q}, \quad \text{for } |\xi| \geqslant R, \tag{8}$$

for some positive constants c_2, R, q *and step size* $h \leqslant \{1/R,\ ^{1/q}(2c_2\zeta(q))^{-1/q}\}$, *where* $\zeta(q)$ *is the Riemann Zeta function.*

(5), h $\sum_{n\neq 0}|\hat{f}(\frac{n}{h})| \leqslant$ (7),

$$\sum_{n\neq 0}\left|\hat{f}\ \{$$

$$S_\infty(r) = \frac{h}{(\beta)} \sum_{n\in\mathbb{Z}} e^{\beta(t_0+nh)} e^{-e^{t_0+nh}r}. \tag{13}$$

$h = h(\ ,\beta)$

$$\sum_{n\neq 0} \frac{|\ (\beta + 2\pi i \frac{n}{h})|}{(\beta)} < \ . \tag{14}$$

3. *Given* $\beta > 0$ *and* $0 < \ \leqslant 1$, *for any step size* h *such that*

$$h \leqslant \frac{2\pi}{3 + \beta \ (\ 1)^{-1} + \ ^{-1}}, \tag{15}$$

and any $t_0 \in \mathbb{R}$ *we have*

$$\frac{|r^{-\beta} - S_\infty(r)|}{r^{-\beta}} \leqslant \ , \quad \text{for all } r > 0, \tag{16}$$

where S_∞ *is given in* (13).

$S_\infty(r)$ (16) r. ,

β,3 9 8 ( ) 4 0 .

4. *For all* $r > 0$,

$$S_F(r) < S_\infty(r) < (\ +1)r^{-\beta}.$$

, $S_F(r)$, *on the whole positive axis*, 12 . 4 9 .

fi f (10) r β. (16). □

S_F β, δ ,

5. *For any* $\beta > 0$, $\delta > 0$, *and* 1 / 7 1 9.7304 0 0 9.7304 303 *ou8309o94 0 TD 0.2518 f4p -33.1058 -2.866 TD 0.0004 0 9.*

B M N $r \in [\delta, 1]$. $f_{\beta,r}(y)$ (9) (10) $y_0 =$ $(r^{-1}\beta)$. M $t_M \leqslant$ β $f_{\beta,r}(y)$ $(-\infty, t_M)$ $r \in (0, 1]$ $\mathcal{T}_M(r)$

$$\mathcal{T}_M(r) \leqslant \frac{r^\beta}{(\beta)} \int_{-\infty}^{t_M} e^{-re^y+\beta y}\, dy \leqslant \frac{1}{(\beta)} \int_{-\infty}^{t_M} e^{-e^y+\beta y}\, dy \tag{27}$$

$$= \frac{1}{(\beta)} \int_0^{e^{t_M}} e^{-s} s^{\beta-1}\, ds = 1 - \frac{(\beta, e^{t_M})}{(\beta)}, \tag{28}$$

$$(\beta, x) = \int_x^\infty e^{-s} s^{\beta-1}\, ds$$

t_M δ.

N

$$t_N \geqslant \left(\delta^{-1}\beta\right), \tag{29}$$

fi

$$\mathcal{T}^N(r) \leqslant \frac{r^\beta}{(\beta)} \int_{t_N}^\infty e^{-re^y+\beta y}\, dy = \frac{1}{(\beta)} \int_{re^{t_N}}^\infty e^{-s} s^{\beta-1}\, ds,$$

$r \in [\delta, 1]$

$$\mathcal{T}^N(r) \leqslant \frac{(\beta, \delta e^{t_N})}{(\beta)}. \tag{30}$$

$$\lim_{x\to 0} \frac{(\beta, x)}{(\beta)} = 1 \quad \lim_{x\to\infty} \frac{(\beta, x)}{(\beta)} = 0,$$

fi t_* t^*

$$1 - \frac{(\beta, e^{t_*})}{(\beta)} = \quad , \tag{31}$$

$$\frac{(\beta, \delta e^{t}\ )}{(\beta)} = \quad . \tag{32}$$

M_* N^* fi S_F, t_*, t^*, h t_0, $(t_* - t_0)/h$ $(t^* - t_0)/h$

7. *For all $\beta > 0$, $\delta > 0$ and $1/e \geqslant \ > 0$, the solution t_* of (31) does not depend on δ and satisfies*

$$t_* \geqslant \frac{(1+\beta)}{\beta} = \frac{1}{\beta} \quad + \quad (1+\beta)^{\frac{1}{\beta}}. \tag{33}$$

The solution t^* of (32) has a weak dependence on and satisfies

$$t^* \leqslant \ \delta^{-1} + \quad {}^{-1} + \ \beta + \frac{1}{2}. \tag{34}$$

t_* t^*, (33) (34). S_F 5. 7 A A.2. (9) (10) $-\infty$ $+\infty$ S_F e^{hn} (21) (n). A 16, (21) 3

8. *For any* $\delta > 0$, *and* > 0, *there exist a step size h and a positive integer M such that*

$$\left|e^{-xy} - G_e(x, y)\right| \leqslant \quad , \quad \text{for } xy \geqslant \delta, \tag{41}$$

where

$$G_e(x, y) = \frac{hx}{2\sqrt{\pi}} \sum_{j=0}^{M} e^{- \; x^2}$$

$\leqslant$

be an approximation of the kernel by Gaussians valid for $\delta \leqslant r \leqslant 1$. *Then, for any bounded, compactly supported function* f *in* D *and* $x \in D$, *we have*

$$\left| \int_{B_1} \| \; \|^{-\alpha} f(\; + \;) \, d \; - \int_{B_1} G_F\big(\|y\|\big) f(\; + \;) \, d \; \right| \leqslant \big(\; + (2 + \;) \delta^{d-\alpha} \big) \frac{\omega_{d-1}}{d-\alpha} \|f\|_\infty .$$

. C B_δ

A. $u(z, \ )$ $z > 0$, $u_{zz} + \ u = 0$, $u(0, \ ) = u_0(\ )$,

$$u(z, \ ) = \int_{\mathbb{R}^d} \mathcal{P}(z, \ - \ ) u_0(\ )\, d \ , \quad z \geqslant 0, \tag{56}$$

$$\mathcal{P}(z, \ ) = \frac{2}{\omega_d} \frac{z}{(z^2 + \| \ \|^2)^{(d+1)/2}}$$

0 3, > 0, $z > 0$, 0

$$S_\infty\left(z^2 + \| \ \|^2\right) = \frac{zh}{\pi^{(d+1)/2}}$$

$0 \leqslant n \leqslant 2N$. … , $a > 0$ … $[0, 1]$ … ffi … w_m … (66) … $\eta_m = \frac{2N}{a} t_m$, … $h(x)$ … $x \in [0, a]$,

$$\left| h(x) - \sum_{m=1}^{M} w_m e^{-\eta_m x} \right| < \;', \tag{67}$$

… h … (66) … (66) … :

- B … $(N+1) \times (N+1)$ … 0 … $_{kl} = h_{k+l}$ … $h_n = h(a\frac{n}{2N})$, $0 \leqslant n \leqslant 2N$.
- … $= (u_0, \ldots, u_N)$, … $= \sigma$ … σ … 0 … (… 16, … 22); … σ … fi … $M+1$ … $\sigma_0 \geqslant \sigma_1 \geqslant \cdots \geqslant \sigma_M$, … σ_M … $\sigma_M/\sigma_0 \approx$ … , … , $M = \mathcal{O}(\;\;^{-1})$ … $M \ll N$.
- C … γ_m … $u($

$x =$ $^{-1}$. , β

$$\tilde{t}^* = \delta^{-1} + \quad ^{-1}. \tag{70}$$

$\beta > 1$,

$$\frac{(\beta, x)}{(\beta)} \leqslant \frac{e}{(\beta)} x^{\beta-1} e^{-x} \leqslant ,$$

fi (69)

$$x > d(\beta - 1) \tag{71}$$

$d = e/(e-1) \approx 1.582$, , 14, x

$$x \geqslant d$$

A.3. Proof of Theorem 5

2.1.1 (20) h (15) t_* t^* 7. $t_0 = 0$

$$\tilde{h} = \frac{10}{2\quad{}^{-1} + \quad \beta + 2}.$$

$$\frac{\tilde{t}^* - \tilde{t}_*}{\tilde{h}},$$

$\tilde{t}^*$, $\tilde{t}_*$ (33) (34).

$$\tilde{t}^* - \tilde{t}_* = \quad \delta^{-1} + \frac{1}{\beta}\quad{}^{-1} + \quad{}^{-1} + \left(\frac{\beta+1}{(1+\beta)^{\frac{1}{\beta}}}\right) + \frac{1}{2}$$

$$\leqslant \quad \delta^{-1} + \frac{1}{\beta}\quad{}^{-1} + \quad{}^{-1} + \frac{3}{2}$$

15.

(23) (27) (28).

15. *Let* $g(x) = \frac{(x+1)^{\frac{1}{x}}}{x+1}$ *for* $x > 0$

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