Fast algorithms for Helmholtz Green's functions

B G EG BE I *, CH I HE C A D CA

2 030 -0 2 ,

- H G '

.

G ’

. , -

G ’

.

- H G ’ ,

= K ,

B E ' (1921) ,
(1.6) (G & (1980) (1998)

Proposition 2.1. ( ) $\in \mathcal{S}(\mathbb{R}\ )$, Λ

, Λ^*

3. Quasi-periodic Green's function via absolutely convergent series

\- G ' (1.6)

( ), (1.1) (1.4) (1.5).

Proposition 3.1. *(3.1)* *(1.2)* *(1.3)* > 0, $\neq 2$ $\boldsymbol{d}$ K , $\boldsymbol{d} \in \Lambda^*$
$\in \mathbb{R}$. ≥ 2.

F (3.1)

$$\text{F} \quad \text{C} = \frac{1}{\ } \sum_{\boldsymbol{d}\in\Lambda^*} \frac{\left(\frac{\text{K } 2 \ \boldsymbol{d}\text{K}\ ^2\text{C}\ ^2}{4\ ^2}\right)}{2\ \boldsymbol{d}\,\text{K}\ ^2\,\text{K}\ ^2} \ \$\, 2\ \boldsymbol{d}\text{K} \quad \text{K}\ \$ = \text{K}\ \$ \quad \text{F},$$

3 3 1

$^2\ \$\boldsymbol{d} = 1 \quad \in \Lambda \quad \boldsymbol{d}$

$$= \frac{1}{2} \rule{1cm}{0.4pt}$$

Remark 3.3. ,
- G ’

I (3.12),

> . 2.1

$$\frac{1}{2^{3/2}} \quad 0 \quad \frac{C \quad ^{2}}{4.^{2}} \quad \$ \quad K, \quad C,^{2}.^{2} \quad \in \Lambda$$

$$= \frac{1}{2} \quad 0 \quad \frac{C \quad ^{2}}{4.^{2}} \quad \sum_{\boldsymbol{d} \in \Lambda^*} \$ \, 2 \quad \boldsymbol{d} K \quad K \frac{2 \quad \boldsymbol{d} K \quad ^{2}}{4.^{2}} \quad \frac{}{3},$$

Λ^* . B

,

$$\frac{1}{2} \sum_{\boldsymbol{d} \in \Lambda^*} \$ \, 2 \quad \boldsymbol{d} K \quad 0 \quad \frac{K, 2 \quad \boldsymbol{d} K \quad ^{2} \, C \quad C \quad ^{2}}{4.^{2}} \quad \frac{}{3}$$

$$= \frac{1}{} \sum_{\boldsymbol{d} \in \Lambda^*} \frac{\$ \, 2 \quad \boldsymbol{d} K \quad \frac{K, 2 \quad \boldsymbol{d} K,^{2}}{}}{}$$

ET 2 4 7 . 5 . 5 6 0 . 5 4 6 9 0 j/ F 2 1 1 7 . 6 7 1 1

4. Fast convolutions with Green's function

- G '
(3.1) (3.2)
.
(3.2) G .
G ' ,
( ) .
, .
.

( ) '

- G '
(3.4).
F ,
F

$$\tilde{}_F = \frac{1}{} \sum_{\substack{d\in\Lambda^* \\ 2\ dK \leq}} \frac{\left(\frac{K\ 2\ dK\ ^2 C\ ^2}{4\ ^2}\right)}{2\ d K\ ^2 K\ ^2} \$\ 2\ dK , \quad 4.1$$

>0 >0

F
' ' (3.3)
G . F , >0
(3.2)

$$\sum_{\substack{\in\Lambda \\ \leq}} \$ \quad C ,$$

, ,
B . ( )
. , G ,

$$= \sum_{=1} K\ ^2 , \quad 4.2$$

>0 $>0.$
(B . ( )). (4.2),

$$\tilde{} = \sum_{\substack{\in\Lambda \\ \leq}} \$ \quad C . \quad 4.3$$

C (4.1) (4.3), - G ’

$$\tilde{} = \tilde{} \quad C \tilde{}_F \, . \qquad 4.4$$

: ( )

( )

(4.2).

,

.

$\tilde{}_F$ F

d dK x 2 2 d K

$$\tilde{}_F \; *, \; = \frac{1}{} \sum_{\substack{d \in \Lambda^* \\ 2 \; dK \leq}} \frac{\left(\frac{K\, 2 \; dK \;^2 C \;^2}{4 \;^2}\right)}{}$$

$> 0 \qquad > 1$

$$\frac{1}{\ } \sum_{\substack{d \in \Lambda^* \\ 2\ d K > }} \frac{\left(\frac{K\ 2\ d K\ ^2 C\ ^2}{4\ ^2}\right)}{2\ d K\ ^2 K\ ^2} \leq \frac{\ }{3}$$

, ,

$$\left\| \ F \qquad K^{\sim} F \qquad \right\|_{1} \leq \frac{\ }{3}. \qquad 4.8$$

$K^{\sim}$

$| \qquad |^{1}$

, (3.1)

$$\frac{\left(\frac{K\ \ ^2K\ ^2}{4\ ^2}\right)}{^2K\ \ ^2} \leq \frac{}{^2\ \ ^2K\,1}.$$

(4.16),

(3.2) [redacted] .

A >1, [redacted]

. F ~ 3, [redacted]

. [redacted]

Remark 4.2. D [redacted]

E ' (. . C (1978) . [redacted]

(2006; [redacted]

) [redacted]

. F , [redacted] (3.2)

( ) [redacted]

- [redacted]

(4.4). [redacted] (4.6)

, [redacted]

$= \tilde{}$ K [redacted]

[redacted] .

I :

( ) . F [redacted] ,

( ) . , . , , .
F , .

B . (2008), F
$\mathcal{O}($ $)$. ,
, ∼ . A ,
∼ C $_2$ ^{K1} .

A :

( ) B . (2008),
(4.7) $\mathcal{O}$ \$ \$ \$. A ,
G (G & 1991; 1991;
G & 1998) ,
. A \$ \$ ∼ ^{K1} ^{3},
,
(, ,
B . (2008)
G). (4.7) $=0$

. ,
(4.7) $\mathcal{O}$ C $_3$ ^{K1}),
^{3} .
( ) $_F$. F
2 $\boldsymbol{d}$K $_{K1}$ ≤
. ∼
(3.1). G
(4.5), FF (D & 1993; B 1995;
& G 2005) . ,
$\mathcal{O}$ C C $\mathcal{O}$, $\mathcal{O}$ C
4 ^{K1} , 4 .
, F ,

(4.17) G ’
. B
, G ’
. F
, G ’
(4.17) (4.17) ,
= Δ C . (, . 2000, (17)) Δ
(1998, (2.49), (2.53) (2.54)) .
I 1, (4.17)
(4.4) $\approx 10^{-9}$. $= 0$

($\approx 10^{-9}$) 4.1.

$$= \frac{2\alpha}{} - \quad {}^{3} \quad -\alpha \quad - \quad C \, {}^{2} \qquad 4.18$$

$\in \Lambda \quad = 1$

$\alpha = 300$, $= (1/3, 4/7)$, ${}_{1} = (0, 0)$, ${}_{2} = (1/10, 1/10)$ ${}_{3} = (-3/$

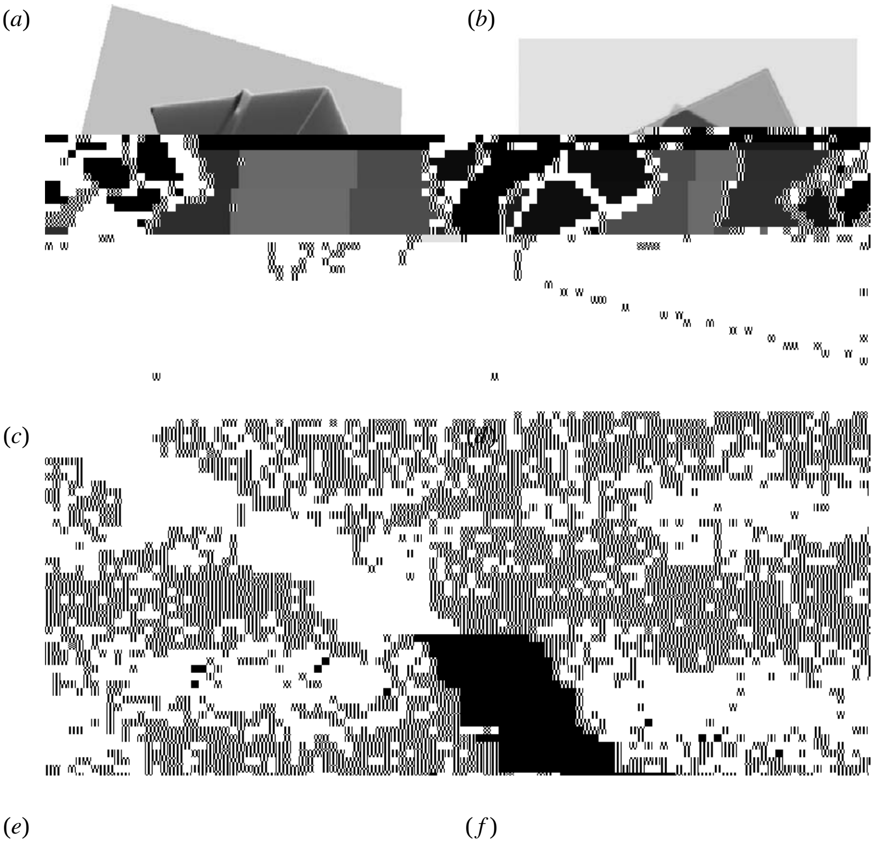
(*a*) (*b*)

F 3. A - G ' $=(3,5)$ $=100$ - $_1 =\ 1, 0$ $_2 =\ 1/2,\ \ 3/2$ K1/2, 1/2 ! K1/2, 1/2 : ( ) ( ) .

F . I 2, . G ' $=10^{K11}$, ^{2}- $_2 \approx 1.76$ - $_2 \approx 1.31\$10^3$. 4.1.

, - G ' . I 3, - - G ' . : ( )

F ( ) G ' . I 4, G ' . ( ) .



5. Green’s functions with boundary conditions on simple domains

G ’

-

G ’ (3.4).

, G ’

- G ’ .

G ’ D ,

F .
I , G
F .
F , - D
$= K1/2, 1/2$! $K1/2, 1/2$.
G , G , (2
),
$\Delta C 4^{2}$ $= K$
(1.3) $=0$. G ,

$$ {}_{1}, {}_{2} = K \frac{1}{4} \sum_{{}_1 = K\infty}^{\infty} \sum_{{}_2 = K\infty}^{\infty} \gamma_0 \; 2 \quad {}_1 C {}_1{}^2 C {}_2 C {}_2{}^2 , $$

G , , (1.6), -
, ${}_1 = 1, 0$ ${}_2 = 0, 1$.
(3.4),

$$ {}_{1}, {}_{2} = \frac{1}{2} \sum_{\in \mathbb{Z}^2} \sum^{\infty} {}_{2} \quad K \; C \; {}^{2} \frac{2}{4} C \, 4^{2} \; {}^{K2} . $$

$$ C \sum_{\in \mathbb{Z}^2} \frac{\overline{K \; {}^2 \; {}^2 C \; {}^2}}{} $$

$$= \left(K\frac{}{4} \quad K \quad C\,2 \quad {}^2\right) K \quad \left(K\frac{}{4} \quad C \quad C\,1\,C\,2 \quad {}^2\right). \qquad 5.3$$

, (5.2)

$$\tilde{D} \quad , \quad = \quad =1 \quad {}_1^2 C \; {}_2^2 \leq \quad K1/2 \quad 2, \; 2 \quad 1/2$$

$$! \quad K1/2 \quad 1 \quad 1, \; 1 \quad 1, \; 2 \quad 1 \quad 2, \quad 1/2$$

§4.

I F , , >1
(4.8)

$$\tilde{D}_F \quad 1, \; 2, \; 1, \; 2 = \quad 2 \quad {}_1^2 C \; {}_2^2 \leq \quad \frac{\frac{K\;^2 \; {}_1^2 C \; {}_2^2 \; C\;^2}{2}}{4\;^2 \; {}_1^2 C \; {}_2^2 \; K\;^2} \quad 1 \; 1 C \; 2 \; 2$$

$$! \left(K \quad 1 \; 1 \; K \quad 1 \; 1 C1 \right) \left(K \quad 2 \; 2 \; K \quad 2 \; 2 C1 \right). \qquad 5.4$$

$$\tilde{D}_F \quad , \quad = \quad 2 \quad {}_1^2 C \; {}_2^2 \leq \quad \frac{\frac{K\;^2 \; {}_1^2 C \; {}_2^2 \; C\;^2}{2}}{4\;^2 \; {}_1^2 C \; {}_2^2 \; K\;^2} \quad 1 \; 1 C \; 2 \; 2$$

$$! \quad \hat{} \quad 1, \quad 2 \; K \quad {}^1\hat{} \quad K \quad 1, \quad 2$$

$$K \quad {}^2\hat{} \quad 1, K \quad 2 \quad C \quad {}_1 C \; {}_2\hat{} \quad K \quad 1, K \quad 2 \;,$$

5.5

$\hat{}$ (4.6). FF (5.5) §4 .

Remark 5.1. A

Remark 5.2. G ' D
$=3$ -
. I ,

, ( ) , .
A -
(
H . 2003, 2004; . 2004 ,).
-
, .
. I ,
G '
.
( ) $=0$.
H , ,
.

A - G '
.
( )
( ) , ,
.

(
),.
F , E '
F
,
- , G ' .

F D -0612358, D E/
4000038129, D E DE-FG02-03E 25583 AF FA9550-07-1-0135.

References

A , . & , I. A. 1970 , 9 . ,
: D .
B , . 1981 : '
. 131, 163–216. (:10.1016/0003-4916(81)90189-5)
B , G. 1995 F
. 2, 363–381. (:10.1006/ .1995.1026)
B , G. & , . . 2002
99, 10 246–10 251. (:10.1073/ .112329799)
B , G. & , . . 2005 A .
. 26, 2133–2159. (:10.1137/040604959)
B , G. & , . 2005
. 19, 17–48. (:10.1016/ . .2005.01.003)
B , G., C , ., F , G. & H , . 2007
. 23, 235–253. (:10.1016/ . .2007.01.001)
B , G., C , . & , F. 2008 F -
. 24, 354–377. (:10.1016/ . .2007.08.001)
B , G., , C. & , . . F H
G ' .
B , . 1953 , : D .

C , . 1978 E :
E . 34, 974–979. (:10.1107/ 0567739478001990)

C , D., , H. & , . ( ) 1992 ,
, :

D , A., H , F. & H , . 2001 - , G '
H . 457, 67–85. (:10.1098/ .2000.0656)

D , A. & , . 1993 F F .
. 14, 1368–1393. (:10.1137/0914081)

E , . 1913 C - . 14,
465–472.

E , . 1921 D . 64,
253–287. (:10.1002/ .19213690304)

G ' , I. . & , G. E. 1964 , . 1.
(. E). , : A .

G , . & , I. 1980 . I
(H. E & D. H), . 67–139. , : A .

G , . 2004 , :
E , I .

G , . & , . 1991 G . 12, 79–94.
(:10.1137/0912004)

G , . & , . 1998 A G . I
, 1 2 , . III (E .), . 575–584. B ,
G : B .

H , F. & , B. 1961 E G '
124, 1786–1796. (:10.1103/ .124.1786)

H , ., F , G., , & B , G. 2003
. I - 2003, . 2660 (. . A. ,
D. A , A. . B , . D , A. . & . E. G), . 103–110.
C . B , G : .

H , ., F , G., , ., G , . & B , G. 2004 :
. . 121, 11 587–11 598. (:10.1063/1.1791051)

, ., , . & , . 1995
, : .

, . E., , G. . & , . 1986 A G '
H .

, C. (.) 1992 , :
, . 1991 G 12, 1131 1139. (:10.1137/0912059)
, ., F , G., G , ., H , . & B , G. 2004 : H F 121, 2866 2876. (:10.1063/1.1768161)
, ., F , G., G , ., H , . & B , G. 2004 : H F . 121, 6680 6688. (:10.1063/1.1790931)
, . & , . 1999 E - G ' . 47, 1050 1055. (:10.1109/8.777130)