APPM 3570 / STAT 3100
Fall 2021
Exam 3
December 13

- This exam has two parts, and you may start on either as long as you follow the instructions for each.

- Notes, your text and other books, cell phones, and other electronic devices are <u>not</u> permitted, except for calculators.

- Calculators <u>are</u> permitted.

- Write your name and sign and date your pledge to the CU Honor Code in the box below.

2. (4 points.) Consider a random vector (X, Y) such that $Y \sim$ Uniform$(0, 1)$ and the conditional distribution of X given that $Y = y$ is Normal with mean y^2 and variance 100. What's the expected value of X?

   (a) 100
   (b) $1/3$
   (c) 101
   (d) $1/4$
   (e) $1/2$
   (f) None of the above

3. (4 points.) If X and Y are discrete random variables such that $p(x) = (1/2)^x$, for $x \geq 0$, and $p(y/x) = 1/(x + 1)$, for $y = 0, \ldots, x$, what's the joint p.m.f. of X and Y?

   (a) $p(x, y) = \begin{cases} \frac{2^x}{x+1} & ; \text{ for } 0 \leq y \leq x \text{ integers} \\ 0 & ; \text{otherwise} \end{cases}$

   (b) $p(x, y) = \begin{cases} (x + 1)2^x & ; \text{ for } 0 \leq y \leq x \text{ integers} \\ 0 & ; \text{otherwise} \end{cases}$

   (c) $p(x, y) = \begin{cases} \frac{1}{(x+1)2^x} & ; \text{ for } 0 \leq y \leq x \text{ integers} \\ 0 & ; \text{otherwise} \end{cases}$

   (d) $p(x, y) = \begin{cases} \frac{x+1}{2^x} & ; \text{ for } 0 \leq y \leq x \text{ integers} \\ 0 & ; \text{otherwise} \end{cases}$

   (e) $p(x, y) = \begin{cases} \frac{1}{x+1} & ; \text{ for } 0 \leq y \leq x \text{ integers} \\ 0 & ; \text{otherwise} \end{cases}$

   (f) None of the above

4. (4 points.) Suppose that $X \sim$ Poisson(λ), with $\lambda > 0$, and $P(Y = 0 / X = k) = \frac{1}{k+1}$. What's the probability that $Y = 0$?

   (a)
   (b) $e - 1$
   (c) $1 - e$
   (d) $\frac{}{2}$
   (e) $\frac{1}{}$
   (f) None of the above

5. (4 points.) A grasshopper jumps one unit to the left with probability $1/3$, and one unit to the right with probability $2/3$. Call a sequence of two consecutive jumps a \waste" if the grasshopper returns to where it was two jumps ago. What's the expected number of wasted jumps in 10 jumps of the grasshopper?

   (a) 4/9
   (b) 40/9
   (c) 9/2
   (d) 5
   (e) 4
   (f) None of the above

6. (4 points.) Imagine cereal boxes that contain one of four possible equally likely but different types of coupons. How many boxes should one expect to buy to collect all four coupon types for the first time?

   (a) 4
   (b) $5/2$
   (c) 40,000
   (d) $25/3$
   (e) $22/3$
   (f) None of the above

7. (4 points.) If X and Y are random variables such that $\mathrm{cov}(X, Y) = 0$, which of the followin..9556astatements must be TRUE?

   (a) X and Y are independent
   (b) X and Y have the same distribution
   (c) X and Y are discrete and have a joint p.m.f.
   (d) X and Y are not independent
   (e) X and Y are continuous and have a joint p.d.f.
   (f) None of the above

8. (4 points.) Let $n \gg 1$ be a very large integer, and $X_1, \ldots, X_n$ i.i.d. random variables with mean 2 and variance 25. Due to the *Law of Large Numbers* (LLN), $\frac{1}{n} \sum_{i=1}^{n} X_i^2$ should be with high probability approximately equal to:

(a) 29

(b) 2

(c) 5

(d) 25

(e) 4

(f) None of the above

9. (4 points.) Let X and Y be continuous random variables with joint p.d.f. $f(x; y) = (2e - 5) x e^y =2$ for $x, y \geq 0$ such that $x + y \leq 1$, otherwise $f(x; y) = 0$. What's the conditional p.d.f. of X given that $Y = 1=2$?

(a) $8x$, for $0 \leq x \leq 1$, otherwise it vanishes

(b) x, for $0 \leq x \leq 1=2$, otherwise it vanishes

(c) $8x$, for $0 \leq x \leq 1=2$, otherwise it vanishes

(d) $(2e - 5) x e^{1=2} =2$, for $0 \leq x \leq 1=2$, otherwise it vanishes

(e) $2x$, for $0 \leq x \leq 1$, otherwise it vanishes

(f) None of the above

10. (4 points.) Let X, Y, Z be random variables such that $\mathrm{cov}(X; Y) = 1$, $\mathrm{cov}(X; Z) = -2$, $\mathrm{cov}(Y; Z) = 3$, and $V(Z) = 5$. What's $\mathrm{cov}(Y + Z; X - Z)$?

(a) 1

(b) -9

(c) -3

(d) 7

(e) -11

(f) None of the above

11.

14. (4 points.) If X is a random variable with variance 3, what's the variance of $1 - 2X$?

    (a) 6

    (b) 12

    (c) -6

    (d) -12

    (e) 5

    (f) None of the above

15. (4 points.) Let $X_1, \ldots, X_m$ be Binomial random variables with parameters $(n, p=\frac{1}{2})$. If for $i \neq j$ the covariance between X_i and X_j is (-1), what's the variance of $\frac{1}{m}\sum_{i=1}^{m} X_i$?

    (a) $\frac{n}{4}$

    (b) $\frac{n}{2} - m + 1$

    (c) $\frac{n}{4} - m + 1$

    (d) $\frac{n}{4} + m - 1$

    (e) $\frac{n}{2} + m - 1$

    (f) None of the above

* One More Questions Ahead! *

INSTRUCTIONS FOR PART II: Use the first pages of your bluebook to answer to the following question. On the front cover of your bluebook, write (i) your name; and (ii) when your class meets (9 AM, or 3 PM). Do all parts of the problem. Box in your answers, and make sure to **show all your work and justify your answers**.

Please draw a grading table with three rows and two columns on the front of your bluebook.

Problem A. (40 points.) Let X and Y be continuous random variables with joint p.d.f. given by the formula:

$$f(x, y) = \begin{cases} \frac{\exp(-x^2/2)}{3} & ; x > 0 \text{ and } x < y < 2x; \\ 0 & ; \text{otherwise}. \end{cases}$$