Write your name below. You must show your work and not give decimal answers (i.e. don't use a calculator or software to compute a decimal answer). You are not allowed to collaborate on the exam or seek outside help, though using your notes, the book, the recorded lectures, or material

(h) Suppose **A** is a symmetric matrix. Then, **Ax** = **b** has a solution if and only if **b** is orthogonal to the corange of **A**.

(i) If **A** is invertible, then **A** is diagonalizable.

(j) The singular values of a nonsingular matrix **A** are the same as the singular values of **A** $^{-1}$.

2. Consider the following linear transformation: $L(x, y) = \begin{pmatrix} x + y \\ 2x \quad y \end{pmatrix}$.

   (a) (4 points) Find the matrix form of $L(x, y)$ with respect to the standard basis.

   (b) (6 points) Find the matrix form of $L(x, y)$ with respect to the following basis: $v_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$; $v_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$. (The same basis is used for both the domain space and the co-domain space.)

3. (20 points) Find the Jordan decomposition of $\mathbf{A} = \begin{bmatrix} 2 & 3 & 3 & 3 \\ 4 & 1 & 1 & 1 \end{bmatrix}$

4. (20 points) Find the singular value decomposition of

$$A = \begin{pmatrix} 3 & 2 & 2 \\ 2 & 3 & 2 \end{pmatrix}.$$

7. (10 points) Find the least-squares solution of the system

$$\begin{pmatrix} 2 & 1 & 1 & 0 \\ 6 & 1 & 0 & 1 \\ 6 & 1 & 0 & 1 \\ 4 & 0 & 1 & 1 \\ & 1 & 1 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 3 \\ 7 \\ 7 \\ 5 \end{pmatrix}, \begin{pmatrix} 2 \\ 6 \\ 6 \\ 4 \end{pmatrix}, \begin{pmatrix} 3 \\ 7 \\ 7 \\ 5 \\ 6 \end{pmatrix} :$$