

This exam is worth 100 points and has 6 problems.

Show all work and simplify your answers! Answers with no justification will receive no points unless otherwise noted.

Please begin each problem on a new page.

DO NOT leave the exam until you have satisfactorily scanned and uploaded your exam to Gradescope.

5. [2360/111622 (20 pts)] Use the Method of Undetermined Coefficients to find the general solution of $\frac{d^2y}{dt^2} - \frac{dy}{dt} = 40 \cos^2 t$. Hint: $2 \cos^2 t = 1 + \cos 2t$.

6. [2360/111622 (20 pts)] Use Laplace Transforms to solve the initial value problem $y'' + 2y' = 12e^{-2t}$; $y(0) = 5$; $y'(0) = 0$. Hint: The following may be handy: $\frac{1}{s(s+2)^2} = \frac{1}{4} \left[\frac{1}{s} - \frac{1}{s+2} - \frac{2}{(s+2)^2} \right]$

Short table of Laplace Transforms: $L\{f(t)g(t)\} = F(s) \int_0^1 e^{-st} f(t) dt$

In this table, a, b, c are real numbers with $c \neq 0$, and $n = 0; 1; 2; 3; \dots$

$$L\{t^n e^{at}\} = \frac{n!}{(s-a)^{n+1}} \quad L\{e^{at} \cos bt\} = \frac{s-a}{(s-a)^2 + b^2} \quad L\{e^{at} \sin bt\} = \frac{b}{(s-a)^2 + b^2}$$

$$L\{t^n f(t)g(t)\} = (-1)^n \frac{d^n F(s)}{ds^n} \quad L\{e^{at} f(t)\} = F(s-a) \quad L\{f(t-c)g(t)\} = e^{-cs} F(s)$$

$$L\{t^n f^{(n)}(t)g(t)\} = F(s) \frac{d^n F(s)}{ds^n} \quad L\{f(t-c) \text{step}(t-c)g(t)\} = e^{-cs} F(s) \quad L\{f(t) \text{step}(t-c)g(t)\} = e^{-cs} L\{f(t+c)g(t)\}$$

$$L\{f^{(n)}(t)\} = s^n F(s) - s^{n-1} f(0) - s^{n-2} f'(0) - \dots - s f^{(n-1)}(0)$$