1. [2360/092122 (35 pts)] Consider the initial value problem $(t + 1)\,y^0 \quad 3(t + 1)\,y + e^{3t} = 0$; $y(0) = \ln 3$; $t > \quad 1$.

   (a) (4 pts) Classif y the equation.

   (b) (2 pts) Does the equation possess an y equilibrium solutions? If so, nd them.

   (c) (7 pts)

(e) FALSE Substituting $x = 0$ and $y = 0$ into the rst equation gives $x' = 2 \neq 0$. Going a bit further, the v-nullcline is $x^2 + y^2 = 2$ and the h-nullcline is $y^2 = x$

(b) (15 pts) Find the general solution to the differential equation.

(c) (5 pts) Solve the initial value problem.

<u>SOLUTION :</u>

(a) Substitute w_p into the differential equation and show that an identity results.

$$x\frac{dw_p}{dx} + (2x+1)\,w_p \overset{?}{=} 2x^2$$

$$x\left(-\frac{1}{2x^2} + 1\right) + (2x+1)\left(\frac{1}{2x} + x - 1\right) \overset{?}{=} 2x^2$$

$$-\frac{1}{2x} + x + 1 + 2x^2 - 2x + \frac{1}{2x} + x - 1 \overset{?}{=} 2x^2$$

$$2x^2 = 2x^2 \quad \checkmark$$

(b) We need the solution, w_h, to the associated homogeneous equation.

$$x\frac{dw_h}{dx} + (2x+1)\,w_h = 0$$

$$\int \frac{dw_h}{w_h} = -\int \frac{2x+1}{x}\,dx = -\int \left(2 + \frac{1}{x}\right) dx$$

$$\ln|w_h| = -2x - \ln|x| + c = -2x - \ln x + c \quad \text{since } x > 0$$

$$|w_h| = e^{-2x - \ln x + c}$$

$$w_h = \frac{C}{xe^{2x}}; \quad C \in \mathbb{R}$$

Now apply the Nonhomogeneous Principle to obtain the general solution as

$$w = w_h + w_p = \frac{C}{xe^{2x}} + \frac{1}{2x} + x - 1$$

(c) Apply the initial condition.

$$w(1) = \frac{C}{e^2} + \frac{1}{2} + 1 - 1 = \frac{3}{2} \implies C = e^2$$

giving the solution to the initial value problem as

$$w(x) = \frac{e^{2-2x}}{x} + \frac{1}{2x} + x - 1$$