

1. [2350/063023 (30 pts)] Penelope the platypus adventures about on the surface of a river, where distances are measured in meters. Suppose the temperature C on the river's surface is given by

$$T(x; y) = (2x^2 - 4)^2 + 4y^2$$

- (a) [10 pts] Penelope's current location is $(1; 3)$. Platypus risk overheating in Australian summers and can only cool down by immersing themselves in cold water.
- [5 pts] Find the direction Penelope should travel to reach lower temperatures the fastest.
 - [5 pts] What is the instantaneous rate of temperature change with respect to distance in this direction?
- (b) [20 pts] Penelope is now at the point $(2; -2)$, a location with a comfortable temperature.
- [6 pts] What is the equation of the level curve for this temperature? Sketch it.
 - [6 pts] What direction(s) should Penelope travel from this point to remain at this temperature? Report your answer(s) as unit vectors.
 - [8 pts] If Penelope were swimming along the path $\mathbf{r}(t) = (t + 2)\mathbf{i} + (t^2 + 2t - 2)\mathbf{j}$, what is the instantaneous rate of change

2. [2350/063023 (23 pts)] The following parts (a) and (b) are not related.

(a) [15 pts] Consider

$$f(x; y) = \frac{y^2 x^4 - 4x^4 y + 4x^4}{y^2}$$

- i. [5 pts] What are the domain and range of $f(x; y)$?
- ii. [5 pts] Find $\lim_{(x,y) \rightarrow (-2;2)} f(x; y)$ or show that it does not exist.

iii.

$f(4; y)$ p ion 5.363 2 Td [()] c58.664 9.9626 RTf 3.113 0 Td [(!)] T1967 6.9738 Tf 5.242 30 Td [

(b)

$$f_x = 8(x - 1) \quad f_y = 2y$$

$$0 = f_x = 8(x - 1) \Rightarrow x = 1$$

$$0 = f_y = 2y \Rightarrow y = 0$$

Thus the only critical point is $(1; 0)$: To classify the critical point, we use the Second Derivatives test.

$$f_{xx} = 8; \quad f_{yy} = 2; \quad f_{xy} = 0$$

$$D(x; y) = f_{xx}(x; y)f_{yy}(x; y) - f_{xy}^2(x; y) = 8(2) - 0^2 = 16$$

$D(1; 0) = 16 > 0$ and $f_{xx} = 8 < 0$ which implies that $(1; 0) = 100$ is a local maximum.

(c) The constraint for the Lagrange multiplier method is $g(x; y) = 3x^2 + y^2 = 9$.

$$r f - \lambda g \Rightarrow h = 8(x - 1) - 2\lambda x; \quad 2y - 2\lambda y$$

$$8(x - 1) - 2\lambda x = 0 \tag{1}$$

$$2y - 2\lambda y = 0 \tag{2}$$

$$3x^2 + y^2 = 9 \tag{3}$$

Equation (2) can be written as $(\lambda - 1)y = 0 \Rightarrow y = 0$ or $\lambda = 1$. In the case $y = 0$, $x = \pm \sqrt{3}$ from Eq. (3). In the case $\lambda = 1$, $x = 4$ from Eq. (1) which implies $y^2 = 9 - 3(16)$ in Eq. (3) which is impossible. Thus the only two critical points on the boundary are $(\sqrt{3}; 0)$ and $(-\sqrt{3}; 0)$.