APPM 2350—Exam 3

Wednesday April 13th, 6:30pm-8pm 2022

This exam has 4 problems. Please start each new problem at the top of a new page in your blue book. Show all your work in your blue book and simplify your answers. Answers with missing or insuffcient justifca-tion will receive no points. You are allowed one 8.5×11-in page of notes (ONE side). You may NOT use a calculator, smartphone, smartwatch, the Internet or any other electronic device.

Problem 1 (20 points)

Suppose the temperature at any point in the xy-plane is given by

$$T(x,y) = xe^{-y^2}$$

Suppose the area of a region R is given by

$$\int_0^3 \int_{x^2}^9 dy\,dx$$

(a) Sketch and shade the region R and clearly label your axes and any intercepts.

(b) Find the average temperature on the region R. Fully simplify your final answer.

SOLUTION:

(a) [redacted]

(b) The average temperature is given by: $$T_{avg} = \frac{\iint_R T(x,y)\,dA}{\iint_R dA} = \frac{\int_0^3 \int_{x^2}^9 xe^{-y^2}\,dy\,dx}{\int_0^3 \int_{x^2}^9 dy\,dx}.$$

Considering each integral separately and starting with the denominator:

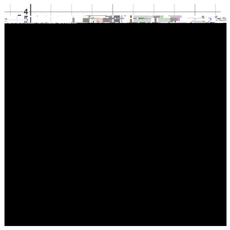
$\int$ 3xy

Problem 2 (30 points)

The volume of an object is given by

$$\int_{0}^{\pi/4}\int_{0}^{1}\int_{\sqrt{3}}^{3} r\,dz\,dr\,d \;+\; \int_{0}^{\pi/4}\int_{1}^{\sqrt{3}}\int_{r\sqrt{3}}^{3} r\,dz\,dr\,d$$

(a) Sketch and shade a 2D cross section of the object in the rz-plane (for any such that $0 \le \quad \le \quad =$



The arrow enters the region when $z = \sqrt{3} \implies \cos = \sqrt{3} \implies = \sqrt{3}\sec$

The arrow exits the region when $z = 3 \implies \cos = 3 \implies = 3\sec$.

To fnd the limits, we note that the smallest on the region occurs when $= 0$.

The largest occurs when $z = \sqrt{3}r \implies \cos = \sqrt{3} \sin$

$\implies \tan = \frac{1}{\sqrt{3}} \implies = = 6$

The limits are the same as in cylindrical: $0 \le \le = 4$

We replace $r\,dz\,dr\,d$ with $^2 \sin\, d\, d\, d$

$$\implies 5 = \iint_R \frac{||\nabla g||}{|\nabla g \cdot \hat{p}|}\, dA$$

Since we are projecting onto the yz-plane, $p = \hat{\imath}$

Let $g(x; y; z) = 3x - 4y + 2z$

$\implies \nabla g = \langle 3; -4; 2 \rangle \implies ||\nabla g|| = \sqrt{3^2 + (-4)^2 + 2^2} = \sqrt{29}$

and $|\nabla g \cdot \hat{\imath}| = 3$

Thus

$$5 = \iint_R \frac{\sqrt{29}}{3}\, dA$$

$$\implies 5 = \frac{\sqrt{29}}{3} \iint_R dA$$

$$\implies \text{Area of } R = \iint_R dA = \boxed{\frac{15}{\sqrt{29}}}$$

Problem 4 (20 points)

Consider the region, R, that is bounded by

$$y = \sqrt{x}; \quad y = \sqrt{x} + 2; \quad y = 4 - \sqrt{x}; \quad y = 6 - \sqrt{x}$$

where x and y are measured in meters.

A sprinkler sprays water on the region R in such a way that the depth of water (in meters) that reaches the point $(x; y)$ in 1 hour is given by

$$g(x; y) = \frac{e^{(y - \sqrt{x})}}{20\sqrt{x}}$$

Use an appropriate uv-transformation to find the total volume of water the sprinkler sprays on the region R in 1 hr. Fully simplify your final answer.

<u>SOLUTION :</u>

$$g$$

$$\text{Total Volume} = \iint_R = \frac{e^{(y - \sqrt{x})}}{20\sqrt{x}}\, dA$$

The original region looks like:

Solving for the xy-transformation yields:

$$x = \left(\frac{v - u}{2}\right)^2 \qquad y = \frac{u + v}{2}$$

Thus our boundaries convert as follows:

Boundary in xy	Corresponding Boundary in uv
$y=\sqrt{x}$	$u=0$
$y=\sqrt{x}+2$	$u=2$
$y=4-\sqrt{x}$	$v=4$
$y=6-\sqrt{x}$	$v=6$