APPM 2350—Exam 3

Wednesday, Nov 17th 6:30pm-8pm 2021

This exam has 5 problems. Please start each new problem at the top of a new page in your blue book. Show all your work in your blue book and simplify your answers. Answers with no justification will receive no points. You are allowed one 8.5×11-in page of notes (ONE side). NO calculators, smartphones/watches, or any other electronic devices allowed.

Problem 1 (17 pts)

A wire lies along the intersection of the surfaces $z = \frac{1}{3}xy$ and $y = \frac{1}{2}x^2$ from $(0, 0, 0)$ to $(4, 8, \frac{32}{3})$. Suppose the charge density at any point on the wire is given by

$$\rho(x, y, z) = x \ \frac{Coulombs}{meter}$$

Find the total charge on the wire. (You can assume distance in xyz-space is measured in meters).

<u>SOLUTION:</u>

$$\text{Charge} = \int_C \rho(x, y, z)\, ds = \int_C x\, ds$$

where C is the path traced out by the wire.

To evaluate, we first need to parameterize the wire.

One simple parameterization:

$$\mathbf{r}(t) = \langle t, \frac{1}{2}t^2, \frac{1}{6}t^3 \rangle, \quad 0 \le t \le 4$$

$$\text{Charge} = \int_0^4 t\, ||\mathbf{r}'(t)||\, dt$$

$$\mathbf{r}'(t) = \langle 1, t, \frac{1}{2}t^2 \rangle$$

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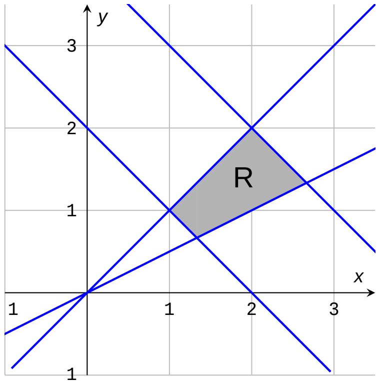
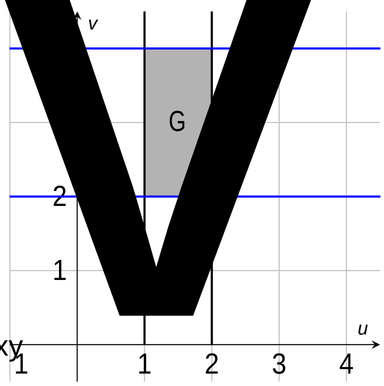
(a)



(b) We will use the following substitution:

$$u = \frac{x}{y} \qquad v = x + y$$



xy

Solving for the *xy*-transformation yields:

$$x = \frac{uv}{1+u} \qquad y = \frac{v}{1+u}$$

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xy

Let $g(x, y, z) = xy - z$

Since the surface is given as a function of x and y it's easiest to project the surface onto the xy-plane Thus, let $\hat{p} = \mathbf{k}$.

$$dS = \frac{\|\nabla g\|}{|\nabla g \cdot \mathbf{k}|}\, dA = \frac{\sqrt{y^2 + x^2 + 1}}{|1|}\, dA$$

$$T_{ave} = \frac{\iint_S 3xyz\, dS}{\iint_S dS} = \iint_R 3xyz \sqrt{xy}$$

(b) 

(c) 

(d) (i)

$$Mass = \int_0^{\frac{a}{\sqrt{2}}} \int_{\sqrt{a^2-y^2}}^{\sqrt{3a^2-y^2}} \int_a^{\sqrt{4a^2-x^2-y^2}} \sqrt{x^2+y^2}\, dz\, dx\, dy + \int_{\frac{a}{\sqrt{2}}}^{a\sqrt{\frac{3}{2}}} \int_y^{\sqrt{3a^2-y^2}} \int_a^{\sqrt{4a^2-x^2-y^2}} \sqrt{x^2+y^2}\, dz\, dx\, dy$$

(ii)

$$Mass = \int_0^{\frac{\pi}{4}} \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \int_{a\csc\phi}^{2a} \rho^3 \sin^2\phi \, d\rho\, d\phi\, d\theta + \int_0^{\frac{\pi}{4}} \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \int_{a\sec\phi}^{2a} \rho^3 \sin^2\phi \, d\rho\, d\phi\, d\theta$$

Problem 5 (17 points)
A neighborhood sits in the region in the xy-plane given by

$$x^2 + y^2 \le 4 \text{ and } x \ge 1$$

(where distances in the xy-plane are measured in miles). An earthquake occurs with epicenter at the origin. Suppose at each point in the neighborhood, the energy density released from the earthquake is given by the function

$$E(x;y) = \frac{10^6}{(d(x;y))^3} \quad \frac{joules}{miles^2}$$

where $d(x;y)$ is the distance from $(x;y)$ to the epicenter.
Find the total amount of energy (in joules) released by the earthquake in this neighborhood.

Solution:

End Of Exam