1. (18 pts) The following two problems are not related.

   (a) Let
   $$w = \frac{2}{x} + \ln(yz); \quad x = r\sin r; \quad y = \frac{r}{2s}; \quad z = \frac{s^2}{r}.$$
   Find $\partial w/\partial s$. Express your answer in terms of r and s.

   (b) Show that $\displaystyle\lim_{(x,y)\to(0,0)} \frac{2x^2y}{3x^4+4y^2}$ does not exist.

Solution:

(a)

$$\begin{aligned}\frac{\partial w}{\partial s} &= \frac{\partial w}{\partial y}\frac{\partial y}{\partial s} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial s}\\ &= \frac{1}{y}\ \frac{r}{2s^2} + \frac{1}{z}\ \frac{2s}{r}\\ &= \frac{r}{2s^2y} + \frac{2s}{rz}\\ &= \frac{r}{2s^2}\ \frac{2s}{r} + \frac{2s}{r}\ \frac{r}{s^2}\\ &= \frac{1}{s} + \frac{2}{s} = \frac{1}{s}\end{aligned}$$

(b) Approaching $(0,0)$ along the x-axis,

$$\lim_{(x,0)\to(0,0)} \frac{2x^2y}{3x^4+4y^2} = \lim_{(x,0)\to(0,0)} \frac{0\ 2x^2}{(3x^4)} = 0.$$

Similarly the limit equals 0 approaching the origin along the y-axis.

0) along the line y [redacted] e result is the same:

$$\lim_{(x,x)\to(0,\ldots)} \ldots \frac{\ldots x^2y}{\ldots 4y^2} = \lim_{(x,x)\to(0,0)} \frac{2x^3}{32} \ldots \lim$$

o(0 ... g the origin along the

2. (24 pts) Let $f(x,y) = x^2y$.

(a) Find the rate of change of f at $Q(-1,3)$ in the direction toward the origin.

(b) Find a unit vector tangent to the level curve $f(x,y) = 3$ at Q.

(c) What is the greatest possible rate of change of f at Q?

(d) Find a vector normal to the surface $z = f(x,y)$ at Q.

Solution:

(a)

$$\nabla f(x,y) = \langle 2xy, x^2\rangle \implies \nabla f(-1,3) = \langle -6, 1\rangle$$

Let O represent the origin and let the direction vector $\mathbf{u} = \dfrac{\vec{QO}}{|\vec{QO}|} = \dfrac{\langle 1,-3\rangle}{\sqrt{10}}$. Then

$$D_{\mathbf{u}}f(-1,3) = \nabla f(-1,3)\cdot\mathbf{u} = \langle -6,1\rangle\cdot\frac{\langle 1,-3\rangle}{\sqrt{10}} = -\frac{9}{\sqrt{10}}.$$

(b) The gradient vector $\nabla f(-1,3) = \langle -6,1\rangle$ is orthogonal to the level curve at Q, so a tangent vector is $\langle 1,6\rangle$ or $\langle -1,-6\rangle$. The corresponding unit vector is $\dfrac{\langle 1,6\rangle}{\sqrt{37}}$ or $\dfrac{\langle -1,-6\rangle}{\sqrt{37}}$.

(c) The greatest possible rate of change is $|\nabla f(-1,3)| = |\langle -6,1\rangle| = \sqrt{37}$.

(d) Let $F(x,y,z) = f(x,y) - z$. Then

3. (26 pts) Let $g(x, y) = x \sin(2y)$.

(a) Find all critical points of g in the open region $R = \left\{ (x, y) \mid |x| < \frac{\pi}{2},\ |y| < \frac{\pi}{2} \right\}$. Use the Second Derivatives Test to classify the points.

(b) Use Taylor series to find a quadratic approximation of g at $(0, 0)$.

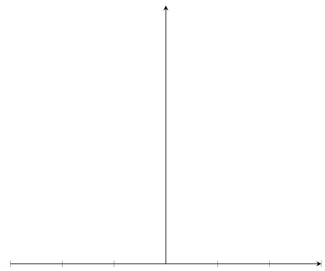
(c) Find the maximum error in the quadratic approximation of g for $|x| \le 0.1$, $|y| \le 0.1$. You may leave the final answer unsimplified.

Solution:

(a)

$$g_x = \sin(2y)$$

4. (20 pts) An archway has the shape of the parabola $y = 15 \quad x^2$ for $y \quad 0$. Use Lagrange multipliers to determine the width and height in units of the largest rectangular box that will fit through the archway.



5. (12 pts) Match the three surfaces to their contour plots. No explanation is necessary. (For each surface, the first octant is facing toward the front.)

Surface 1 Surface 2 Surface 3