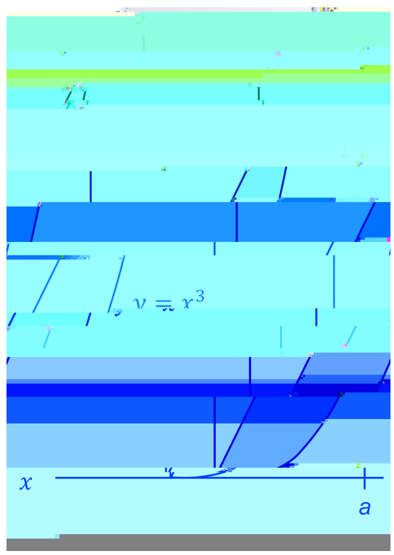
1. (10 points) Solve the following initial value problem:

$$\frac{dy}{dx} = x^2 \csc(y); \qquad y(2) = 0:$$

Write your answer in the form $y = f(x)$.

2. (12 pts) Consider the lamina depicted below, which is bounded above by a line through the origin and below by the curve $y = x^3$ on the interval $0 \le x \le a$. The line and the curve intersect at $x = 0$ and at $x = a$. The lamina has a uniform density of : What value of a is needed so that $x = 1$?



3. (28 pts) Consider the region R, in quadrant I, bounded by the x-axis, the y-axis, $y = 2$, and $y = \ln(2x)$.

   (a) Use the grid below to sketch and shade the region R. Label the coordinates of the intersections of two curves. (You may find it helpful to know that $e^2 \approx 7:4:$)

   (b) Set up but <u>do not evaluate</u> expressions involving integrals to determine each of the following:

      I. The volume of revolution found by revolving the given region about the y-axis using cylindrical shells.
      II. The area of the surface generated by rotating the curve $f(x) = \ln(2x)$ with $0 \le y \le 2$ about the y-axis.
      III. The perimeter of R. (That is, find the arc length of the entire perimeter of R.)

4. (27 pts) Determine if each of the following converges or diverges. Be sure to fully justify your answers using the teeter of R

(c) $$\sum_{n=2}^{\infty} \ln \left(\frac{n^2 + n}{4 - 9n + 5n^2} \right)$$

5.