1. (34 points) Find the requested information. The problems are unrelated.

   (a) Evaluate $\displaystyle\int \frac{\tan^{-1}(x)}{x^2}\,dx$ (Hint: Start with IBP)

   (b) Find y as a function of x given that $\dfrac{dy}{dx} = 2x\sqrt{1-y^2}$ and $y(0) = 1$

   (c) Find the sum of the series $-\dfrac{x^6}{3!} + \dfrac{x^{10}}{5!} - \dfrac{x^{14}}{7!} + \cdots$

   (d) For what values of θ does the series $\displaystyle\sum_{n=0}^{\infty} \sin^2(\theta)\cos^{2n}(\theta)$ converge? Find the sum for those values of θ.

2. (16 points) Decide whether the following quantities are convergent or divergent. Explain your reasoning and name any test you use.

   (a) The sequence given by $a_n = \left(1 - \dfrac{\ln(3)}{n}\right)^n$, for $n = 1, 2, \ldots$

   (b) $\displaystyle\int_1^{\infty} \frac{1}{x^2}\sqrt{1 + \frac{3}{x^3}}\,dx$

3. (12 points) Consider the series $\displaystyle\sum_{k=1}^{\infty} b_k$. Suppose the n^{th} partial sum of the series is $s_n = 2 - \dfrac{2}{n+1}$.

   (a) What is s_3?

   (b) Find a simple formula for b_n.

   (c) What does $\{b_n\}$ converge to?

   (d) What is the sum of the series $\displaystyle\sum_{k=1}^{\infty} b_k$?

4. (25 points) Recall $\cosh(x) = \dfrac{e^x + e^{-x}}{2}$.

   (a) Find the MacLaurin series of the hyperbolic cosine function.

   (b) Find the interval of convergence for the power series from part (a).

   (c) Find $T_3(x)$, the Taylor polynomial of order 3, of the hyperbolic cosine centered at $a = 0$. Use the Taylor Remainder formula to find an upper bound for the absolute error if $T_3(x)$ is used to approximate $\cosh(1)$.

   (d) Use the MacLaurin series (not L'Hôpital!) to evaluate the following limit:

$$\lim_{x \to 0} \frac{\cosh(x) - \left(1 - \frac{x^2}{2}\right)}{x^4}.$$

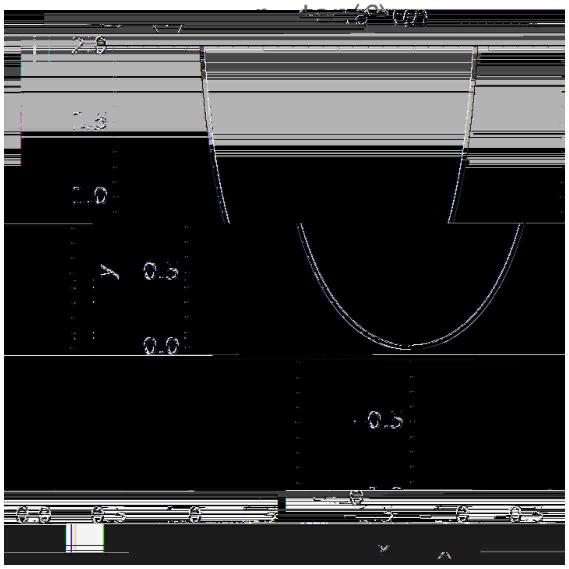
5. (18 points) Suppose $g(x)$ equals the power series $\displaystyle\sum_{n=2}^{\infty} \frac{(n+1)(x+b)^n}{c^{2n}}$, where b and c are constants, and the series has an interval of convergence of $-6 < x < 2$.

   (a) Find the center and radius of convergence of the series.

(b) Evaluate $\int g(x)\,dx$ as a power series.

(c) Given the interval of convergence, find possible values for b and c. Justify your answer using appropriate test(s).

6. (25 points) For this problem, let $r = \tan(\theta)$ for $-\pi/2 < \theta < \pi/2$. The polar graph (in the xy plane) is given below. Answer the following questions.



(a) Find an equation for the tangent line at $\theta = \pi/4$.