Name: _____

APPM 1350
Summer 2023

Exam 2

June 30

Instructions:

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1. (18 pts) Given the curve

$$y\sin^2(x) = \cos(x) + y^3$$

   (a) Find the derivative $\frac{dy}{dx}$ in terms of x and y.

   (b) Find the slope of the tangent line to the curve at $(\frac{\pi}{2}; 1)$.

2. (24 pts) Let $g(x) = \sqrt{25 - x}$

   (a) Find the linearization of $g(x)$ for $a = 9$ and use it to approximate $\sqrt{15.6}$. Your answer should be in decimal form.

   (b) Find the value(s) c that satisfy the conclusion of the Mean Value Theorem for $g(x)$ on $[0; 9]$. Leave your answer as an improper fraction.

3. (22 pts) Let x and y be two positive numbers under the constraint that $x + 2y = 4$. Determine the values of x and y that would maximize $(x + 1)(y + 2)$. Verify that it is a maximum by using either the first derivative test or the second derivative test.

4. (36 pts) Consider the function $y = x(x - 4)^3$,
   its simplified derivative $y' = 4(x - 4)^2(x - 1)$,
   and its simplified second derivative $y'' = 12(x - 4)(x - 2)$.
   Justify your answers for each of the following problems.

   (a) Find the $x-$ and $y-$intercepts.

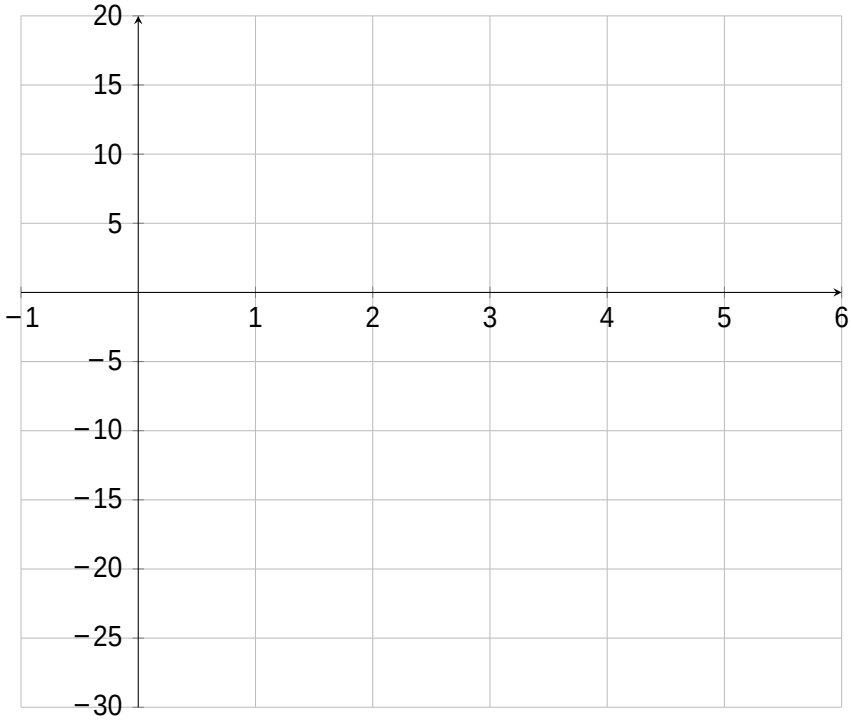
   (b) On what intervals is the function increasing? On what intervals is it decreasing?

   (c) Find the $(x; y)$ coordinates of any local maximum and local minimum values, if they exist. If none exist, state this.

   (d) On what intervals is the function concave up? On what intervals is it concave down?

   (e) Find the $(x; y)$ coordinates of any inflection points, if they exist. If none exist, state this.

   (f) Use the empty plot located on the next page to sketch the graph of this function. Carefully label all key features such as any intercepts, maximum(s), minimum(s), and inflection point(s). (Hint: there are no asymptotes for this function).



THIS IS THE END OF THE EXAM