APPM 1345
Exam 3
Spring 2024

Name		
Instructor	Richard McNamara	**Section** 150

1. (23 pts) Parts (a) and (b) are unrelated.

   (a) Find the inverse function of $f(x) = \dfrac{\ln(2x)}{1 + \ln(2x)}$ for $x \quad \dfrac{1}{2}$.

   Express your answer in the form $f^{-1}(x)$. (You do not have to identify the inverse function's domain.)

(b) Consider the function $g(x) = 2x \quad \cos x$.

i. Explain why g is invertible, based on its derivative.

ii. Find an equation of the line that is tangent to the curve $y = g^{-1}(x)$ at the point $(4 \quad 1; 2)$.
Hint: Do not attempt to identify the function $g^{-1}(x)$.

2. (25 pts) Parts (a) and (b) are unrelated.

   (a) If a substance undergoing exponential decay has a half-life of 50 years, how many years would it take for a sample of that substance to decay to 1 percent of its original amount?

(b) Consider the function $p(t) = p_0 e^{kt}$, which represents an exponential growth model for a population, where the constant p_0 represents the initial population size and the constant k represents the population's relative growth rate. Suppose $p(10) = 2$ and $p(50) = 6$.

i. Find the value of k.

ii. Find the value of p_0.

3. (26 pts) Evaluate the following derivatives using properties of logarithms and/or logarithmic differentiation. Do **not** fully simplify your answers, although they must be expressed as functions of x.

(a) $\dfrac{d}{dx}\left[\ln \dfrac{(10 \cos^2 x)\sqrt{x^4+6}}{e^{x \sin x}}\right]$

(b) $\frac{d}{dx}\left(e^{x} + e^{x}\right)^{x}$

4. (26 pts) Evaluate the following integrals.

(a) $\int_1^2 \frac{2^x}{9 \quad 2^x}\, dx$

(b) $\int \frac{x}{x \quad 1}\, dx$

END OF EXAM

Your Initials _____

ADDITIONAL BLANK SPACE

If you write a solution here, please clearly indicate the problem number.