APPM 1345

Exam 1

Spring 2024

1. (29 pts) Parts (a) and (b) are unrelated.

(a) Find the most general form of $u(x)$ such that $u''(x) = \sin x + x^{3=4} + 1$.

Solution:

$$u'(x) = \quad \cos x + \frac{x^{7=4}}{7=4} + x + C_1 = \quad \cos x + \frac{4}{7}x^{7=4} + x + C_1$$

$$u(x) = \quad \sin x + \frac{4}{7}\;\frac{x^{11=4}}{11=4} + \frac{x^2}{2} + C_1 x + C_2 = \quad \sin x + \frac{4}{7}\;\frac{4}{11}x^{11=4} + \frac{x^2}{2} + C_1 x + C_2$$

$$u(x) = \boxed{\quad \sin x + \frac{16}{77}x^{11=4} + \frac{x^2}{2} + C_1 x + C_2}$$

(b) Consider a particle that is moving along a linear path. Let t represent the time in seconds, let $s(t)$ represent the position in meters, let $v(t)$ represent the velocity in m=s, and let $a(t)$ represent the acceleration in m=s^2. Suppose the particle is accelerating at a constant rate of $a = 4$ m=s^2, its initial velocity is $v(0) = 2$ m=s, and its initial position is $s(0) = 6$ meters. How many seconds would it take for the particle to move from a position of $s = 6$ meters to a position of $s = 30$ meters? Show the full derivation of your results (do not simply use a formula from a different course).

Solution:

$a(t) = 4$

The velocity function is an antiderivative of the acceleration function.

$v(t) = 4t + C_1$ and $v(0) = 2$, so $C_1 = 2$. Therefore, $v(t) = 4t + 2$.

The position function is an antiderivative of the velocity function.

$s(t) = 2t^2 + 2t + C_2$ and $s(0) = 6$, so $C_2 = 6$. Therefore, $s(t) = 2t^2 + 2t + 6$.

We need to solve for the value of t for which $s(t) = 30$.

$s(t) = 2t^2 + 2t + 6 = 30$

$2t^2 + 2t \quad 24 = 0$

$2(t^2 + t \quad 12) = 2(t \quad 3)(t + 4) = 0 \quad) \quad t = \quad 4; 3$

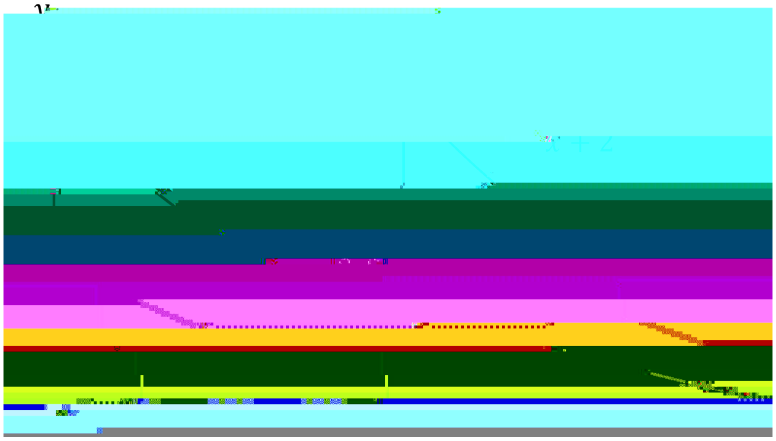
Movement in the positive direction with an acceleration that is always positive implies that t must be positive.

$\boxed{t = 3 \text{ seconds}}$

2. (22 pts) The rectangle shown has one side on the positive x-axis, one side on the positive y-axis, and its upper right corner at the point $(a; b)$, which lies on the curve

$$y = \frac{5}{x+2} \quad 1$$

Identify the point $(a; b)$ that produces the rectangle with the largest area, and use the Second Derivative Test to confirm that your result is a local maximum value of the area function.



Solution:

The area of the rectangle whose upper right corner is at the point $(x; y)$, where $y = \dfrac{5}{x+2} \quad 1$ is

$$A = xy = x \quad \frac{5}{x+2} \quad 1 \quad = \frac{5x}{x+2} \quad x \quad) \quad A(x) = \frac{5x}{x+2} \quad x$$

Next, we find the critical numbers of the area function $A(x)$

3. (23 pts) Suppose Newton's Method is used to estimate the value of a root of $y = p(x) = 0{:}05(2x^3 \quad 3x^2 \quad 12x + 5)$.

(a) Write the expression for Newton's Method for the specified function $p(x)$. Your answer should be an expression for x_{n+1} in terms of x_n.

Solution:

The general equation for Newton's Method when applied to a function $p(x)$ is $x_{n+1} = x_n \; {}_{p}(\; \frac{p(x_n)}{p^{\prime}(x_n}$

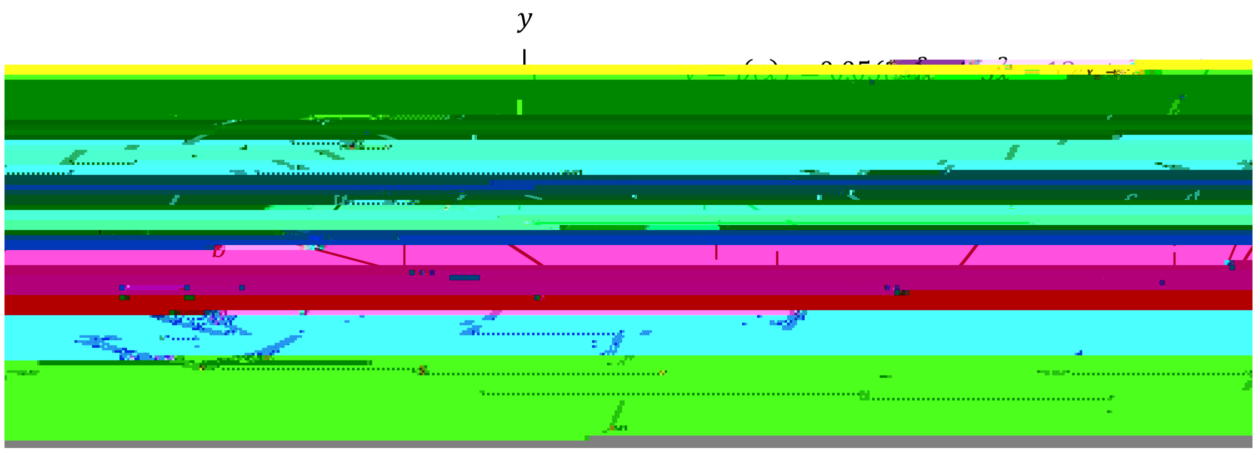
p

Soltton:

(c) The following graph of $y = p(x)$ labels four particular x values as constants a through d. The line tangent to $y = p(x)$ at each of those four x values is also shown.

i. Determine the actual numerical values of the constants b and d, which correspond to horizontal tangent lines.

ii. Consider using each of the four labeled x values (a through d) as the value of x_0 in Newton's Method. For which of those choices would Newton's Method fail to converge? List all that apply, and in each case, briefly explain why it would not converge.



Solution:

i. Part (a) indicates that $p'(x) = (0.05)(6)(x^2 - x - 2) = 0.3(x^2 - x - 2)$. The curve $y = p(x)$ has a horizontal tangent at $x = b$ and at $x = d$, the numerical values of which can be found by setting $p'(x) = 0$ and solving for x.

$p'(x) = 0.3(x^2 - x - 2) = 0.3(x - 2)(x + 1) = 0$

The solutions are $x = -1$ and $x = 2$, so that $\boxed{b = -1}$ and $\boxed{d = 2}$

ii. Newton's Method would not converge for $\boxed{x_0 = b}$ and $\boxed{x_0 = d}$ because the curve $y = p(x)$ has horizontal tangent lines at those locations.

Newton's Method would not converge for $\boxed{x_0 = c}$ because its tangent line leads to a value of $x_1 = b$, and there is a horizontal tangent line at that location.

4. (26 pts) Let $f(x) = (x \quad 1)\sin x + \cos x$ on the interval $[0; \quad]$. Answer the following for the specified interval.

(a) Identify all critical numbers of $f(x)$.

(b) For which values of x is $f(x)$ increasing and for which values of x is $f(x)$ decreasing? Express your answers using interval notation.

(c) Identify the x-coordinate of each local maximum and minimum value of $f(x)$ (if any). Use the First Derivative Test to classify each one.

Solution:

(a)

Your Initials _________

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If you write a solution here, please clearly indicate the problem number.