

APPM 1345

Exam 3

Spring 2023

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| Name | | |
| Instructor | Richard McNamara | Section 150 |

1. (23 pts) Parts (a) and (b) are unrelated.

(a) Find the inverse function of $g(x) = 6x^5 - 1$.

Solution:

$$y = g(x) = 6x^5 - 1$$

$$6x^5 = y + 1$$

$$x^5 = \frac{y + 1}{6}$$

2. (27 pts) Parts (a), (b) and (c) are unrelated.

- (a) Suppose $\frac{1}{3}$ of a radioactive substance remains after decaying exponentially for 10 years. Find the half-life of the substance, including the correct unit of measurement. Fully support your answer.

Solution:

Since the substance is undergoing exponential decay, the amount of the substance at time t years can be represented by $y(t) = y_0 e^{kt}$, where $y_0 = y(0)$ is the amount of the substance at time $t = 0$ and k is the relative rate of change. Therefore, for $t = 10$, we have the following:

$$y(10) = y_0 e^{10k} = \frac{y_0}{3} \quad) \quad 10k = \ln \frac{1}{3} \quad) \quad k =$$

3. (24 pts) Evaluate the following derivatives using properties of logarithms and/or logarithmic differentiation. Do **not** fully simplify your answers, although they must be expressed as functions of x .

(a) d

4. (26 pts) Evaluate the following integrals. Fully simplify your answers.

$$(a) \int_4^9 \frac{dx}{x(1-2\sqrt{x})}$$

Solution:

Let $u = 1 - 2\sqrt{x} = 1 - 2x^{1/2}$, which implies that $du = -x^{-1/2} dx = -\frac{dx}{\sqrt{x}}$.

$$x = 4 \quad \Rightarrow \quad u = 1 - 2\sqrt{4} = -3$$

$$x = 9 \quad \Rightarrow \quad u = 1 - 2\sqrt{9} = -5$$

$$\int_4^9 \frac{dx}{x(1-2\sqrt{x})} = \int_{-3}^{-5} \frac{du}{u} = \int_{-5}^{-3} \frac{du}{u} = \ln|u| \Big|_{-5}^{-3} = \ln 3 - \ln 5 = \ln \frac{3}{5}$$

Your Initials _____

ADDITIONAL BLANK SPACE

If you write a solution here, please clearly indicate the problem number.