

1. (32 pts) The position function of a particle is given by $s(t) = 4t^2$ on the interval $1 \leq t \leq 16$, where position is in meters and time is in seconds.

(a) Determine the particle's velocity function $v(t)$. Include the correct unit of measurement.

Solution:

$$s(t) = 4t^2 \quad t = 4t^{1=2} \quad t$$

$$v(t) = s'(t) = 2t^{1=2} \quad 1 = \boxed{\frac{2}{t} \quad 1 \text{ m/s}}, \quad 1 < t < 16$$

(b) Determine the total distance traveled by the particle on the interval $1 \leq t \leq 16$.

- (c) i. Verify that all hypotheses of the Mean Value Theorem are satisfied for the given position function $s(t) = 4t^2 - t^3$ on the interval $1 \leq t \leq 16$.

Solution:

$s(t)$ is continuous on $[1; 16]$ and differentiable on $(1; 16)$

- ii. Use the Mean Value Theorem to determine all time values c on the interval $1 \leq t \leq 16$, if any, for which the instantaneous velocity of the particle equals the average velocity of the particle on that interval. Include the correct unit of measurement.

Solution:

The Mean Value Theorem states that since the hypotheses have been satisfied, there exists at least one number c on the interval $(1; 16)$ such that

$$s'(c) = \frac{s(16) - s(1)}{16 - 1} = \frac{0 - 3}{15} = -\frac{1}{5}$$

Therefore, using the result from part (a), we have

$$v(c) = \frac{2}{c} - 1 = -\frac{1}{5}$$

$$\frac{2}{c} = \frac{4}{5}$$

$$c = \frac{5}{2}$$

$$c = \frac{25}{4} \text{ sec}$$

2. (11 pts) Let v represent a person's walking speed, expressed in miles per hour, and let p represent the corresponding walking pace, expressed in minutes per mile. The pace can be expressed as the following function of speed:

$$p(v) = \frac{60}{v}; \quad v > 0$$

- (a) Find the linearization $L(v)$ that approximates $p(v)$ near $v = 4$.

Solution:

$$p(v) \quad L(v) = p(4) + p'(4)(v - 4)$$

$$p(4) = \frac{60}{4} = 15$$

$$p'(v) = \frac{60}{v^2} \quad) \quad p'(4) = \frac{60}{4^2} = \frac{15}{4}$$

$$L(v) = \boxed{15 + \frac{15}{4}(v - 4)}$$

- (b) Use your linearization from part (a) to estimate the walking pace of a person moving at 4.2 miles per hour. Include the correct unit of measurement. You must use linearization to earn credit.

Solution:

$$p(4.2) \quad L(4.2) = 15 + \frac{15}{4}(4.2 - 4) = 15 + \frac{15}{4} \cdot \frac{1}{5}$$

3. (26 pts) Consider the function $f(x) = \sin x + \cos^2 x$ on the interval $[0; 2\pi]$.

(a) Identify all critical numbers of f on the specified interval.

Solution:

Critical numbers are values of x in the domain of f such that $f'(x) = 0$ or $f'(x)$ does not exist. There are no critical numbers of the latter type for this function.

$$f'(x) = \cos x + 2 \cos x (-\sin x) = \cos x(1 - 2 \sin x) = 0$$

$x = \pi/2$ is the only value of x

6. (15 pts) Determine $g'(x)$ for the function $g(x) = \sqrt[3]{3x + 1}$ by using the definition of derivative. You must obtain g' by evaluating an appropriate limit to earn credit.

Solution:

The definition of derivative indicates that $g'(x) = \lim_{h \rightarrow 0} \frac{g(x+h) - g(x)}{h} = \lim_{h \rightarrow 0} \frac{\sqrt[3]{3(x+h) + 1} - \sqrt[3]{3x + 1}}{h}$

7. (22 pts) Consider the function $h(x) = \frac{\sin x}{x(x-2)}$.

(a) Find the $(x; y)$