1. (28 pts) The position function of a particle is given by $s(t) = -10t^2 + 40t + 50$ on the interval $[0, 5]$, where t is measured in seconds and s is measured in feet.

   (a) i. Find all critical numbers of $s(t)$ on the given interval and the corresponding function values.

   Solution:

   $s'(t) = -20t + 40$

   Critical numbers are values of t such that $s'(t) = 0$ or $s'(t)$ does not exist. There are no critical numbers of the latter type for this function.

   $s'(t) = -20t + 40 = 0 \quad \Rightarrow \quad t = 2$

   $\boxed{t = 2}$ is the only critical number of this function, and it does lie on the given interval.

   $s(2) = (-10)(2^2) + (40)(2) + 50 = -40 + 80 + 50 = \boxed{90}$

   ii. Identify the absolute maximum and minimum values of $s(t)$ on the given interval and the corresponding values of t at which they occur.

   Solution:

   The Closed Interval Method compares the function values at the critical numbers and the boundaries of the interval. From part (i) we have the function value at the only critical number:

   $s(2) = 90$

   The function values at the boundaries are as follows:

   $s(0) = (-10)(0^2) + (40)(0) + 50 = 0 + 0 + 50 = 50$
   $s(5) = (-10)(5^2) + (40)(5) + 50 = -250 + 200 + 50 = 0$

   A comparison of the three preceding function values leads to the following abosolute maximum and minimum values of $s(t)$ on $[0, 5]$:

   Absolute maximum: $\boxed{s(2) = 90}$

   Absolute minimum: $\boxed{s(5) = 0}$

(b) i. Determine the particle's velocity and acceleration at $t = 3$

2. (24 pts) Parts (a) and (b) are not related.

(a) Find the equations of the tangent and normal lines to the curve $5y + \sin y + 1 = x^2$ at the point $(\ \ 1; 0)$.

Solution:

Use implicit differentiation.

$$\frac{d}{dx}[5y + \sin y + 1] = \frac{d}{dx}\ x^2$$

$$5\ \frac{dy}{dx} + \cos y\ \frac{dy}{dx} = 2x$$

$$(5 + \cos y)\frac{dy}{dx} = 2x$$

$$\frac{dy}{dx} = \frac{2x}{5 + \cos y}$$

$$\frac{dy}{dx}_{(\ \ 1;0)} = \frac{(2)(\ \ 1)}{5 + \cos(0)} = \frac{\ \ 2}{5 + 1} = \ \ \frac{1}{3}$$

Tangent line: $\boxed{y = \ \ \frac{1}{3}(x + 1)}$

Normal line: $\boxed{y = 3(x + 1)}$

(b) Evaluate $\frac{d}{dx}[x \tan^4 x]$.

Solution:

Use product rule and chain rule.

$$\frac{d}{dx}[x \tan^4 x] = (x)[4 \tan^3 x\ \ \sec^2 x] + \ \ \tan^4 x\ \ (1) = \boxed{\tan^3 x(4x \sec^2 x + \tan x)}$$

3. (35 pts) A 10-foot ladder is initally resting against a vertical wall. Suppose the bottom of the ladder slides away from the wall at a constant rate of 2 feet per second. Determine the values of the following quantities when the top of the ladder is 6 feet above the floor. Include the correct units of measurement.

   (a) The speed at which the top of the ladder is sliding down the wall

      Solution:

      Figure 1 depicts the geometry of the problem, with variables assigned to represent various physical attributes.

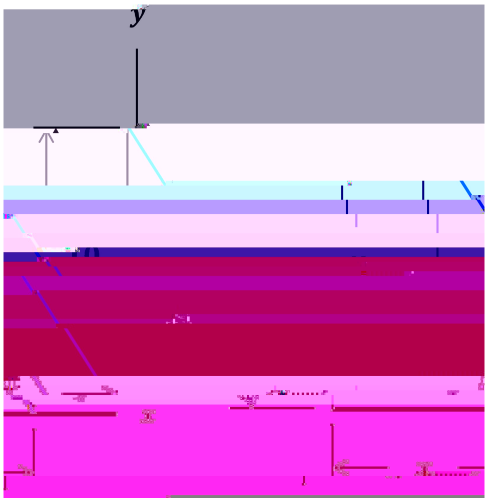

Figure 1

In the context of the assigned variables, $dx=dt$ represents the speed at which the bottom of the ladder is moving across the floor, which is given to be a constant 2 feet per second.

The objective of part (a) involves determining the value of $dy=dt$ when $y = 6$. In order to relate the unknown $dy=dt$ to the known $dx=dt$, we need a relationship between y and x, which is provided by the Pythagorean Theorem: $x^2 + y^2 = 10^2$.

$$\frac{d}{dt}[x^2 + y^2] = \frac{d}{dt}[100]$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0 \quad) \quad \frac{dy}{dt} = \frac{x}{y}\frac{dx}{dt}$$

Figure 2 depicts the situation when $y = 6$.

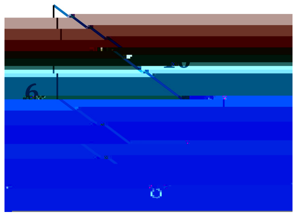

Figure 2

The Pythagorean Theorem has been used to determine that $x = \sqrt{10^2 \quad 6^2} = 8$.

$$\left.\frac{dy}{dt}\right|_{y=6} = \quad \frac{8}{6}(2) = \quad \frac{8}{3}$$

Since *dy=dt*, which represents the rate of change in *y* with respect to *t*, is negative, the speed at which the top of the ladder is sliding *down* the wall is 8=3 ft/s

(b) The rate at which the angle between the ladder and the floor is decreasing

(*Hint:* The distance between the wall and the bottom of the ladder is related to the angle between the ladder and the floor. Start by writing an equation that represents that relationship.)

Solution:

The objective of part (b) involves determining the value of *d* =*dt* when $y = 6$. In order to relate the unknown *d* =*dt* to the known *dx=dt*

4. (20 pts) The side of a cube is measured to be 2 cm with a possible error in measurement of up to 0.1 cm.

(a) Identify the function $V(x)$ representing the volume of the cube, where x represents the length of a side.

Solution:

$$V(x) = \boxed{x^3}$$

(b) Find the linear approximation of $V(x)$ about $x = 2$.

Solution:

$$V(x) \approx L(x) = V(2) + V'(2)(x - 2)$$

$$V(2) = 2^3 = 8$$

$$V'(x) = 3x^2 \quad \Rightarrow \quad V'(2) = (3)(2^2) = 12$$

$$L(x) = \boxed{8 + 12(x - 2)}$$

(c) Use differentials to estimate the maximum possible error in computing the volume of the cube. Include the correct units of measurement.

Solution:

$$\frac{dV}{dx} = 3x^2 \quad \Rightarrow \quad dV = 3x^2\,dx$$

$$dV\,\Big|_{x=2} = (3)(2^2)(0.1) = \boxed{1.2 \text{ cm}^3}$$

5. (15 pts) Verify that the hypotheses of the Mean Value Theorem are satisfied for $g(x) = x + \frac{1}{x}$ on the interval

6.

ii. Find the equation of every vertical asymptote of $y = r(x)$, if any exist. Support your answer by evaluating the appropriate limits.

Solution:

Based on the factorization in part (i), $r(\quad 3)$ is undefined due to division by zero, and the numerator is nonzero. This indicates the possibility of a vertical asymptote at $x = \quad 3$.

$$\lim_{x\,!\quad 3}$$