

INSTRUCTIONS: **Simplify** and **box** all your answers. Write neatly and **justify all answers**. A correct answer with incorrect work or no justification may receive no credit. Books, notes, electronic devices, other unauthorized devices, and help from another person are not permitted while taking the exam. The exam is worth 100 points.

Name: _____

(c) Determine the end behavior of $R(x)$.

Solution:

We notice that the Numerator and the Denominator of $R(x)$ both have degree 2. Hence as $x \rightarrow \infty$ the function approaches the ratio of the leading coefficients 2 and thus has a Horizontal Asymptote given by

$$y = 2$$

(d) Find all vertical asymptote(s) of $R(x)$. If there are none write NONE.

Solution:

Looking at the reduced function $R(x) = \frac{2(x+2)}{(x+1)}$, there is Vertical Asymptote when the denominator is zero. It is given by the vertical line

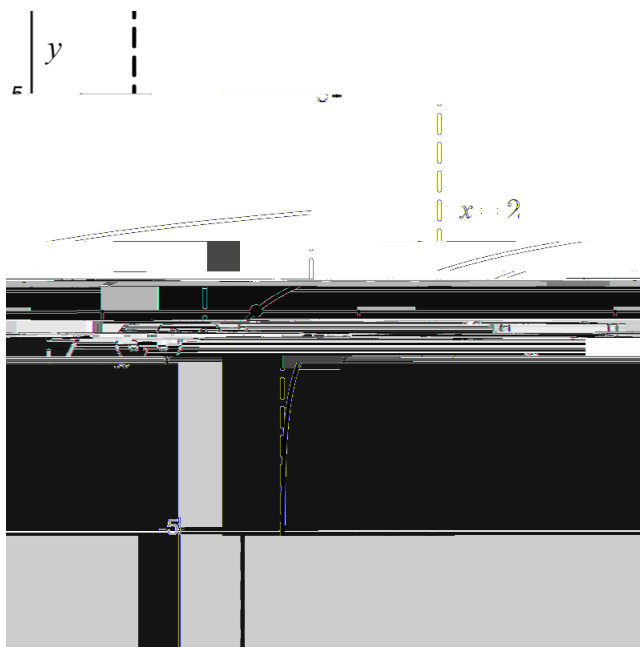
$$x = -1$$

3. The following parts are unrelated:

(a) Graph the following function. Please label any asymptotes and x , y -intercepts on your axes. (4 pts)

$$f(x) = \ln(x - 2)$$

Solution:



(b) Find the function of the form $y = b^x$ whose graphx

(c) Express in terms of sums and differences of logarithms without exponents: $\ln \frac{e^2}{z^4 z^{\frac{9}{2}}}$

Solution:

$$\ln \frac{e^2}{z^4 z^{\frac{9}{2}}} = \ln \frac{e^2}{z^4 z^{\frac{9}{2}}} \quad (12)$$

$$= \ln \frac{e^2}{z^{\frac{9}{2}}} \quad (13)$$

$$= \ln(e^2) - \ln(z^{\frac{9}{2}}) \quad (14)$$

$$= 2 \ln(e) - \frac{9}{2} \ln(z) \quad (15)$$

$$= \boxed{2 - \frac{9}{2} \ln(z)} \quad (16)$$

6. Solve the following equations. (10 pts)

(a) $2 = \log(3x + 1)$

Solution:

Converting to exponential form, we obtain

$$3x + 1 = 10^2 \quad (17)$$

$$\quad) \quad 3x = 10^2 - 1 \quad (18)$$

$$\quad) \quad 3x = 99 \quad (19)$$

$$\quad) \quad x = \boxed{33} \quad (20)$$

(b) $5^{3x+2} = 5^{x-1}$

Solution: Since exponential are one-to-one functions, we can write

$$5^{3x+2} = 5^{x-1} \quad (21)$$

$$\quad) \quad 3x + 2 = x - 1 \quad (22)$$

$$\quad) \quad 2x = -3 \quad (23)$$

$$\quad) \quad x = \boxed{-\frac{3}{2}} \quad (24)$$

7. Solve the following equations. (10 pts)

(a) $\log(x) + \log(x - 4) = \log(3x)$

Solution:

$$\log(x) + \log(x - 4) = \log(3x) \quad (25)$$

$$\log(x(x - 4)) = \log(3x) \quad (26)$$

$$x(x - 4) = 3x \quad (27)$$

$$x^2 - 4x - 3x = 0 \quad (28)$$

$$x(x - 7) = 0 \quad (29)$$

$$x = 0; 7 \quad (30)$$

However, $x = 0$ is not in the domain of $\log(x)$ and hence is an extraneous solution. Also, plugging in $x = 7$ to both sides of the equation, we find that both sides match. Hence the valid solution is

$$\boxed{x = 7}$$

(b) $2^{x+1} = 3^x$

Solution:

$$2^{x+1} = 3^x \tag{31}$$

$$\log_2(2^{x+1}) = \log_2(3^x) \tag{32}$$

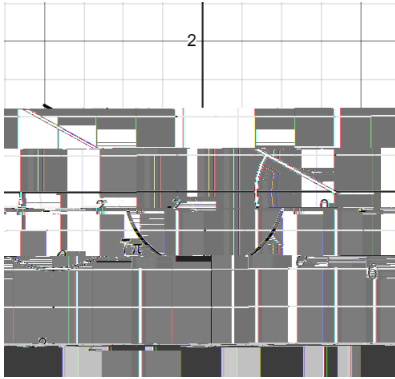
$$x + 1 = x \log_2(3) \tag{33}$$

$$x(1 - \log_2(3)) = -1 \tag{34}$$

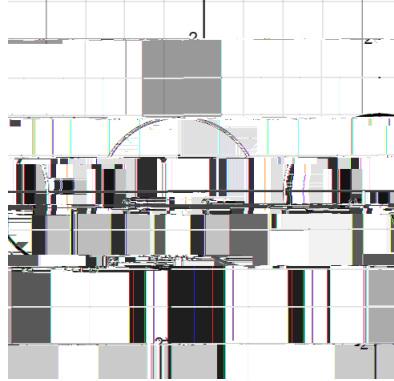
$$x = \frac{1}{\log_2(3) - 1} \tag{35}$$

9. Sketch each angle, θ , in standard position on the x,y -axes. Give a separate graph for each.

(a) $\theta = \frac{7\pi}{6}$ (3 pts)



(b) $\theta = 270^\circ$ (3 pts)



10. Evaluate the following: (15 pts)

Solution:

The following can be obtained from a Unit Circle

(a) $\sin(0)$

$$\sin(0) = 0$$

(b) $\tan(120^\circ)$

$$\tan(120^\circ) = -\sqrt{3}$$

(c) $\sin \frac{5\pi}{4}$

$$\sin \frac{5\pi}{4} = -\frac{\sqrt{2}}{2}$$

(d) $\cos \frac{2\pi}{3}$

$$\cos \frac{2\pi}{3} = -\frac{1}{2}$$

(e) $\csc \frac{\pi}{3}$

$$\csc \frac{\pi}{3} = \frac{2}{\sqrt{3}}$$

11. A squirrel clings to the trunk of a tree, and she sees a peanut on the flat ground some distance away.

13. Knowing that $\cos^2 \theta + \sin^2 \theta = 1$ and $\tan^2 \theta + 1 = \sec^2 \theta$ write $\tan \theta$ in terms of $\sin \theta$ if θ is in Quadrant I (4 pts)

Solution:

$$\tan^2 \theta + 1 = \sec^2 \theta \quad (49)$$

$$\tan^2 \theta + 1 = \frac{1}{\cos^2 \theta} \quad (50)$$

$$\tan^2 \theta + 1 = \frac{1}{1 - \sin^2 \theta} \quad (51)$$

$$\tan^2 \theta = \frac{1}{1 - \sin^2 \theta} - 1 \quad (52)$$

$$\tan^2 \theta = \frac{\sin^2 \theta}{1 - \sin^2 \theta} \quad (53)$$

$$\tan \theta = \pm \frac{\sin \theta}{\sqrt{1 - \sin^2 \theta}} \quad (54)$$

In Quadrant I, $\tan \theta > 0$. Hence we take the positive square root and obtain

$\tan \theta = \frac{\sin \theta}{\sqrt{1 - \sin^2 \theta}}$
--