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F ④ f f rG f f , f
A f , f 4 f ④ ④ ④ f G f f f ④ f f f ④ ④
G ④ f ④ f ④ f ④ f

$$\mathbf{A}^{-1} \mathbf{N}^{-1} \mathbf{W}^{-1} \mathbf{A}^{-1} \mathbf{f} = \mathbf{A}^{-1} \mathbf{U} \mathbf{F} \mathbf{F}^T \mathbf{A}^{-1} \mathbf{f} \quad (8)$$

$$\delta \mathbf{f} = \mathbf{A}^{-1} \mathbf{U} \mathbf{F} \mathbf{F}^T \mathbf{A}^{-1} \mathbf{f} \quad (1)$$

$$\mathbf{F} = \mathbf{f} \quad (3)$$

$$\mathbf{U} = \mathbf{F} \quad (3)$$

$$G_{\ell} = \frac{\pi}{2} \sum_{\ell'} \frac{\pi^2 \omega^2}{2\pi \omega_{\ell'}} \dots \omega \quad (3)$$

$$\mathbf{C} = \mathbf{G} \quad (8)$$

$$\mathbf{f} = \mathbf{G} \mathbf{f} \quad (8)$$

$$\xi < \frac{\ell}{2\pi} \frac{\epsilon}{\pi} \quad (9)$$

$$\mathbf{W} = \mathbf{f} \quad (4)$$

$$\mathbf{U} = \mathbf{A} \quad (3)$$

$$g_n = \mathbf{A} \quad (26)$$

$$G_{\ell} \approx \tilde{G}_{\ell} \quad (12)$$

$$\mathbf{C} = \mathbf{G} \quad (12)$$

$\xi < \frac{1}{4}$ G A f

$$\frac{\pi}{\omega} \frac{\omega^2}{vN^2 \lambda} \omega$$

$$\frac{vN}{\kappa} \lambda \pi \times \frac{\lambda^2 v^2 N^2 \pi \omega^2 \lambda^2 \pi^2 \lambda v^2 N^2 \omega^2}{\kappa}$$

$$\times 2 \frac{\pi^2 \lambda^2 v^4 N^2 \pi \lambda^2 v^3 N^2}{\kappa}$$

$$\kappa \pi^2 \lambda v^2 N^2 \pi^2 \lambda^2 v^4 N^2 \quad 14$$

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$$\tilde{G}_f \frac{vN}{\kappa} \lambda \pi \frac{\pi^2 \lambda v^2 N^2 \omega^2 \pi^2 \lambda^2 v^4 N^2}{\kappa} \omega^2$$

$$\times \tilde{f} \frac{\lambda^2 v^2 N^2 \pi \omega^2}{\kappa} \frac{2 \pi \lambda^2 v^3 N^2}{\kappa}$$

1

$$\tilde{f} \tilde{f} \frac{\lambda \pi^2 \omega^2}{\kappa}$$

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 \textcircled{f}

4 199 84 6 1 9626 9 9626 81 2283491 1 6 1 4 4 (-) 2 31 (433) -3331 (138 064434 3 -E

$M = \{ (f, f) \in W \times W \mid \|f\|_{L^2} \leq \epsilon \}$

$$\|f\|_{L^2} \leq \frac{\epsilon}{\sqrt{2}}$$

$\Gamma = \{ (f, f) \in M \mid \|f\|_{L^2} \leq \frac{\epsilon}{\sqrt{2}} \}$

W f @ f f f @ f FF f @ f
 W f @ f f @ f f f
 N = 12 M f FF f , @ f
 f N = 128 FF f N = 8 92 @ f FF f
 F f 1 @ 3, -
 ' 1, a < 138 f , f 1 -
 f

5. Conclusions

W -8 8 8 W 8 8-4 4 (W)-239 (8) 3 f ,

$$\begin{aligned}
 & \int_{\mathbb{R}^d} \left(\int_{\mathbb{R}^d} f(x) \delta(x - \xi) dx \right) d\xi = \int_{\mathbb{R}^d} f(x) dx, \\
 & \int_{\mathbb{R}^d} \left(\int_{\mathbb{R}^d} f(x) \delta(x - \xi) dx \right) \delta(\xi - \eta) d\xi = \int_{\mathbb{R}^d} f(x) \delta(x - \eta) dx, \\
 & \int_{\mathbb{R}^d} \left(\int_{\mathbb{R}^d} f(x) \delta(x - \xi) dx \right) \delta(\xi - \eta) d\xi = \int_{\mathbb{R}^d} f(x) \delta(x - \eta) dx, \\
 & \int_{\mathbb{R}^d} \left(\int_{\mathbb{R}^d} f(x) \delta(x - \xi) dx \right) \delta(\xi - \eta) d\xi = \int_{\mathbb{R}^d} f(x) \delta(x - \eta) dx,
 \end{aligned}$$

A.4

A f_{ξ}^{ν} , f_{ξ}^{ν} (A 9)

$$\mathcal{P}^{\nu} = \sum_{-\infty}^{\infty} 2\pi i \hat{\nu} N \xi \hat{\gamma}_{\lambda} \xi, \quad \text{(A 10)}$$

$$\mathcal{P}^{\nu} = \sum_{-\infty}^{\infty} 2\pi i \xi \hat{\nu} N \xi, \hat{\gamma}_{\lambda} \xi, \xi. \quad \text{(A 11)}$$

\mathbb{W}_{ξ}^{ν} (A 10), f_{ξ}^{ν} f_{λ} , $1/\nu$

$$F_{\xi}^{\nu} = \sum_{-\infty}^{\infty} \mathcal{P}^{\nu} 2\pi i \xi, \quad \text{(A 12)}$$

$$F_{\xi}^{\nu} = \sum_{-\infty}^{\infty} \hat{\nu} N \xi, \hat{\gamma}_{\lambda} \xi, \xi. \quad \text{(A 13)}$$

$$\mathbb{W}_{\xi}^{\nu} \quad \text{(A 13)} \quad \xi$$

Then

$$E_\infty \leq 1 - \hat{\phi}(\alpha) \frac{1}{C^{\alpha, \alpha}}, \quad C^{\alpha, \alpha} \in \mathbb{Z}^+, \quad \alpha \in \mathbb{Z}^+.$$

$$\hat{\phi}(\alpha) = \sum_{l=1}^{\infty} \frac{f_l(\alpha)}{l}, \quad f_l(\alpha) = \sum_{\substack{\xi \in \mathbb{Z}^+ \\ \xi \leq \frac{\alpha}{l}}} \hat{\phi}(\xi), \quad 1 \leq \alpha < \infty.$$

$$F_{\lambda}(\alpha) = \sum_{\substack{\xi \in \mathbb{Z}^+ \\ \xi < \frac{\alpha}{\lambda}}} \hat{\phi}(\xi), \quad \lambda < 1, \quad \alpha \in \mathbb{Z}^+, \quad \alpha \leq \frac{1}{2\lambda}.$$

$\lambda < 1, 4 \leq \lambda < 14$ $F_{\lambda}(\alpha) = \sum_{\substack{\xi \in \mathbb{Z}^+ \\ \xi < \frac{\alpha}{\lambda}}} \hat{\phi}(\xi)$ $\alpha \leq \frac{1}{2\lambda}$

$\hat{\phi}(\xi) \pm 1$
 f_{λ}

16,16
 23 f

A 9)

$\lambda < 1, 4 \leq \lambda < 14$
 $F_{\lambda}(\alpha) = \sum_{\substack{\xi \in \mathbb{Z}^+ \\ \xi < \frac{\alpha}{\lambda}}} \hat{\phi}(\xi)$

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,

(2) C

$$F(\xi) = \int_{-\frac{vN}{2}}^{\frac{vN}{2}-1} \mathcal{P}^v e^{-2\pi i \xi t} dt$$

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(3) $\rightarrow F(\xi_j) = \int_{-\frac{vN}{2}}^{\frac{vN}{2}-1} \mathcal{P}^v e^{-2\pi i \xi_j t} dt$

A.2.2. Fast evaluation of the Fourier series at unequally spaced points

$$L = \int_{-\frac{vN}{2}}^{\frac{vN}{2}-1} \mathcal{P}^v e^{-2\pi i \xi t} dt, \quad \xi = \frac{f}{T}, \quad f = \frac{v}{T} \xi$$

$$\xi = \frac{F \xi/v}{\lambda \xi/v}$$

$$F(\xi) = \int_{-\frac{vN}{2}}^{\frac{vN}{2}-1} \mathcal{P}^v e^{-2\pi i \xi t} dt = a_\lambda(\xi) \quad \text{A 12) } \quad \text{A 4) C}$$

$$\hat{f}(\xi) = \int_{-\infty}^{\infty} \dots \delta(\xi - \xi_j) d\xi$$

$$f(\xi) = \sum_{\nu=-\infty}^{\infty} \hat{G} \frac{\xi}{\nu} \tilde{\mathcal{P}}^{\nu}(\xi) \quad \text{A .20}$$

$$\hat{G} \frac{\xi}{\nu} = \frac{1}{\nu} \sum_{\lambda=-\frac{N}{2}}^{\frac{N}{2}-1} f(\lambda) e^{-i \frac{2\pi \xi \lambda}{\nu}} \quad \text{A .21}$$

Using (19) and (20),

$$f(\xi) = \sum_{\nu=-\infty}^{\infty} \sum_{\lambda \in \mathbb{Z}} \hat{G} \frac{\xi}{\nu} \gamma_{\lambda} \nu N \tilde{f}(\lambda) e^{-i \frac{2\pi \xi \lambda}{\nu}} = N \sum_{\lambda \in \mathbb{Z}} \hat{G} \frac{\xi}{\nu} \gamma_{\lambda} \nu \tilde{f}(\lambda) \quad \text{A .22}$$

$$\tilde{f}(\lambda) = \sum_{\nu \in \mathbb{Z}} \hat{G} \frac{\xi}{\nu} \gamma_{\lambda} \nu \xi \quad \text{A .23}$$

For $\lambda \in \mathbb{Z}$, $\tilde{f}(\lambda) = \sum_{\nu \in \mathbb{Z}} \hat{G} \frac{\xi}{\nu} \gamma_{\lambda} \nu \xi$ is a function of ξ and λ . For $\lambda \in \mathbb{Z}$, $\tilde{f}(\lambda) = \sum_{\nu \in \mathbb{Z}} \hat{G} \frac{\xi}{\nu} \gamma_{\lambda} \nu \xi$ is a function of ξ and λ .

$$\tilde{f}(\lambda) = \sum_{\nu \in \mathbb{Z}} \frac{1}{\nu} \gamma_{\lambda} \nu \xi, \quad \frac{N}{2} \leq \lambda \leq \frac{N}{2} - 1; \quad \text{A .24}$$

Algorithm 2. For $\lambda \in \mathbb{Z}$, $\tilde{f}(\lambda) = \sum_{\nu \in \mathbb{Z}} \hat{G} \frac{\xi}{\nu} \gamma_{\lambda} \nu \xi$ is a function of ξ and λ .

Algorithm 2.

- (1) Compute $\tilde{f}(\lambda)$ for $\lambda \in \mathbb{Z}$ using (24)
- (2) Compute FFT of $\tilde{f}(\lambda)$ to get $\hat{G} \frac{\xi}{\nu}$
- (3) Compute $f(\xi)$ using (23)

A.2.3. Evaluation of unequally spaced FFT at unequally spaced points

For $\lambda \in \mathbb{Z}$, $\tilde{f}(\lambda) = \sum_{\nu \in \mathbb{Z}} \hat{G} \frac{\xi}{\nu} \gamma_{\lambda} \nu \xi$ is a function of ξ and λ .

$$\tilde{f}(\lambda) = \sum_{\nu \in \mathbb{Z}} \hat{G} \frac{\xi}{\nu} \gamma_{\lambda} \nu \xi, \quad \lambda \in \mathbb{Z}, \quad \xi \in \left\{ \frac{N}{2}, \frac{N}{2} - 1, \dots, \frac{N}{2} - \lambda \right\} \quad \text{A .14}$$

Algorithm 3.

- (1) Compute $\tilde{f}(\lambda)$ for $\lambda \in \mathbb{Z}$ using (14)
 - (2) Compute FFT of $\tilde{f}(\lambda)$ to get $\hat{G} \frac{\xi}{\nu}$ for $\nu \in \mathbb{Z}$
- $$\hat{G} \frac{\xi}{\nu} = \sum_{\lambda \in \mathbb{Z}} \tilde{f}(\lambda) e^{-i \frac{2\pi \xi \lambda}{\nu}}, \quad \nu \in \mathbb{Z}, \quad \xi \in \left\{ \frac{v^2 N}{2}, \dots, \frac{v^2 N}{2} - 1 \right\} \quad \text{A .2}$$

